Scattering of water waves by thin floating plates

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Abstract

This work centres on problems involving the interaction of water waves with thin rigid or flexible plates which can either be fixed or freely-floating on the surface of the fluid. A Fourier transform method is key to developing integral equations can subsequently be efficiently solved numerically using a Galerkin approach. A large class of problems can be considered using this approach, including scattering by rectangular and rhomboidal-shaped plates and eigenvalue problems for sloshing modes in ice holes.

Keywords: Water waves, floating plates, Integral equations

1 Introduction

The reflection and transmission of surface gravity waves by a rigid plate or 'dock' on the surface of a fluid is a classical problem in the study of linearised water waves. For example, when the plate covers the half-plane – the so-called semi-infinite dock problem – an explicit expression for the reflection coefficient can be found using the Wiener-Hopf technique (see [1]). For a plate that is infinitely-long and of uniform constant width – the so-called 'finite dock problem' – exact solutions are no longer possible and various techniques have been employed all leading to approximations of the reflection and transmission. See for example, [2], [3].

In §2 we outline how a Fourier transform method may be applied to this two-dimensional scattering problem that results efficient and accurate numerical results. We do not claim that this approach is superior to existing methods, but it does allow a number of extensions to be considered and some of these are outlined in §3 and §4. Further extensions will be presented in the talk.

2 A two-dimensional scattering problem and its solution

To illustrate the main features of the approach, we consider scattering of obliquely-incident plane waves by a rigid thin plate fixed in the free surface of water of infinite depth.

Cartesian coordinates are used with $z = 0$ in the mean free surface and the fluid extending into $z < 0$. A rigid horizontal plate is placed on the surface, $z = 0$, and extends uniformly in the y-direction with $-a < x < a$.

Assuming time-harmonic incident waves of angular frequency ω making an angle θ_0 with respect to the positive x direction, the velocity field components are found from the gradient of $\Re\{\phi(x, z)\mathrm{e}^{\mathrm{i}(\beta_0 y-\omega t)}\}$ where the velocity potential $\phi(x, z)$ satisfies

$$(\nabla^2 - \beta_0^2)\phi(x, z) = 0, \qquad z < 0, \tag{1}$$

with $\beta_0 = K \sin\theta_0$, $K = \omega^2/g$ and

$$\phi_z(x, 0) - K\phi(x, 0) = 0, \tag{2}$$

on the free surface and $|\nabla\phi| \to 0$ as $z \to -\infty$. The zero-velocity condition to be applied on the plate is

$$\phi_z(x, 0) = 0, \qquad |x| < a. \tag{3}$$

and at the ends of the plate (as x approaches $\pm a$) the potential should be bounded. Finally radiation conditions are required and we write

$$\phi(x, z) \sim \begin{cases} \phi_i(x, z) + R\phi_i(-x, z), & x \to -\infty \\ T\phi_i(x, z), & x \to \infty \end{cases} \tag{4}$$

where R and T are the complex reflection and transmission coefficients and $\phi_i(x, z) = \mathrm{e}^{\mathrm{i}\alpha_0 x}\mathrm{e}^{Kz}$ with $\alpha_0 = K \cos\theta_0$.

The Fourier transform of the scattered part of the potential is defined by

$$\overline{\phi}(\alpha, z) = \int_{-\infty}^{\infty} (\phi(x, z) - \phi_i(x, z))\mathrm{e}^{-\mathrm{i}\alpha x}\, dx \tag{5}$$

and the contour of integration in the inverse transform will be defined in order to satisfy the radiation condition.

The application of the Fourier transform yields the following integral equation for the unknown function $\phi(x, 0)$

$$\phi(x, 0) + (\mathcal{K}_0\phi)(x) = \mathrm{e}^{\mathrm{i}\alpha_0 x} \tag{6}$$

for $|x| < a$ where

$$\mathcal{K}_0\phi = \frac{K}{2\pi}\int_{-\infty}^{\infty}\frac{e^{i\alpha x}}{k_0 - K}\int_{-a}^{a}\phi(x',0)e^{-i\alpha x'}\,dx'\,d\alpha.$$

and $k_0^2 = \alpha^2 + \beta_0^2$. R and T can be expressed in terms of simple integral relations of ϕ.

The unknown in (6) is expanded in terms of a set of prescribed functions,

$$\phi(x,0) = \tfrac{1}{2}\sum_{n=0}^{\infty} i^n a_n P_n(x/a), \quad |x| \le a \qquad (7)$$

with unknown complex-valued coefficients a_n where P_n are orthogonal Legendre polynomials.

The expansion (7) is substituted into (6) which is multiplied through by $p_m^*(x/a)$ and integrated over $-a < x < a$. This Galerkin procedure results in the following infinite system of equations for the unknown coefficients a_n:

$$\frac{a_m}{2(2m+1)} + \sum_{n=0}^{\infty} a_n K_{m,n} = j_m(\alpha_0 a), \qquad (8)$$

$m = 0, 1, 2, \ldots$ where

$$K_{m,n} = \frac{Ka}{2\pi}\int_{-\infty}^{\infty}\frac{j_n(\alpha a)j_m(\alpha a)}{k_0 - K}\,d\alpha. \qquad (9)$$

and $j_m(x)$ is the spherical Bessel function. The integral passes under the pole at $k_0 = K$.

Numerical results show that a truncation to just 1 term works well over a large range of Ka and accuracy increases rapidly with increasing truncation size.

3 Extensions to three-dimensional scattering by finite docks

The main focus of the talk will be on using extensions of this method for 3-dimensional scattering problems. The figures illustrate examples of the results one can obtain. We show maximum surface amplitudes for monochromatic plane incident wave (left to right) scattering by rectangular rigid plates and rhomboidal plates.

4 Eigenvalue problems

A second extension of the approach is in solving geometrically complementary problems where the surface is covered by a rigid plate apart from a, say, rectangular section in which the fluid forms a free surface. Mathematically we have an eigenvalue problem in which the sloshing modes and their frequencies can be determined from a homogenous system of equations.

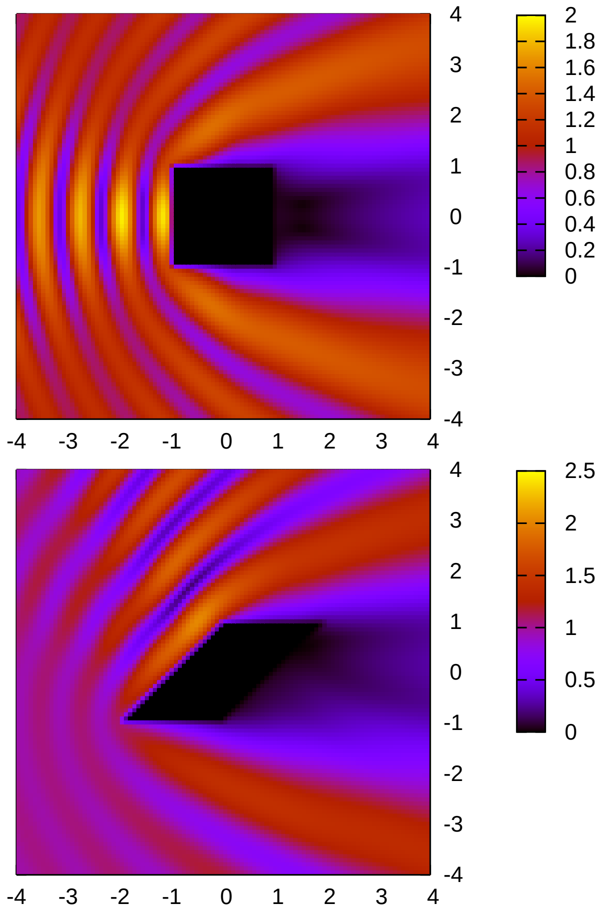

Figure 1: Wave amplitudes for left-right incident wave scattering by rigid plates in the surface.

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