Proc. 18th Int. Conf. on Low Temperature Physics, Kyoto, 1987
Japanese Journal of Applied Physics, Vol. 26 (1987) Supplement 26-3

Nonuniversal Critical Behavior along the λ-line of ^{4}He

R. SCHLOMS, J. EGGERS, V. DOHM

Institut für Theoretische Physik, Technische Hochschule Aachen, Germany

An accurate representation of the renormalization-group (RG) β-function of the Ginzburg-Landau Hamiltonian is given. The nonuniversal parameters of the theory are determined from a few data points of the specific heat above T_λ. Application to the specific heat below T_λ and to the Landau-Placzek ratio of scattered light explains the measured nonuniversal temperature and pressure dependence of these quantities without adjustable parameters.

1. Introduction and strategy

Recent interest in the critical properties of ^{4}He has been focused on nonasymptotic features rather than the pure powerlaw behaviour close to the λ-line. Starting point of our field-theoretic approach is the Ginzburg-Landau Hamiltonian

$$H = \int d^dx \left[\frac{1}{2} r_o\phi_o^2 + \frac{1}{2} (\nabla\phi_o)^2 + u_o\phi_o^4 \right] \quad (1)$$

or the extended Hamiltonian of model C [1]. The traditional calculations of correction-to-scaling amplitudes by means of the RG ε-expansion [2] has led to discrepancies with experiments [3]. These discrepancies are resolved by our nonasymptotic approach. Further applications are the explanation of the observed nonuniversal critical behaviour of the superfluid density [4] and of the Landau-Placzek ratio.

Within RG theory physical quantities can be expressed by noncritical background functions and RG transformations which in turn are determined by RG functions. The basic idea is to calculate the dominant nonlinear RG functions with high accuracy by means of Borel resummation methods [4], whereas the background functions are approximated with sufficient accuracy by a low-order loop expansion [1,4]. We find that the most important RG function β_u (for n=2, d=3) can be represented as

$$\beta_u(u) = -u + 40\, u(1+12.82u)\,(1+31.94u)^{-1} \quad (2)$$

in the range $0 \leq u \leq u^* = 0.037$, within the error bars of our calculation. For other RG functions see [4].

2. Application to thermodynamics

The main results of our procedure are the following

(i) The ratio of the correction-to-scaling amplitudes of the specific heat below T_λ and the superfluid density is $D^-/D_\rho = -0.06$ (rather than + 0.667 obtained by the two-loop ε-expansion [2]) in good agreement with the experimental data [3]. Thus an old discrepancy is resolved.

(ii) Using the specific heat data [5] above T_λ in a small temperature intervall $10^{-4} \leq t = (T-T_\lambda)/T_\lambda \leq 10^{-3.5}$ is sufficient to determine the pressure dependence of the renormalized parameters of (1) and of the model C Hamiltonian. Their values at $t = 10^{-2}$ are listed in the following table (last row in J/mol K).

P[bar]	SVP	6.85	14.73	22.3	28
u	0.036	0.035	0.033	0.030	0.028
γ^2	0.096	0.103	0.112	0.109	0.128
$k_B\chi_o Z_m V_\lambda$	20.9	17.7	15.1	14.0	12.1

(iii) With these nonuniversal parameters all other thermodynamic quantities can be predicted without further adjustments as function of the pressure in the entire temperature range including the precritical region where the description in terms of correction-to-scaling terms is insufficient. This has been carried out for the specific heat and the superfluid density [4]. The agreement between our predictions and the experimental data is shown in Fig. 1. We find the range of validity of model (1) to be $-10^{-3} \lesssim t \lesssim 10^{-2}$ for all pressures.

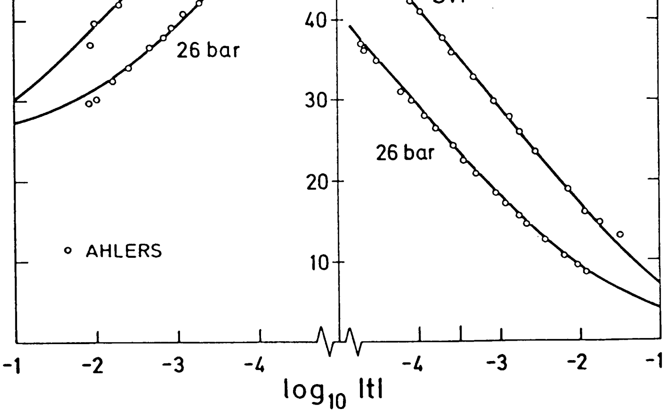

Fig. 1. Specific heat at two pressures (representative data) [6]. Solid curves represent the theoretical result where adjustments have been made only above T_λ in $10^{-4} \leq t \leq 10^{-3.5}$.

3. Application to light scattering

This application is readily made possible by generalizing the thermodynamic results to finite wave numbers k by means of an appropriately defined RG flow parameter $\ell(t,k)$ and by a calculation of k dependent correlation functions. Of particular importance is the Landau-Placzek ratio

$$\frac{I_2(k)}{I_1(k)} = \frac{C_p(k)}{C_v(k)} - 1 \qquad (3)$$

where the k dependent specific heats C_p and C_v can be calculated within model (1) or model C. With the nonuniversal parameters taken from the Table presented above our RG calculations lead to predictions for I_2/I_1 without adjustable parameters shown as full curves in Fig. 2.

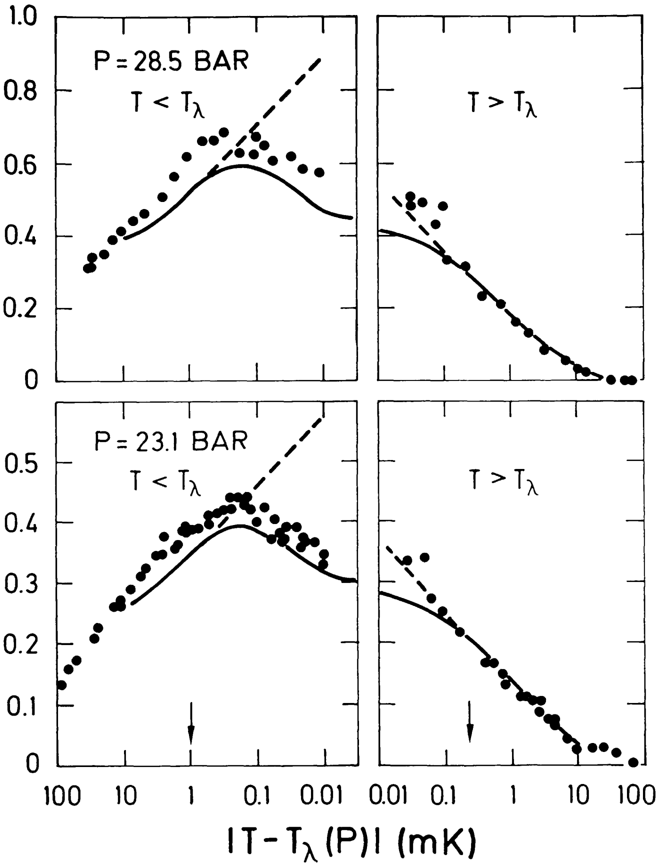

Fig. 2. Temperature dependence of the Landau-Placzek ratio for $k = 1.79 \cdot 10^5\ cm^{-1}$. The solid curves are the prediction of our theory, the dashed lines are k = 0 extrapolations. The data are from Ref. 7.

Good overall agreement with the data [7,8] is found but systematic differences exist below T_λ. Corresponding purely experimental differences exist between the light-scattering data and thermodynamic specific-heat measurements [6,8] whose extrapolations are shown as dashed lines in Fig. 2. Our finite k theory is based on k = 0 thermodynamics, therefore the differences with the data for $T < T_\lambda$ in Fig. 2 was to be expected. A possible solution to this puzzle is a complete calculation of a dynamically defined ratio I_2/I_1 within an appropriate model including first sound [9].

Our results differ significantly from those of a previous theory [10]. Their results are inconsistent with thermodynamic data [6]; furthermore the data presented in [10] for $T > T_\lambda$ at 23 bar do not agree with the original data [7].

A further application of the theory is the calculation of the frequency dependence of the Rayleigh part of the spectrum. It has turned out that the temperature dependence of the total intensity I_2 is not adequately described within the standard model F. Furthermore there exist significant differences between the calculated and measured halfwidths whose origin should be studied within a more complete model [9]. The details of the results discussed here will be published elsewhere[11].

REFERENCES

(1) V. Dohm: Z. Phys. B 60, 61 (1985).
(2) A. Aharony, G. Ahlers, Phys. Rev. Lett. 44, 782 (1980; M.C. Chang, H. Houghton: Phys. Rev. B 23, 1473 (1981).
(3) A. Singsaas, G. Ahlers: Phys. Rev. B 30, 5103 (1984).
(4) R. Schloms, V. Dohm: Europhys. Lett. 3, 413 (1987) and to be published.
(5) W.Y. Tam, G. Ahlers: Phys. Rev. B 33, 183 (1986).
(6) G. Ahlers: Phys. Rev. A 3, 696 (1971); Phys. Rev. A 8, 530 (1973).
(7) J.A. Tarvin, F. Vidal, T.J. Greytak: Phys. Rev. B 15, 4193 (1977).
(8) W.F. Vinen, D.L. Hurd: Adv. Phys. 27, 533 (1978).
(9) J. Pankert, V. Dohm: Europhys. Lett. 2, 775 (1986).
(10) R.A. Ferrell, J.K. Bhattacharjee in: Recent Advances in Theoretical Physics, edited by R. Ramachandran (World Scientific, 1984) p. 257.
(11) J. Eggers, V. Dohm, to be published.