Exercise B.1. Prove that a measurable $f : X \to X$ is ergodic with respect to the measure m if and only if for every measurable set A such that $m(A) > 0$, we have

$$m \left(\bigcup_{n \geq 0} f^{-n}(A) \right) = 1$$

Exercise B.2 Prove that a measurable $f : X \to X$ is ergodic with respect to the measure m if and only if for every pair of measurable sets A and B, we have

$$\frac{1}{N} \sum_{n=0}^{N-1} m(f^{-n}(A) \cap B) \to m(A)m(B)$$

Conclude that mixing implies ergodicity.

Exercise B.3. Prove that the shift $\sigma : \Sigma_2^+ \to \Sigma_2^+$ is mixing with the measure defined in class.