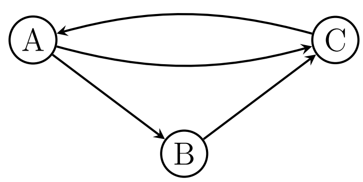
COMS21103: Problems set 9

PageRank, skip lists and Bloom filters

The starred problems below are **optional**, more challenging and hopefully interesting. If any of the problems seems unclear, please post a question on the forum.

1. Either by hand or with a computer, perform 3 iterations of the PageRank algorithm to approximate the PageRank of the vertices in the following graph on 3 vertices (with $p = 0.15$). Optional: use a computer to calculate the PageRank exactly. Does your answer give a reasonable ranking of the vertices?



2. Justify the claim in lecture that one iteration of PageRank computation can be performed on a graph with N vertices and L links in $O(N + L)$ time.

3. Imagine the sequence of integers $5, 4, 11, 15, 9, 10, 3, 12, 6, 13$ are inserted into a skip list. By tossing a fair coin, simulate the operations involved. Draw a diagram of the resulting skip list.

4. Could the time complexities of the various operations discussed in lecture achieved by storing elements in a linked list be improved by storing the elements in sorted order? Why or why not?

5. Using the analysis from lecture, if we want to use a Bloom filter for a Web cache which stores up to 1,000 URLs picked from a universe of 1,000,000 URLs, with a 0.1% probability of false positives, how many bits of storage should we use for the Bloom filter?

6. ($\star$) Imagine we want to extend the Bloom filter to allow deletion of elements. How could this be done? What are the problems with your modification (if any)?

7. ($\star$) The Unequal Quadruples problem is defined as follows. We are given a list of quadruples of variables x_i, possibly negated. We have to output "yes" if there exists an assignment of bits (0 or 1) to the variables such that, for each quadruple, not all of the variables in each quadruple evaluate to the same value, and "no" otherwise. An example instance of Unequal Quadruples is $(x_1, \neg x_2, x_3, \neg x_4), (\neg x_2, x_3, x_3, x_4)$ to which the answer is "yes" (witnessed by $x_1 = x_2 = x_3 = 0$, $x_4 = 1$, for example). Prove that Unequal Quadruples is NP-complete by reducing 3-SAT to Unequal Quadruples.