Dynamical systems

Expanding maps on the circle. Coding

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ICTP

2018

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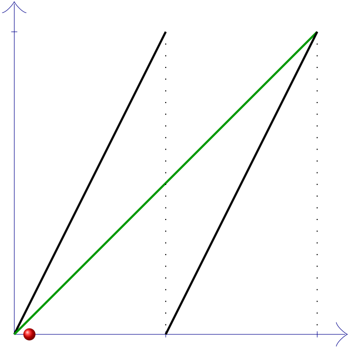
coding

Consider $f : \mathbb{S}^1 \to \mathbb{S}^1$ such that $f(x) = 2x \mod 1$



$\underline{x} =$

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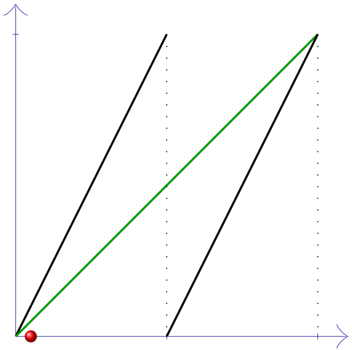

$\underline{x} = 0$

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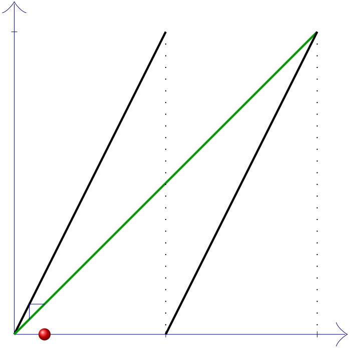

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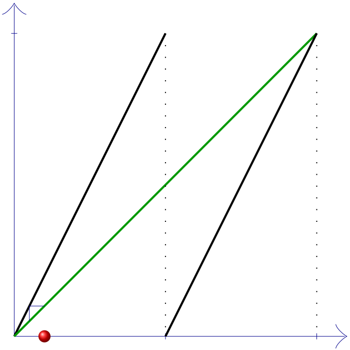

$\underline{x} = 00$

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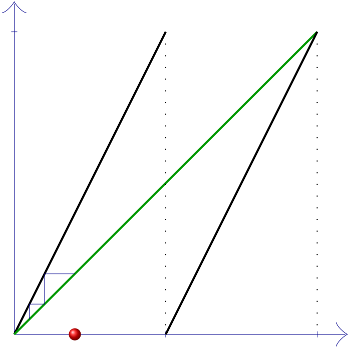
Consider $f : \mathbb{S}^1 \to \mathbb{S}^1$ such that $f(x) = 2x \mod 1$



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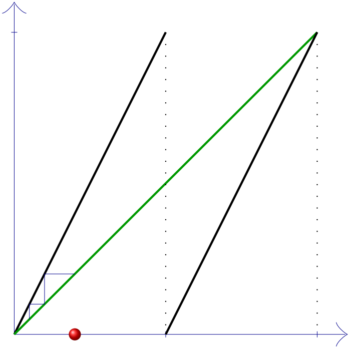

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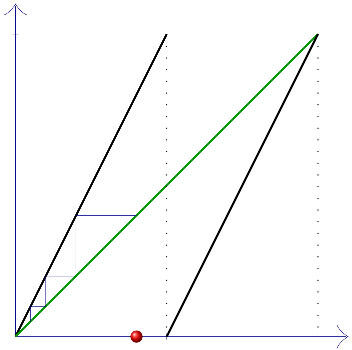

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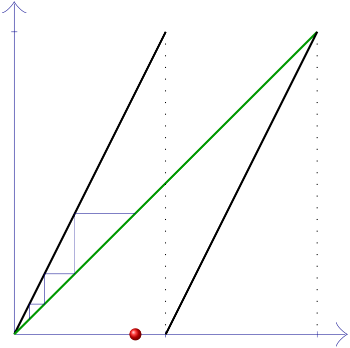

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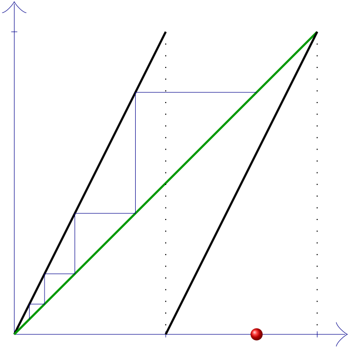

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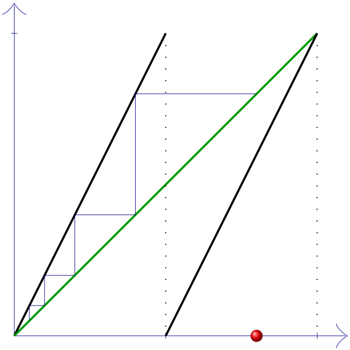

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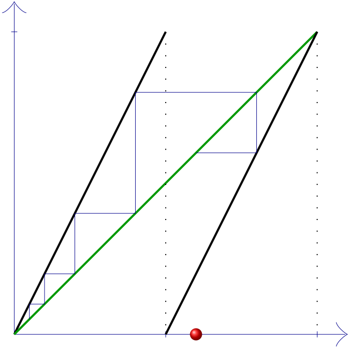

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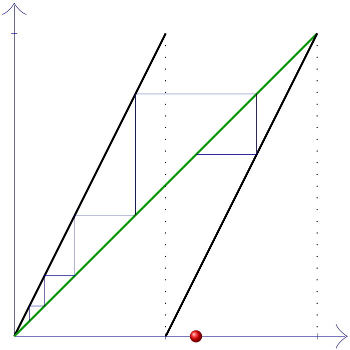

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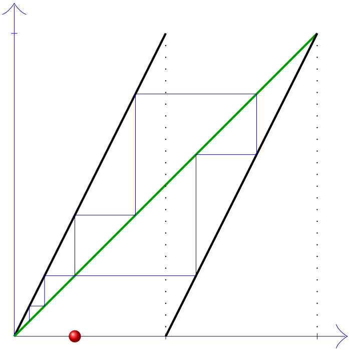

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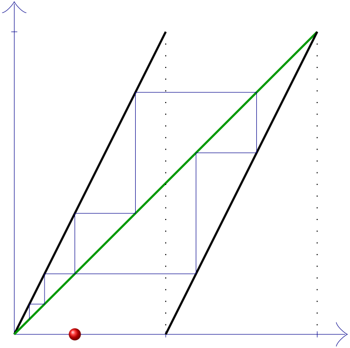

$\underline{x} = 0000110$

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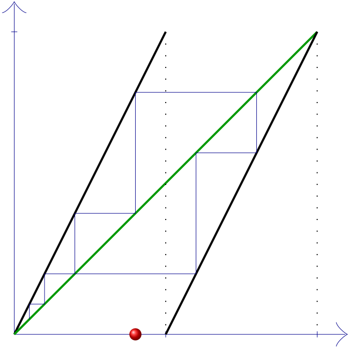

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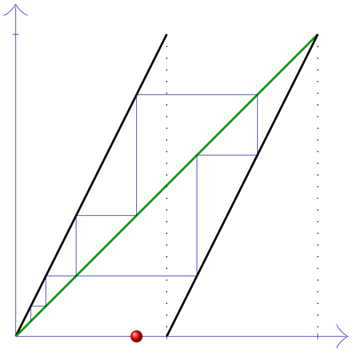

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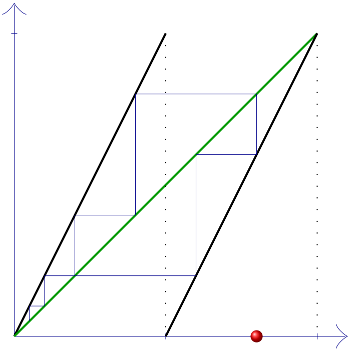

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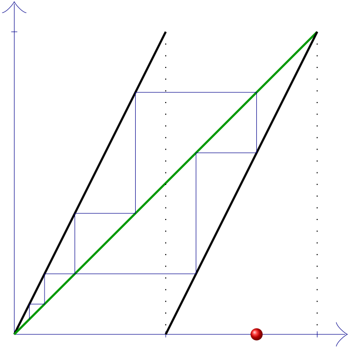

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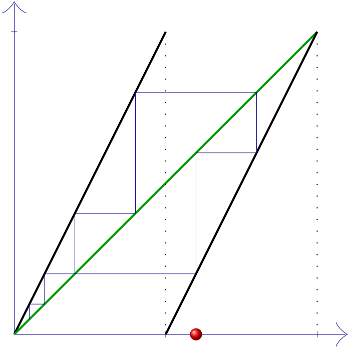

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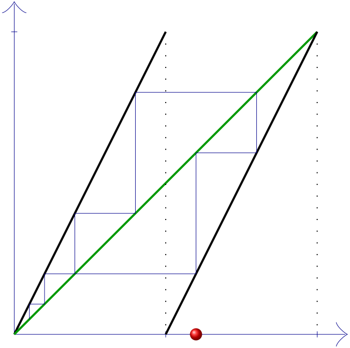

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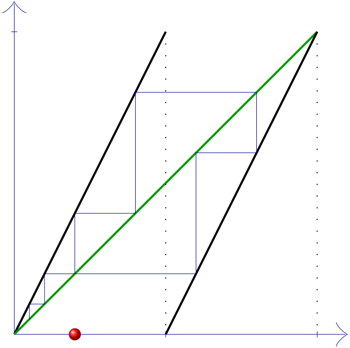

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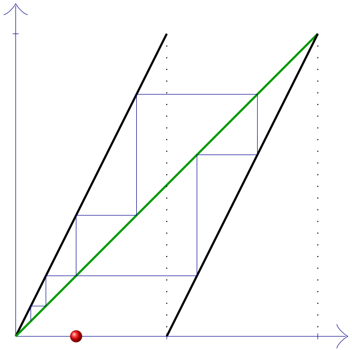

$\underline{x} = 00001100110011$

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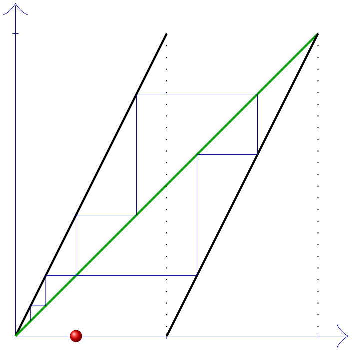

$\underline{x} = 00001100110011\ldots$

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the space Σ_2^+

Σ_2^+

the space Σ_2^+

$$\Sigma_2^+ = \{0, 1\}^{\mathbb{N}}$$

the space Σ_2^+

$$\Sigma_2^+ = \{0, 1\}^{\mathbb{N}}$$

x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	...
1	0	1	1	0	0	1	0	0	0	1	1	0	...

the space Σ_2^+

$$\Sigma_2^+ = \{0, 1\}^{\mathbb{N}}$$

x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	...
1	0	1	1	0	0	1	0	0	0	1	1	0	...
0	1	1	1	1	0	1	0	1	0	0	0	1	...

the space Σ_2^+

$$\Sigma_2^+ = \{0,1\}^{\mathbb{N}}$$

x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	...
1	0	1	1	0	0	1	0	0	0	1	1	0	...
0	1	1	1	1	0	1	0	1	0	0	0	1	...
0	0	1	0	1	1	0	0	0	1	1	0	1	...

the space Σ_2^+

$$\Sigma_2^+ = \{0, 1\}^{\mathbb{N}}$$

x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	...
1	0	1	1	0	0	1	0	0	0	1	1	0	...
0	1	1	1	1	0	1	0	1	0	0	0	1	...
0	0	1	0	1	1	0	0	0	1	1	0	1	...
1	0	1	1	0	0	1	1	1	0	0	1	1	...

the space Σ_2^+

$$\Sigma_2^+ = \{0, 1\}^{\mathbb{N}}$$

x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	...
1	0	1	1	0	0	1	0	0	0	1	1	0	...
0	1	1	1	1	0	1	0	1	0	0	0	1	...
0	0	1	0	1	1	0	0	0	1	1	0	1	...
1	0	1	1	0	0	1	1	1	0	0	1	1	...
⋮						⋮							

a metric on Σ_2^+

We can define a metric on Σ_2^+:

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$$d(\underline{x}, \underline{y}) =$$

a metric on Σ_2^+

We can define a metric on Σ_2^+:

$$d(\underline{x}, \underline{y}) = \sum_{n=0}^{\infty} \frac{|x_n - y_n|}{3^{n+1}}$$

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$$d(\underline{x}, \underline{y}) = \sum_{n=0}^{\infty} \frac{|x_n - y_n|}{3^{n+1}}$$

Proposition

- (Σ_2^+, d) is a compact metric space

a metric on Σ_2^+

We can define a metric on Σ_2^+:

$$d(\underline{x}, \underline{y}) = \sum_{n=0}^{\infty} \frac{|x_n - y_n|}{3^{n+1}}$$

Proposition

- (Σ_2^+, d) is a compact metric space
- $d(\underline{x}, \underline{y}) < {}^{1}/_{3^{n+1}} \quad \Leftrightarrow \quad x_i = y_i$ for $i = 0, \ldots, n$

example

example

points in $B(\underline{1}, {}^1/_{36})$

example

example

points in $B(\underline{1}, {}^{1}\!/_{36})$

x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	...
1	1	1	1	1	1	1	0	0	0	1	1	0	...

example

example

points in $B(\underline{1}, {}^{1}/_{36})$

x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	...
1	1	1	1	1	1	1	0	0	0	1	1	0	...
1	1	1	1	1	1	1	0	1	0	0	0	1	...

example

example

points in $B(\underline{1}, {}^1/_{36})$

x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	...
1	1	1	1	1	1	1	0	0	0	1	1	0	...
1	1	1	1	1	1	1	0	1	0	0	0	1	...
1	1	1	1	1	1	0	0	0	1	1	0	1	...

example

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points in $B(\underline{1}, {}^1/_{36})$

x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	...
1	1	1	1	1	1	1	0	0	0	1	1	0	...
1	1	1	1	1	1	1	0	1	0	0	0	1	...
1	1	1	1	1	1	0	0	0	1	1	0	1	...
1	1	1	1	1	1	1	1	1	0	0	1	1	...

example

example

points in $B(\underline{1}, {}^{1}/_{36})$

x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	...
1	1	1	1	1	1	1	0	0	0	1	1	0	...
1	1	1	1	1	1	1	0	1	0	0	0	1	...
1	1	1	1	1	1	0	0	0	1	1	0	1	...
1	1	1	1	1	1	1	1	1	0	0	1	1	...
⋮						⋮							

the shift transformation

the shift transformation

the shift transformation $\sigma : \Sigma^+ \to \Sigma^+$

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the shift transformation $\sigma : \Sigma^+ \to \Sigma^+$
is defined by

the shift transformation

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the shift transformation $\sigma : \Sigma^+ \to \Sigma^+$
is defined by

$$[\sigma(\underline{x})]_n = x_{n+1}$$

example

example

$\underline{x}$	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	...
$\underline{x}$	1	0	1	1	0	0	1	0	0	0	1	1	...

example

example

$\underline{x}$	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	...
$\underline{x}$	1	0	1	1	0	0	1	0	0	0	1	1	...
$\sigma(\underline{x})$	0	1	1	0	0	1	0	0	0	1	1	0	...

example

example

$\underline{x}$	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	$\ldots$
$\underline{x}$	1	0	1	1	0	0	1	0	0	0	1	1	$\ldots$
$\sigma(\underline{x})$	0	1	1	0	0	1	0	0	0	1	1	0	$\ldots$
$\sigma^2(\underline{x})$	1	1	0	0	1	0	0	0	1	1	0	1	$\ldots$

example

example

$\underline{x}$	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	...
$\underline{x}$	1	0	1	1	0	0	1	0	0	0	1	1	...
$\sigma(\underline{x})$	0	1	1	0	0	1	0	0	0	1	1	0	...
$\sigma^2(\underline{x})$	1	1	0	0	1	0	0	0	1	1	0	1	...
$\sigma^3(\underline{x})$	1	0	0	1	0	0	0	1	1	0	1	1	...

example

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$\underline{x}$	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	...
$\underline{x}$	1	0	1	1	0	0	1	0	0	0	1	1	...
$\sigma(\underline{x})$	0	1	1	0	0	1	0	0	0	1	1	0	...
$\sigma^2(\underline{x})$	1	1	0	0	1	0	0	0	1	1	0	1	...
$\sigma^3(\underline{x})$	1	0	0	1	0	0	0	1	1	0	1	1	...
$\sigma^4(\underline{x})$	0	0	1	0	0	0	1	1	0	1	1	0	...

example

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$\underline{x}$	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	...
$\underline{x}$	1	0	1	1	0	0	1	0	0	0	1	1	...
$\sigma(\underline{x})$	0	1	1	0	0	1	0	0	0	1	1	0	...
$\sigma^2(\underline{x})$	1	1	0	0	1	0	0	0	1	1	0	1	...
$\sigma^3(\underline{x})$	1	0	0	1	0	0	0	1	1	0	1	1	...
$\sigma^4(\underline{x})$	0	0	1	0	0	0	1	1	0	1	1	0	...
$\sigma^5(\underline{x})$	0	1	0	0	0	1	1	0	1	1	0	1	...

example

example

$\underline{x}$	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	$\dots$
$\underline{x}$	1	0	1	1	0	0	1	0	0	0	1	1	$\dots$
$\sigma(\underline{x})$	0	1	1	0	0	1	0	0	0	1	1	0	$\dots$
$\sigma^2(\underline{x})$	1	1	0	0	1	0	0	0	1	1	0	1	$\dots$
$\sigma^3(\underline{x})$	1	0	0	1	0	0	0	1	1	0	1	1	$\dots$
$\sigma^4(\underline{x})$	0	0	1	0	0	0	1	1	0	1	1	0	$\dots$
$\sigma^5(\underline{x})$	0	1	0	0	0	1	1	0	1	1	0	1	$\dots$
$\vdots$						$\vdots$							

Index

fixed points

fixed point

$\underline{x}$ is a fixed point if $\sigma(\underline{x}) = \underline{x}$

fixed points

- $\underline{x}$ is a fixed point

fixed points

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- $\Rightarrow x_{n+1} = x_n$ for all $n \geq 0$
- two cases:
  1. $x_0 = 0$
  2. $x_0 = 1$
- 0000

fixed points

- $\underline{x}$ is a fixed point
- $\Rightarrow \sigma(\underline{x}) = \underline{x}$
- $\Rightarrow [\sigma(\underline{x})]_n = x_n$ for all $n \geq 0$
- $\Rightarrow x_{n+1} = x_n$ for all $n \geq 0$
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periodic points

periodic point

$\underline{x}$ is a periodic point if $\exists N \geq 0$ such that

$$o(\underline{x}) : \qquad \underline{x}, \sigma(\underline{x}), \sigma^2(\underline{x}), \ldots, \sigma^N(\underline{x}) = \underline{x}$$

periodic points of period 2

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  2. $x_0 x_1 = 01$

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  2. $x_0 x_1 = 01$
  3. $x_0 x_1 = 10$

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- (2) 01

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- (2) $010101\ldots$

periodic point are dense

periodic points are dense

the periodic points for the shift transformation are dense in Σ_2^+

transitivity

transitivity

the shift transformation is transitive

hint

hint

there is $\underline{x}$ with dense orbit:

hint

there is $\underline{x}$ with dense orbit:

$$\underline{x} =$$

hint

there is $\underline{x}$ with dense orbit:

$$\underline{x} = 0$$

hint

there is $\underline{x}$ with dense orbit:

$$\underline{x} = 0\ 1$$

hint

there is $\underline{x}$ with dense orbit:

$$\underline{x} = \boxed{0}\boxed{1}\boxed{00}$$

hint

there is $\underline{x}$ with dense orbit:

$$\underline{x} = 0\ 1\ 00\ 01$$

hint

there is $\underline{x}$ with dense orbit:

$$\underline{x} = 0\ 1\ 00\ 01\ 10$$

hint

there is $\underline{x}$ with dense orbit:

$$\underline{x} = 0\ 1\ 00\ 01\ 10\ 11$$

hint

there is $\underline{x}$ with dense orbit:

$$\underline{x} = 0\ 1\ 00\ 01\ 10\ 11\ 000$$

hint

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hint

there is $\underline{x}$ with dense orbit:

$$\underline{x} = 0\ 1\ 00\ 01\ 10\ 11\ 000\ 001\ 010\ 011\ 100$$

hint

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$$\underline{x} = 0\ 1\ 00\ 01\ 10\ 11\ 000\ 001\ 010\ 011\ 100 \ldots$$

hint

there is $\underline{x}$ with dense orbit:

$$\underline{x} = 0\ 1\ 00\ 01\ 10\ 11\ 000\ 001\ 010\ 011\ 100\ \ldots$$

$\underline{x}$ contains all finite sequences of 0's and 1's