Fast inverse square-root program

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Why would one need to compute $1/\sqrt{x}$?

The inverse square root is used to normalise vectors. Normalised vectors are needed for 3D graphics programs to determine angles of incidence and reflection.

As a reminder a vector $\mathbf{v} = (v_1, v_2, v_3) \in \mathbb{R}^3$ can be normalised by dividing by it's norm, that is

$$\hat{\mathbf{v}} = \frac{\mathbf{v}}{||\mathbf{v}||} = \frac{\mathbf{v}}{\sqrt{v_1^2 + v_2^2 + v_3^2}}.$$

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3D graphics programs must normalise millions of vectors every second to simulate lighting. In the early 1990's, the code that did this for decimal numbers was computationally expensive, especially when dealing with a large amount of vectors.

Quake 3 Arena

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Now why is this product placement necessary? Well within the source code in the following quite interesting code to compute the inverse square root.

Magic code

```c
float Q_rsqrt( float number )
{
    long i;
    float x2, y;
    const float threehalfs = 1.5F;

    x2 = number * 0.5F;
    y  = number;
    i  = * ( long * ) &y;                       // evil floating point bit level hacking
    i  = 0x5f3759df - ( i >> 1 );               // what the fuck?
    y  = * ( float * ) &i;
    y  = y * ( threehalfs - ( x2 * y * y ) );   // 1st iteration
//  y  = y * ( threehalfs - ( x2 * y * y ) );   // 2nd iteration, this can be removed

    return y;
}
```

Newton's method

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$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

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will converge to x quadratically.
For nice-enough functions, the hardest part is finding the initial guess.

Newton's method for $1/\sqrt{x}$

For our case we wish to calculate $\frac{1}{\sqrt{x}}$. To do this we consider the function $f(y) = \frac{1}{y^2} - x$, whose positive root is exactly $\frac{1}{\sqrt{x}}$.

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$$y_{n+1} = \frac{y_n\left(3 - xy_n^2\right)}{2} = y_n\left(\frac{3}{2} - \frac{x}{2}y_n^2\right).$$

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This is the step occurring in the line:

```
y  = y * ( threehalfs - ( x2 * y * y ) );   // 1st iteration
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Examples in C include **int** for 16-bit integer, **char** for text characters/strings and **float** for 32-bit decimal numbers.

Long data type

The data type for integers appearing in the code is **long**, which stores whole numbers in 32-bits of memory. The first bit is used to store the sign of the number, 0 for $+$ and 1 for $-$ and the other bits are used to number.

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This allows us to store all integers in the range
$[-2^{31}+1, 2^{31}-1] = [-2,147,483,647, +2,147,483,647]$.

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$[-2^{31}+1, 2^{31}-1] = [-2,147,483,647, +2,147,483,647]$.
On computers we actually store negative numbers in **two's complement** form. Since we'll only be dealing with positive numbers we wont need to look at it here.

How to store decimal values?

The first simplest way to thing about doing this is to just also consider negative powers of 2. For example allow the first 16-bits to be positive powers of 2 and the rest be negative powers of 2 to give you the decimal digits.

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Thankfully some clever people at the Institute of Electrical and Electronic Engineers(IEEE) came up with a standard to store these numbers more efficiently.

IEEE 754-1985 Floating point numbers

The main idea is that we already have a way to minimise how we write numbers by using scientific notation. That is we tend to write a number as

$$x = \pm d_0.d_1 d_2 d_3 \ldots \times 10^e$$

where $1 \leq d_0 < 10$.

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In base 2 we would write our number as

$$x = \pm 1.b_1 b_2 b_3 \ldots \times 2^{e_x} = \pm(1 + m_x)2^{e_x}$$

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where e_x is an integer. We call e_x the **exponent** and $1 + m_x$ the **mantissa** or **significand**. On a computer we would only need to store the numbers e_x and m_x and the sign since we will only ever work in base 2.

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The IEEE 754 standard states that if you want to store a number in this way in 32-bits:

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This means we only need to 3 integers that are all positive E_x, M_x and the sign.

Floating point example

Let's look at $x = \pi = 3.1415926535897 93\ldots$. In fixed point arithmetic in base 2 we have $x = 11.001001000011111101 1011\ldots$.

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$x = 11.001001000011111101101 1\ldots$.
In the normalised form, we would get

$$\begin{aligned} x &= +(1 + 0.5707964897155761 7188\ldots) \times 2^1 \\ &= +(1 + 0.10010010000111111011011\ldots) \times 2^1. \end{aligned}$$

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Hence $e_x = 1$, which means $E_x = 1 + B = 128 = 10000000_2$ and $m_x = 0.10010010000111111011011\ldots$, which gives $M_x = m_x \times L = 10010010000111111011011 = 4788187$ after rounding.

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Initialisation step

```
long i;
float x2, y;
const float threehalfs = 1.5F;

x2 = number * 0.5F;
y  = number;
```

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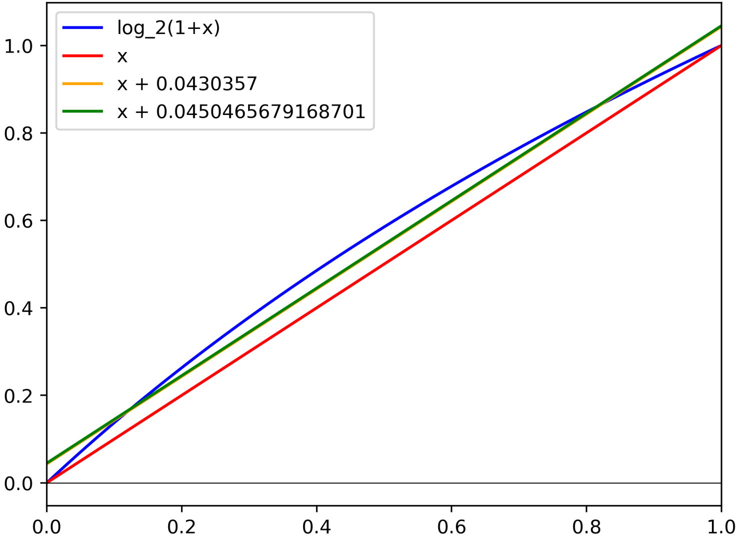
where σ is a free parameter used to tune the approximation.
It turns out that $\sigma \approx 0.0430357\ldots$ gives the best approximation for the uniform error along the interval. For historical purposes we shall let $\sigma = 0.0450465679168701$.

$\log_2(1 + m_x)$ error



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We now do the "evil floating point bit level hack" and interpret the floating point bits of x as a long type. Since the mantissa is already an integer it doesn't change, so all we really do is add on the exponent to the 24-th bit onwards, numerically this means just multiplying it by $L = 2^{23}$.

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Mathematically, this means

$$\begin{aligned} I_x &= E_x L + M_x \\ &= L(e_x + B + m_x) \\ &= L(e_x + m_x + \sigma + B - \sigma) \\ &\approx L \log_2(x) + L(B - \sigma) \end{aligned}$$

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Rearranging, we see that I_x is a linear approximation to $\log_2(x)$

$$\log_2(x) \approx \frac{I_x}{L} - (B - \sigma).$$

WTF

If we let $y = \frac{1}{\sqrt{x}}$, the magic number that I alluded to earlier, actually isn't that magic, it just comes from the identity

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Using our approximation from before we can write this as

$$\frac{I_y}{L} - (B - \sigma) \approx -\frac{1}{2}\left(\frac{I_x}{L} - (B - \sigma)\right)$$

which yields

$$I_y \approx \frac{3}{2}L(B - \sigma) - \frac{1}{2}I_x.$$

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in base 16 or hexadecimal. This is written in the code in the line

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Then the last line

```
y  = * ( float * ) &i;
```

just converts I_y back to y and gives us the initial guess of $y = \frac{1}{\sqrt{x}}$.

How good is this approximation?

Let's consider again $x = 3.14159\ldots$, our memory for y looks like

0	1	0	0	0	0	0	0	0	1	0	0	1	0	0	1	0	0	0	0	1	1	1	1	1	1	0	1	1	0	1	1

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Now interpreting this as an long type and storing this in i, the memory address for i looks like

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Then subtracting this from our "magic number" 0x5f3759df, we get

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Finally reinterpreting this as a float and storing this back in *y* we get

0	0	1	1	1	1	1	1	0	0	0	1	0	0	1	0	1	1	0	1	0	0	0	1	1	1	1	1	0	0	1	0

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Shifting the bits by one place to the right, i.e the line $i >> 1$, i becomes

0	0	1	0	0	0	0	0	0	0	1	0	0	1	0	0	1	0	0	0	0	1	1	1	1	1	1	0	1	1	0	1

Then subtracting this from our "magic number" 0x5f3759df, we get

0	0	1	1	1	1	1	1	0	0	0	1	0	0	1	0	1	1	0	1	0	0	0	1	1	1	1	1	0	0	1	0

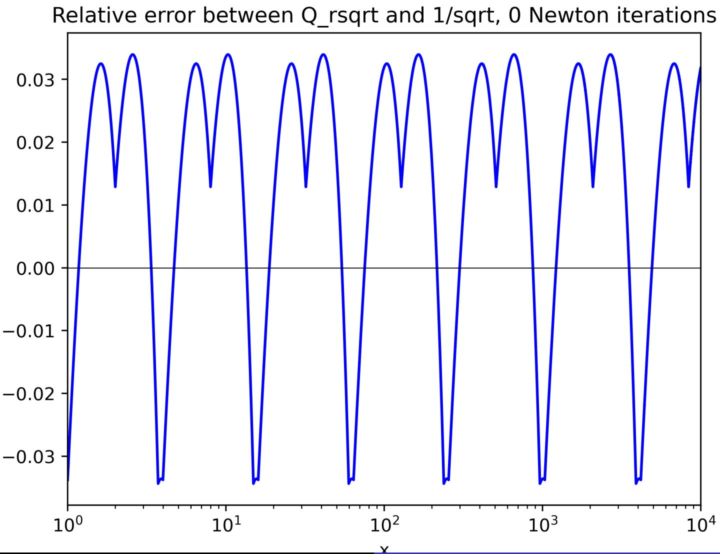
Finally reinterpreting this as a float and storing this back in y we get

0	0	1	1	1	1	1	1	0	0	0	1	0	0	1	0	1	1	0	1	0	0	0	1	1	1	1	1	0	0	1	0

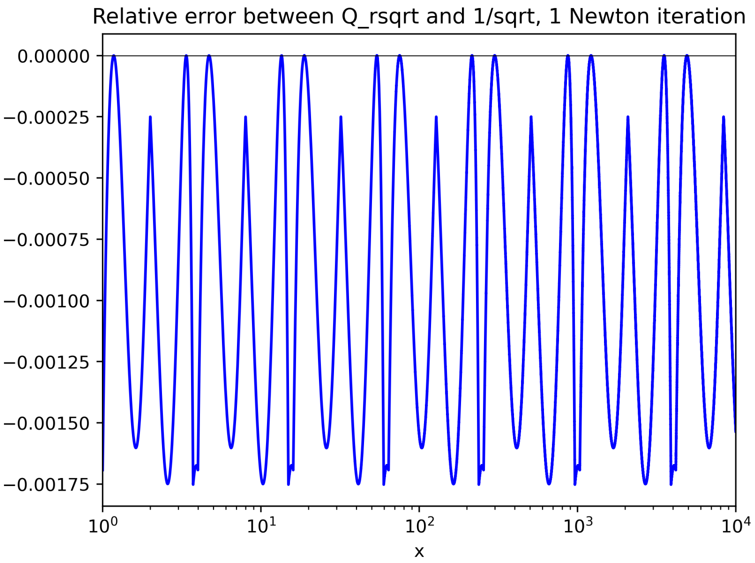
As a decimal number this gives $y = 0.5735160112$. After 1 Newton iteration we get $y = 0.5639570355$. The actual value is $0.564189583547750\ldots$.

Error graph

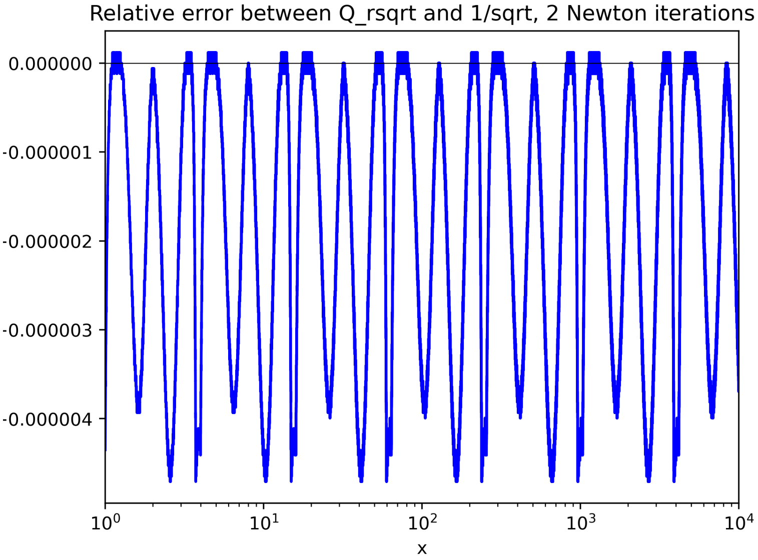
Relative error $= \frac{v_A - v_E}{v_E}$



Better Error graph



Even Better Error graph



The big question, should you use this?

No.

The big question, should you use this?

No. Probably.

Reasons not to use this code

1. The main reason is this is no longer the fastest method. In 1999 the “Streaming SIMD Extensions” (SSE) were added to x86 architecture CPU, effectively allowing certain operations, like square-rooting directly on the CPU without needing to do anything in software. One of the functions include “rsqrtss” which computes the inverse-square root considerably faster and to the full 11 decimal accuracy. Most modern compilers will automatically choose the SSE functions even when you type the software version.

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2. A second slightly less important reason is that this reinterpreting floats and integers using pointers is considered undefined behaviour and some computers probably wont like it. (As of a few months ago C++ defined correct behaviour using C++20's std::bit_cast function).

History

- John Carmack

History

- John Carmack
- Terje Mathisen

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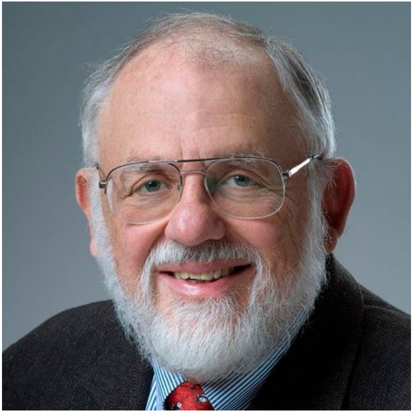
- Greg Walsh

Greg Walsh's inspiration



Cleve Moler

Thanks for listening!

Main references:

- FAST INVERSE SQUARE ROOT - Chris Lomont
- M.Robertson: A Brief History of InvSqrt, Bachelor Thesis, Univ. of New Brunswick 2012.