Sequential measurements, disturbance and property testing

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18 February 2017
I knew who I was this morning but I've changed a few times since then.

❤️♠️♣️♦️
In this talk I will describe an algorithm that solves the following problem.

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- $M$ accepts with probability $\|P\psi\|^2$ and otherwise rejects.

- If $M$ accepts (resp. rejects), the new state of the system is
  
  $$\frac{P\psi}{\|P\psi\|}, \text{ resp. } \frac{(I - P)\psi}{\|(I - P)\psi\|}.$$
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Restating the previous problem mathematically:

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- There exists $i$ such that $\|P_i\psi\|^2 = \Omega(1)$ ("yes" case);
- For all $i$, $\|P_i\psi\|^2 = o\left(\frac{1}{n}\right)$ ("no" case).

Our task is to determine which is the case.

This problem can be seen as a quantum version of computing the OR of the measurement outcomes.

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The quantum anti-Zeno effect

Set

\[ \psi_k = \left( \cos \left( \frac{\pi k}{2n} \right), \sin \left( \frac{\pi k}{2n} \right) \right)^T \]

and set \( M_k = \{ I - \psi_k \psi_k^\dagger, \psi_k^\dagger \psi_k \} \) (first outcome: acceptance, second outcome: rejection).
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- So if we perform \( M_1, \ldots, M_n \) on initial state \( \left( \begin{array}{c} 1 \\ 0 \end{array} \right) = \psi_0 \), then \( \Pr[\text{ever accept}] = O(1/n) \).
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- So if we perform \( M_1, \ldots, M_n \) on initial state \( \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \psi_0 \),
  then \( \Pr[\text{ever accept}] = O(1/n) \).

- But if the final measurement \( M_n \) were performed on \( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \),
  it would accept with certainty.
Combating the quantum anti-Zeno effect

We give two procedures with similar parameters that combat this effect and solve the above problem:

- One procedure is based on Marriott-Watrous gap amplification and has better constants and a more elegant correctness proof.

- The other procedure has more direct intuition and is easier to describe in a talk...
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- Testing measurements in order doesn’t work if the final state is far away from the initial state.
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The intuition behind the second procedure:

- Testing measurements in order doesn’t work if the final state is far away from the initial state.
- So why not just test for this disturbance?
A quantum OR bound by testing disturbance

Algorithm (informal)
Repeat the following $O(n)$ times:

1. With probability $O(1/n)$, do a disturbance test on the current state and return the result.
2. Pick $k$ at random and perform measurement $M_k$. Accept if the measurement accepts. Reject.

The disturbance test accepts whp if the current state is far from the initial state, and rejects whp if it is close to the initial state.

Proof intuition: In a "yes" case, if the current state is close to the initial state, the test in step 2 will accept whp. Otherwise, the test in step 1 will accept whp. So in either case we accept with prob. $\Omega(1/n)$ in each iteration.
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**Theorem**

For any set of permutations $G$, there is a quantum $\epsilon$-tester for $G$-isomorphism which makes $O((\log |G|)/\epsilon)$ queries.
### Consequences

Assume $\epsilon = \Omega(1)$. Then we have the following query complexity bounds:

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\(^1\)[Alon et al. ’13] \(^2\)[Fischer and Matsliah ’08] \(^3\)[Friedl et al. ’09]

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- Note that the quantum algorithms achieving the complexities above are not time-efficient.
Connecting the OR bound to property testing

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So we can apply the quantum algorithm to $k = O(\log |G|)$ copies of $\psi$ and the sequence of measurements $\{M_h\}$. 
Other consequences

We obtain some other consequences too, e.g.:

- Efficient testing of properties of quantum states. If \( P \) is a finite subset of the unit sphere, there is a quantum \( \epsilon \)-tester for membership in \( P \) using \( O\left(\frac{\log |P|}{\epsilon^2}\right) \) copies of the input state.
- Testing genuine multipartite entanglement of a state of \( n \) systems using \( O\left(\frac{n}{\epsilon^2}\right) \) copies of the state.
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Summary and further reading

- Given a quantum state and a sequence of measurements, we can test whether one of them accepts whp.

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Open questions:
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- This has applications to property testing, including **exponential reductions** in quantum query complexity.

Open questions:
- Can we find **time-efficient** quantum algorithms for these property testing problems?

- Are there other applications of the quantum OR bound?