A little bit more about ergodicity

Jana Rodriguez Hertz

ICTP

2018

mark and recapture method

mark and recapture

mark and recapture method

mark and recapture

mark and recapture method

mark and recapture

↓

- ecology

mark and recapture method

mark and recapture

↓

- ecology
- epidemiology

mark and recapture method

mark and recapture

↓

- ecology
- epidemiology

↓

mark and recapture method

mark and recapture

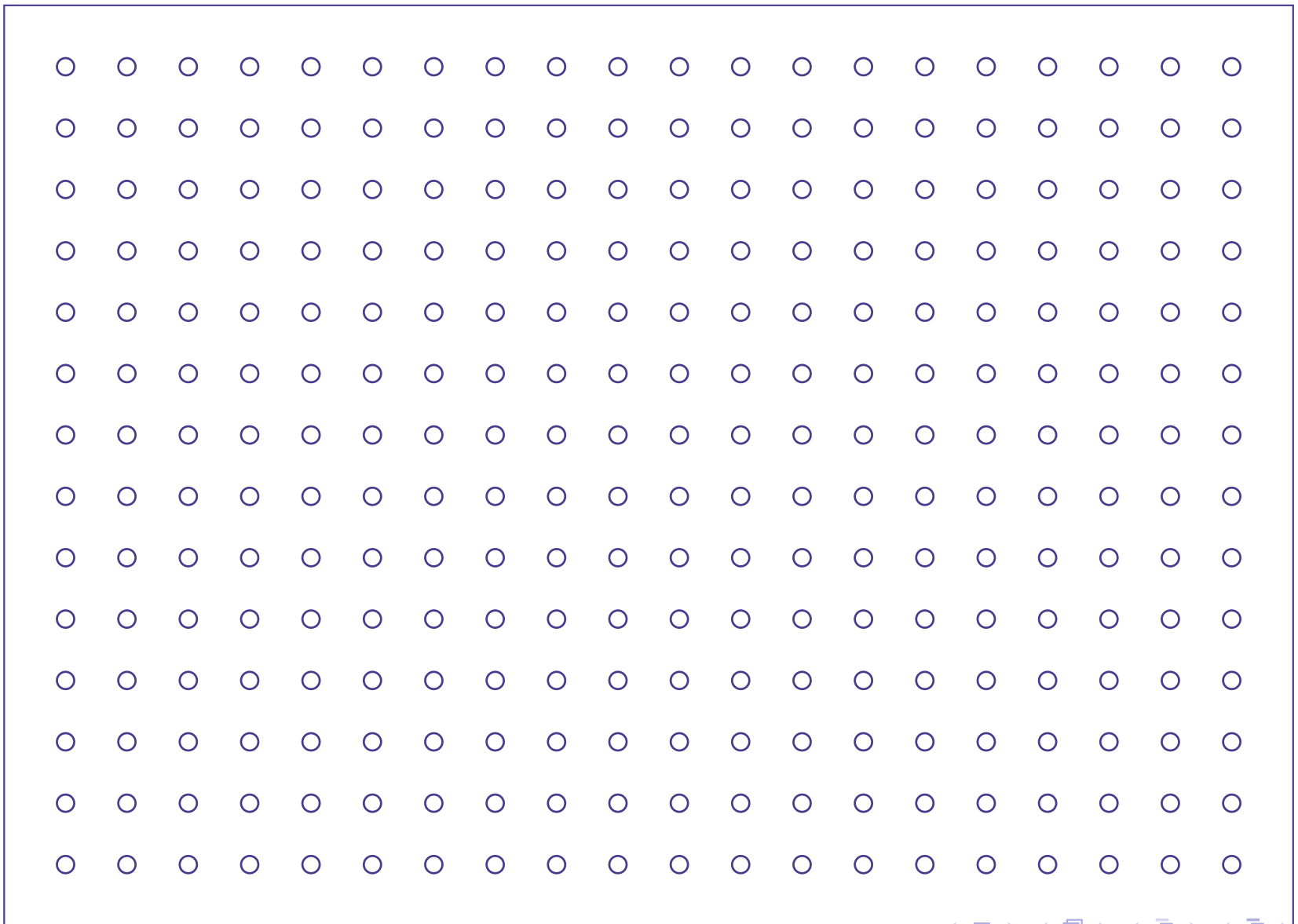
↓

- ecology
- epidemiology

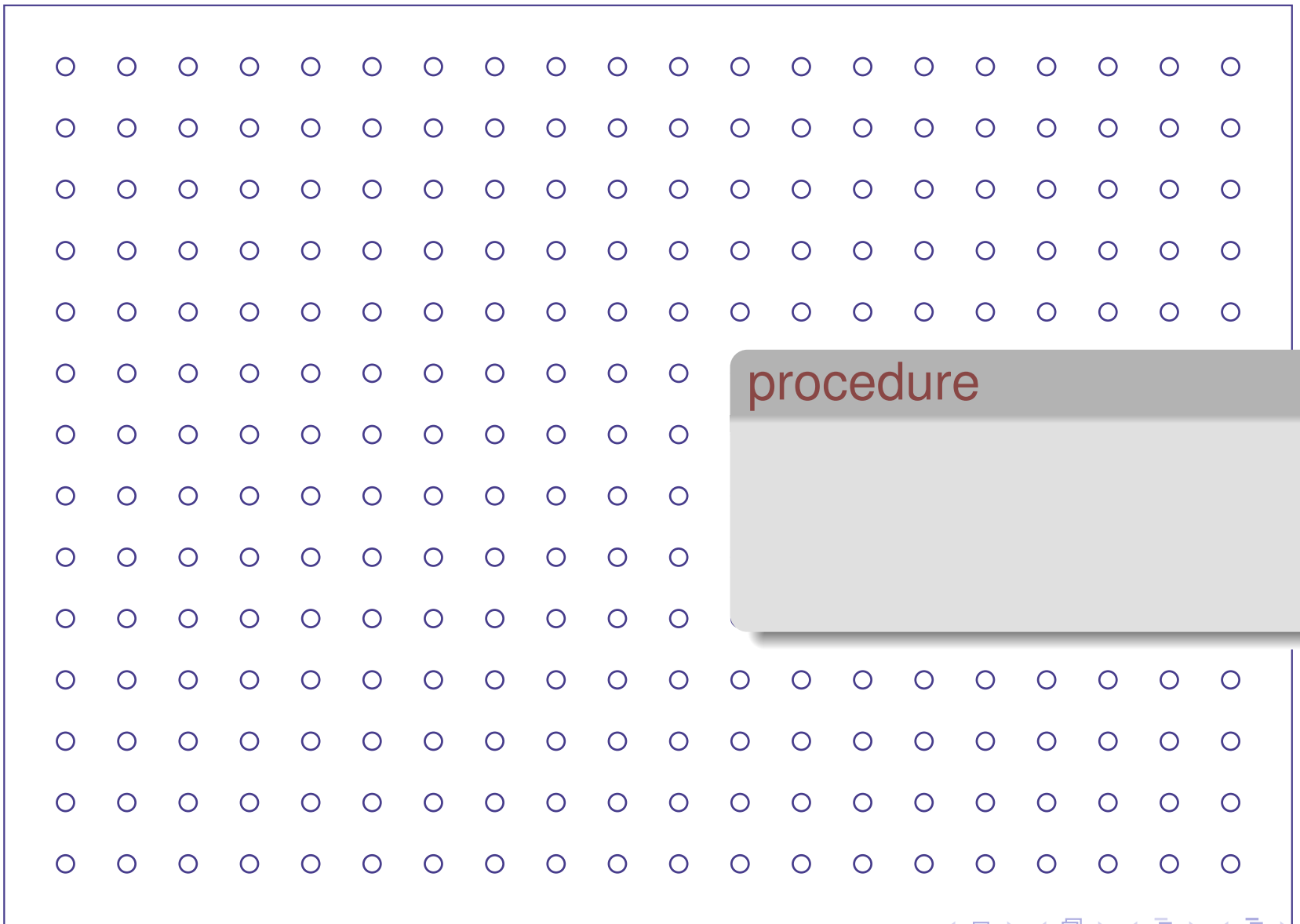
↓

estimate populations

counting fish

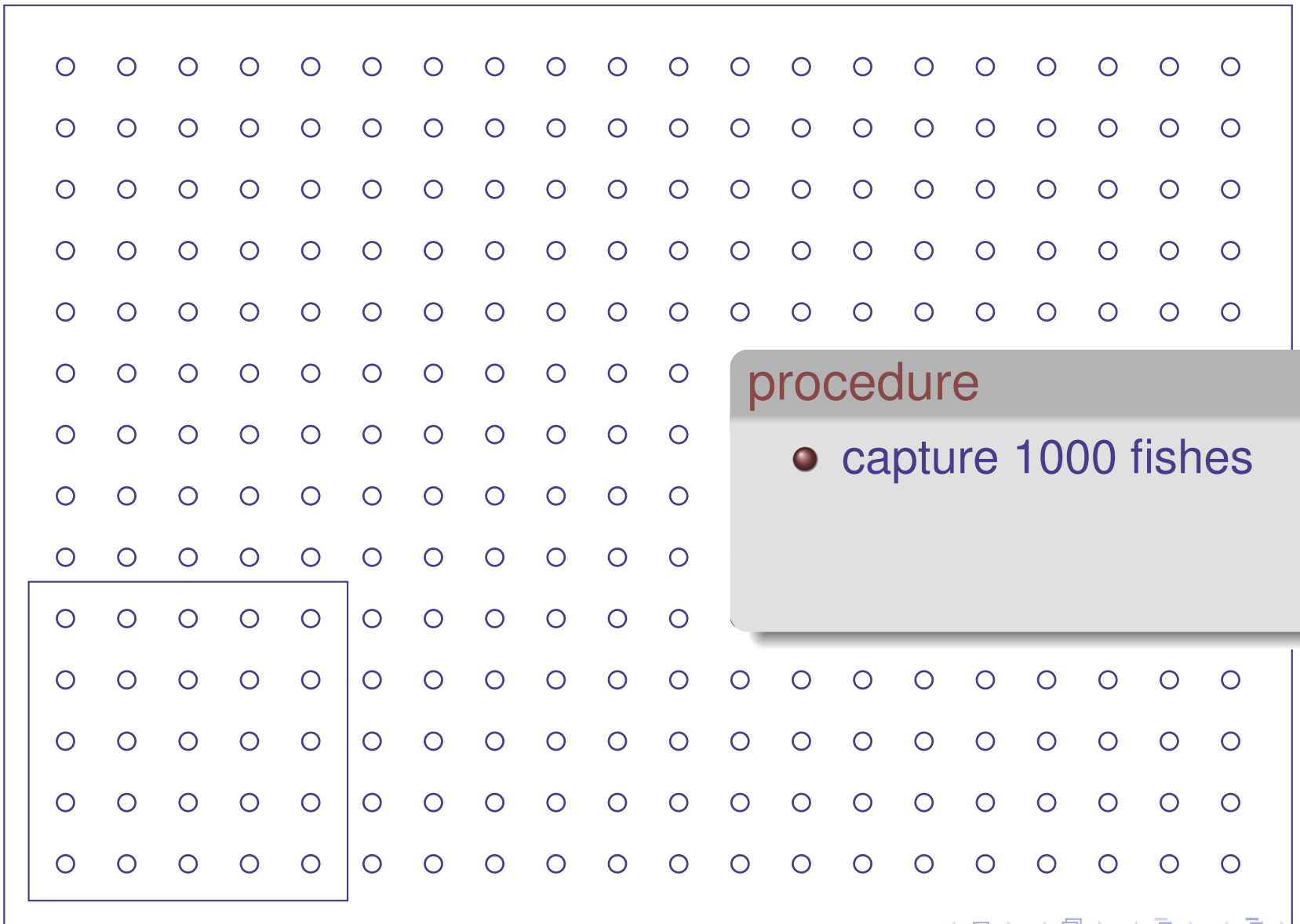


counting fish



procedure

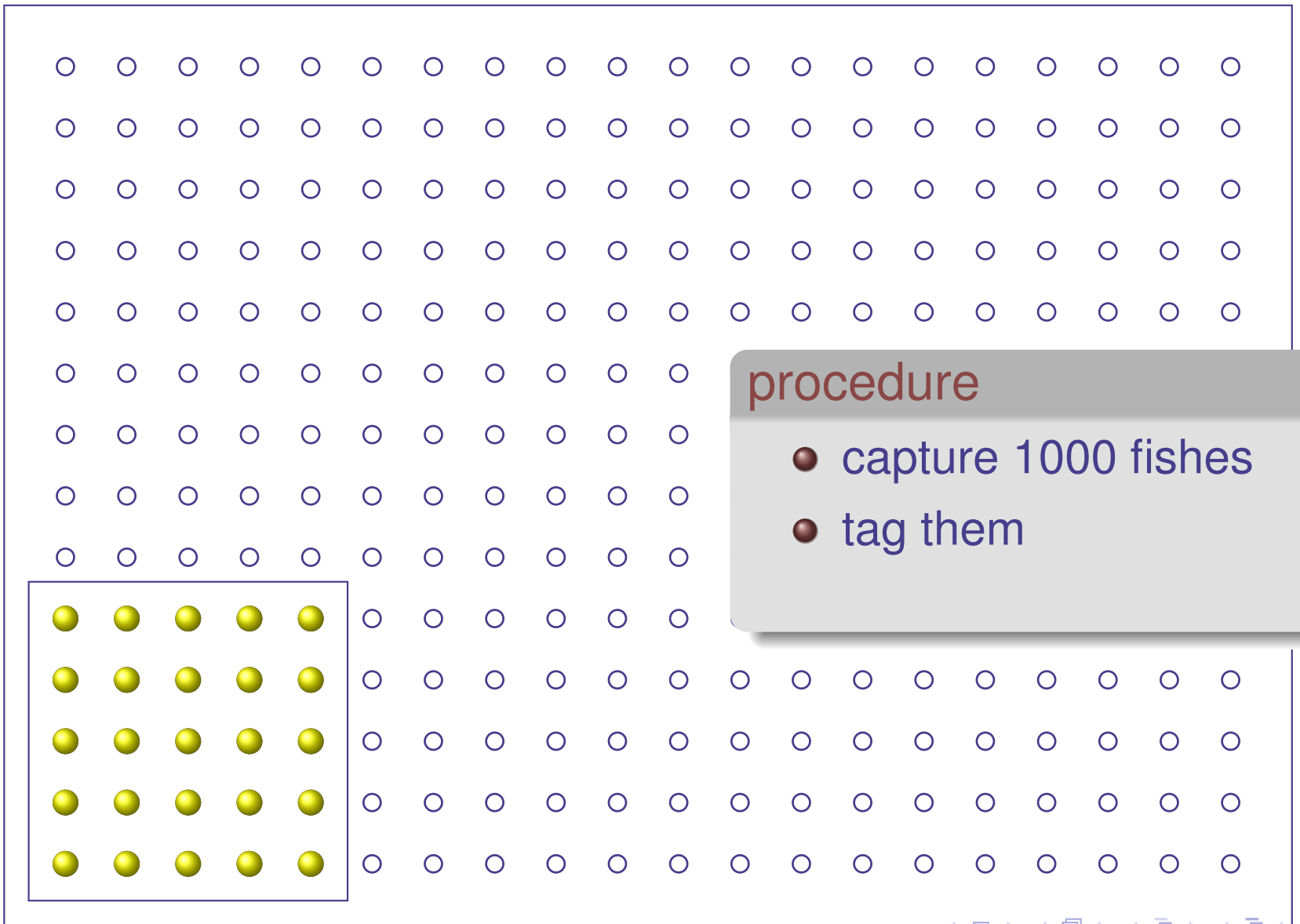
counting fish



procedure

- capture 1000 fishes

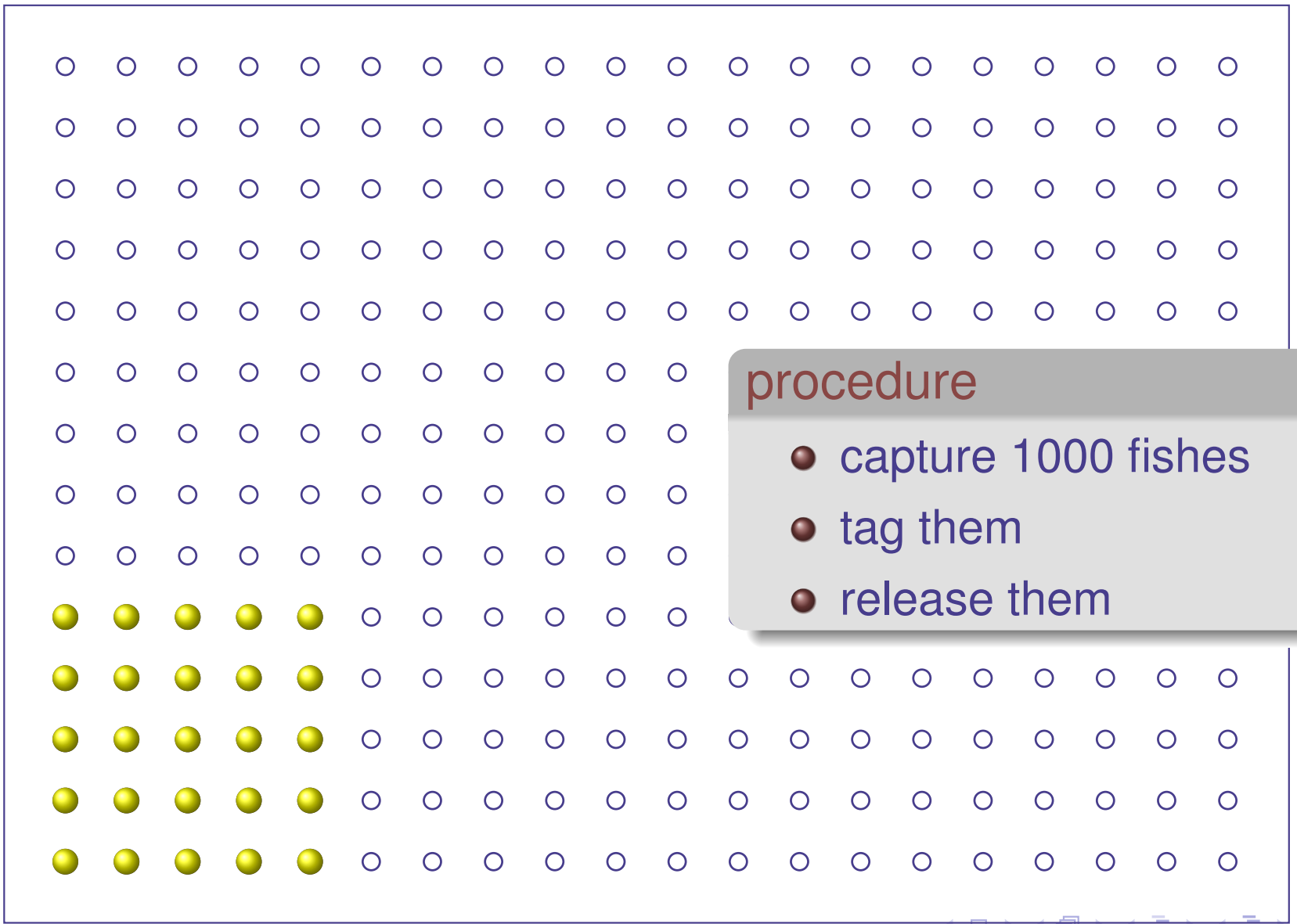
counting fish



procedure

- capture 1000 fishes
- tag them

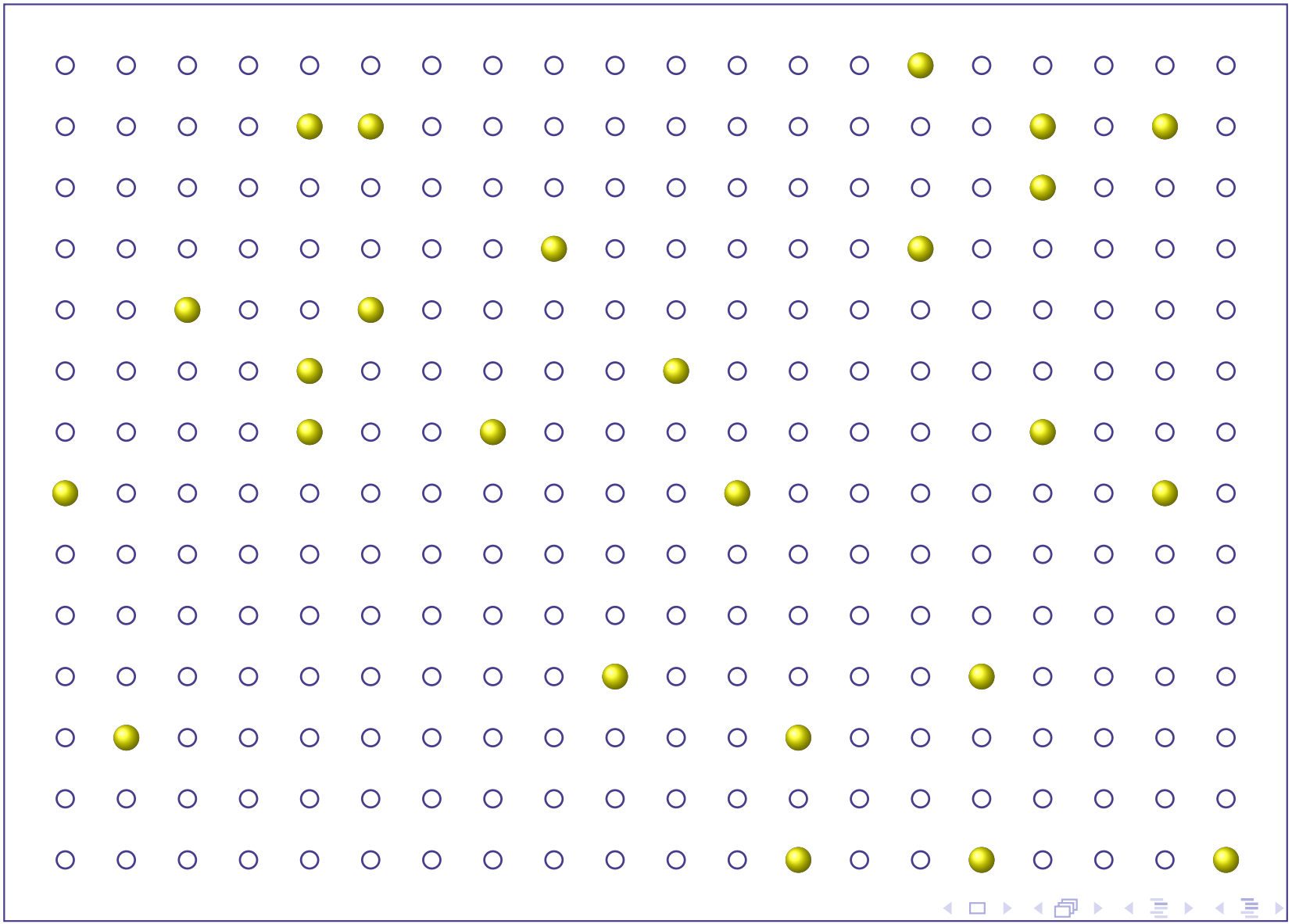
counting fish



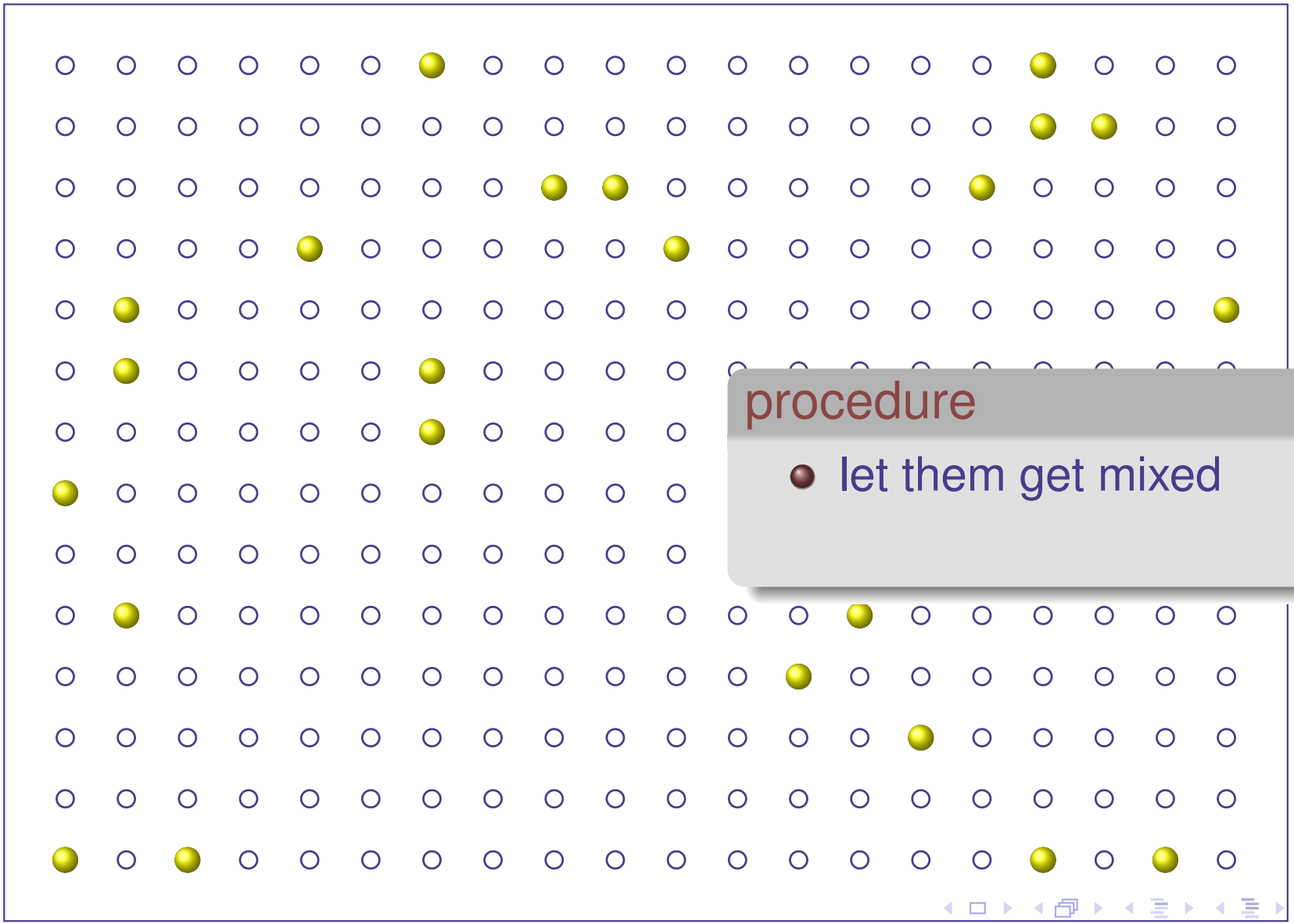
procedure

- capture 1000 fishes
- tag them
- release them

counting fish



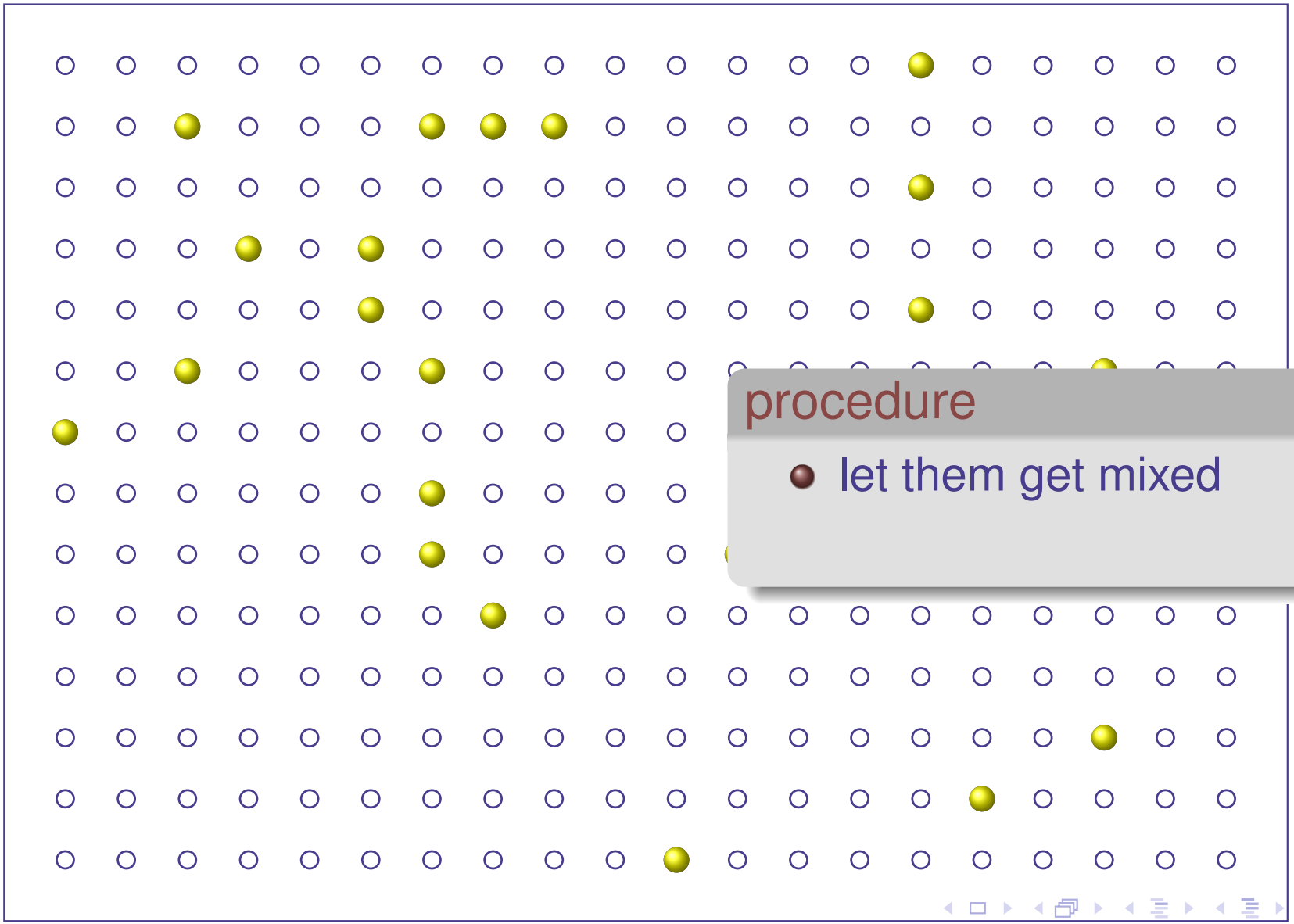
counting fish



procedure

- let them get mixed

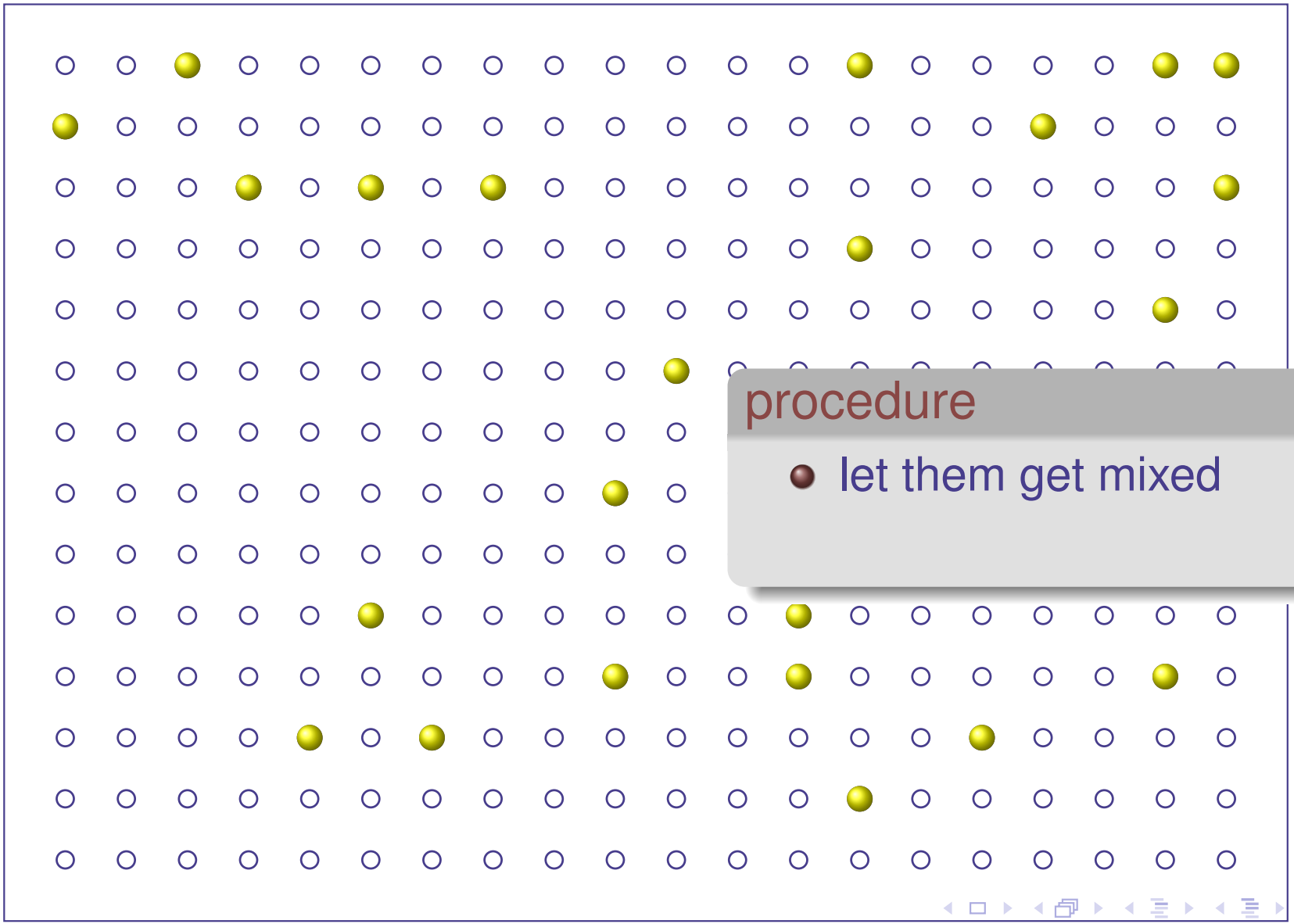
counting fish



procedure

- let them get mixed

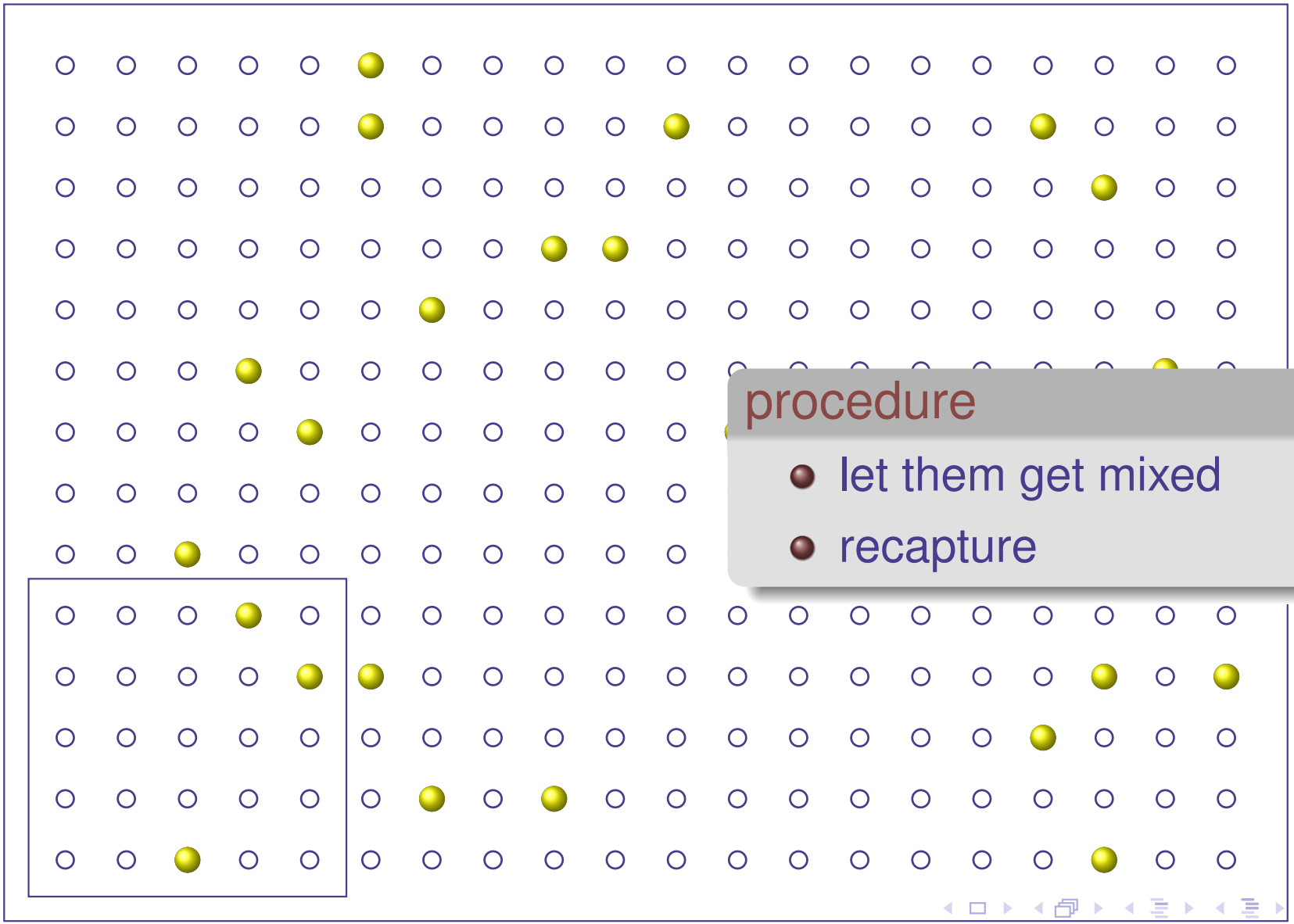
counting fish



procedure

- let them get mixed

counting fish

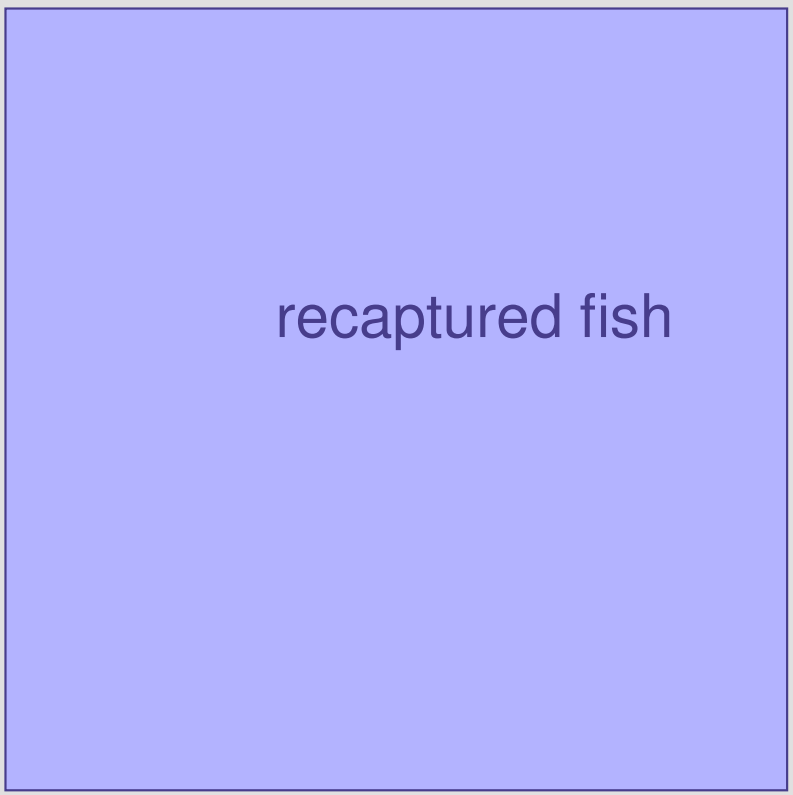


procedure

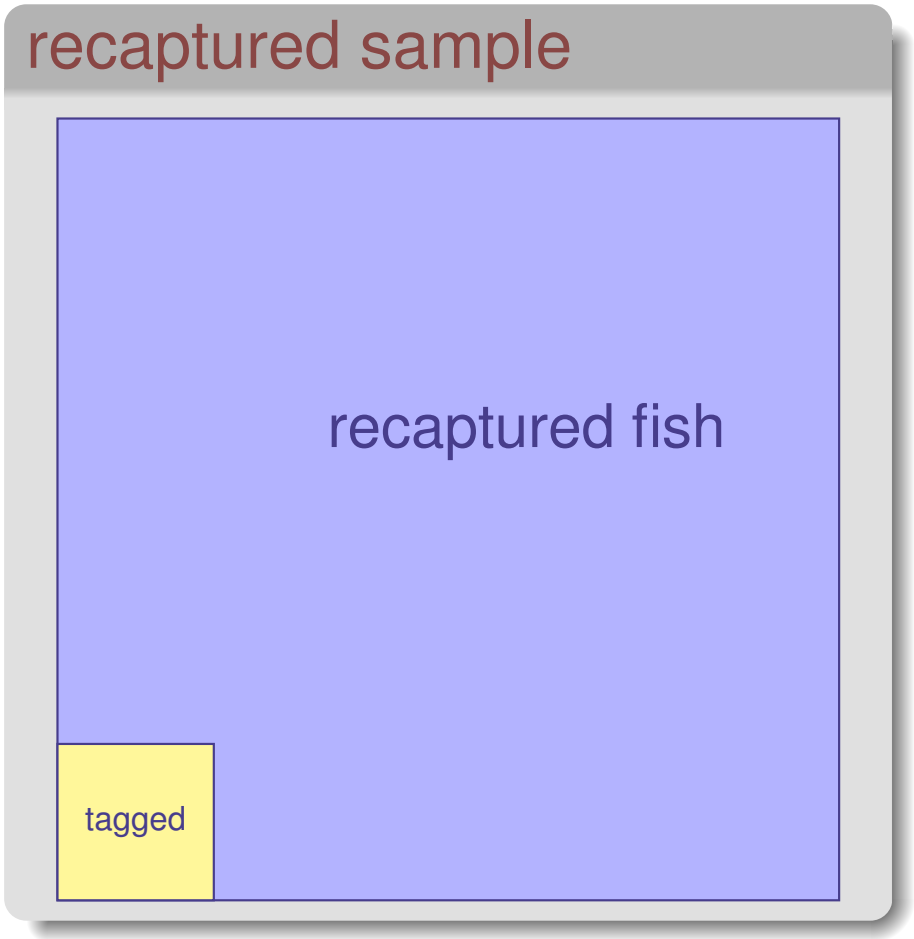
- let them get mixed
- recapture

estimate

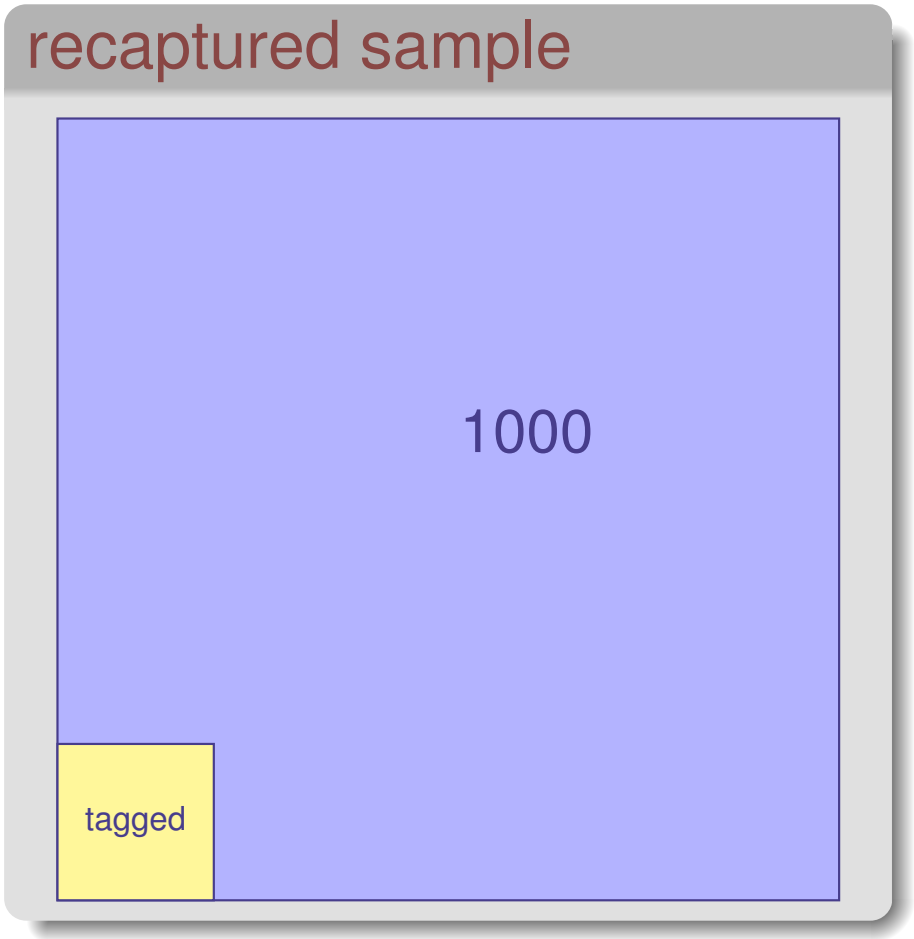
recaptured sample



estimate

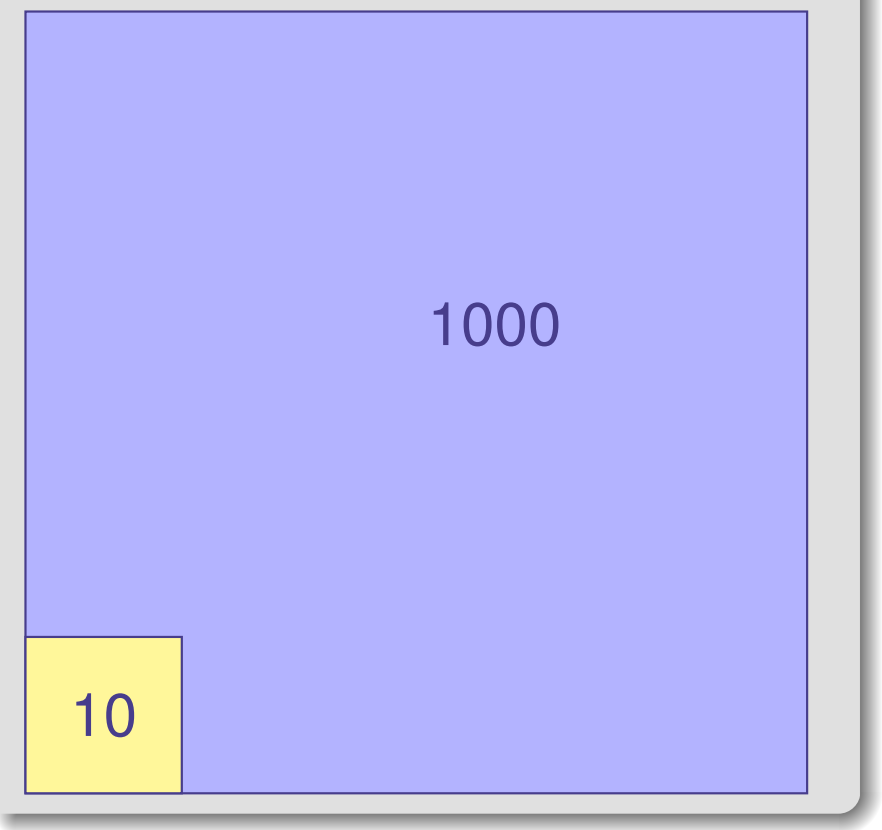


estimate



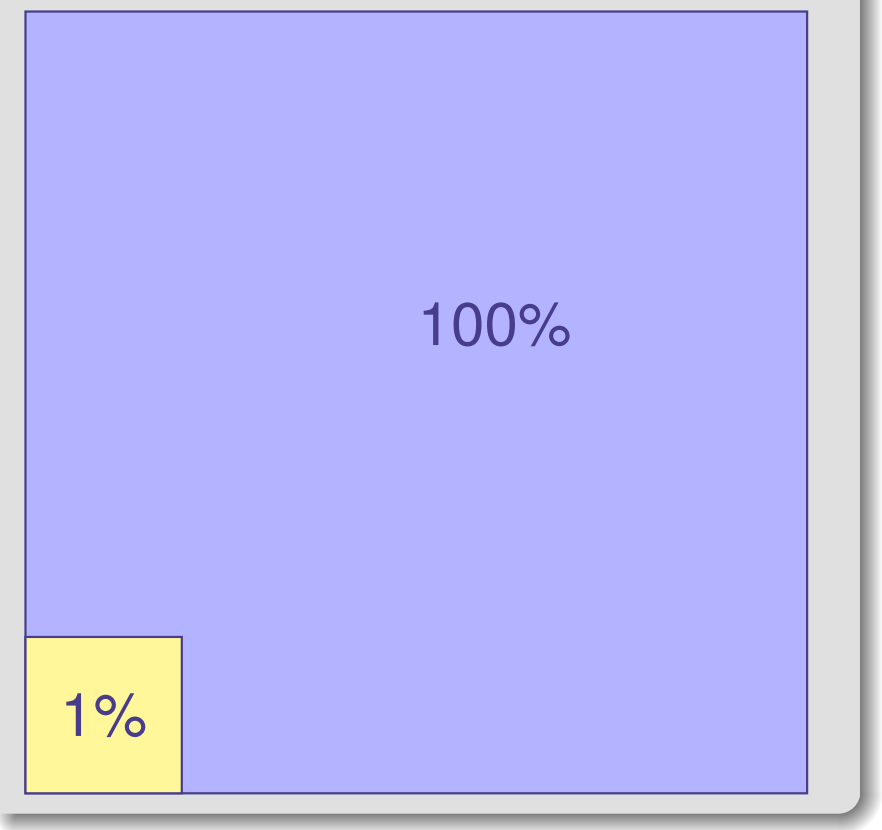
estimate

recaptured sample

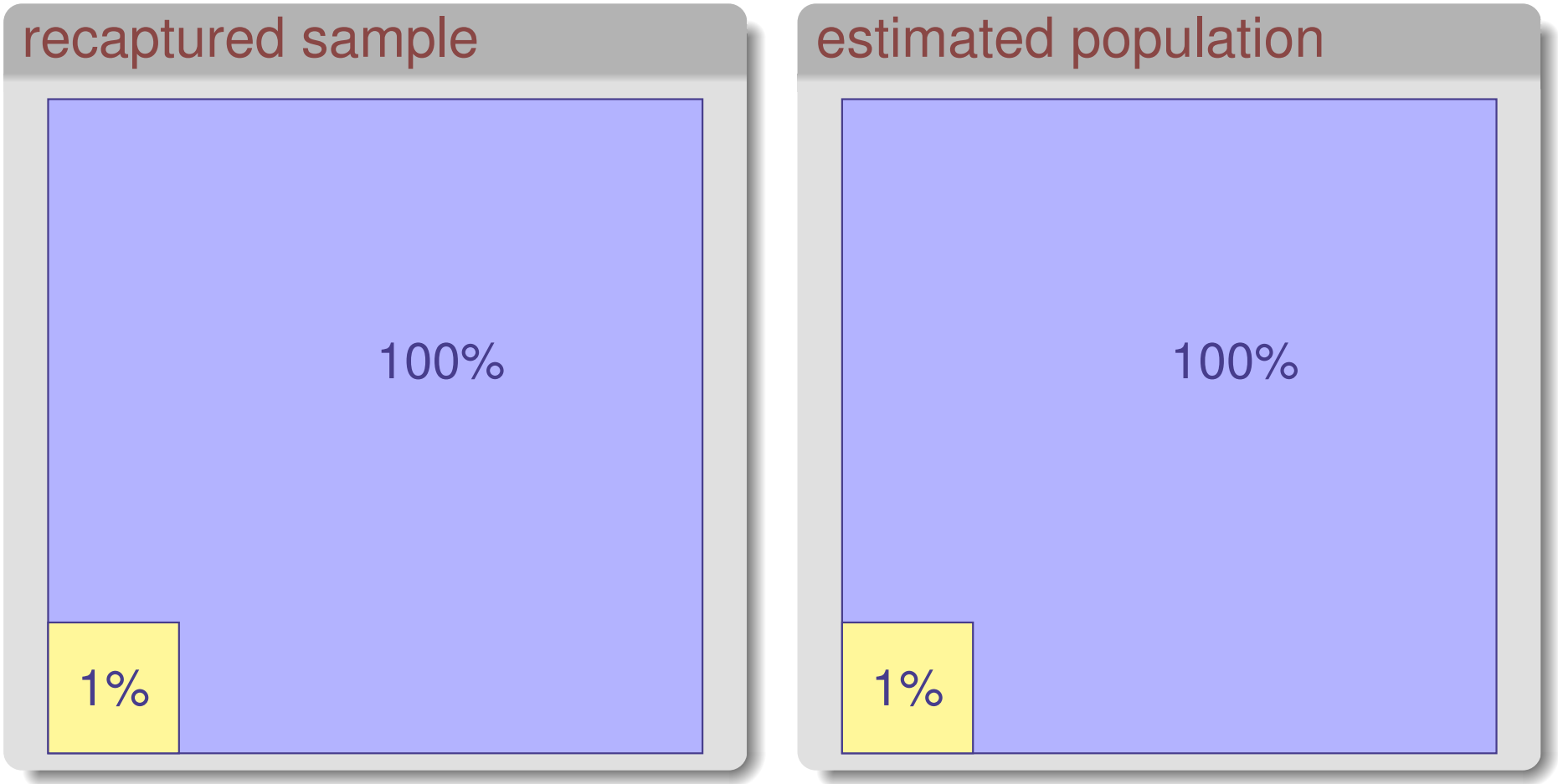


estimate

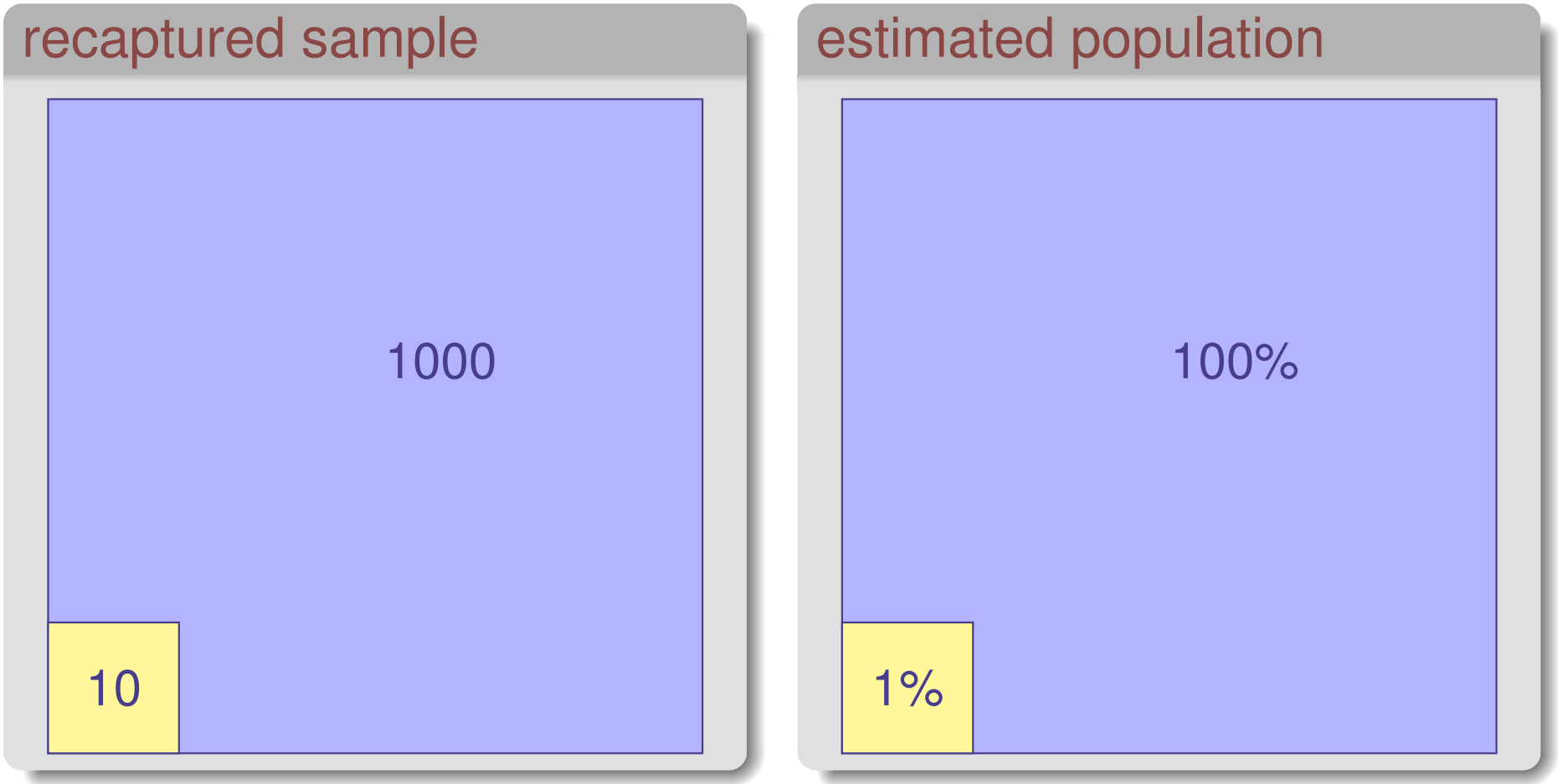
recaptured sample



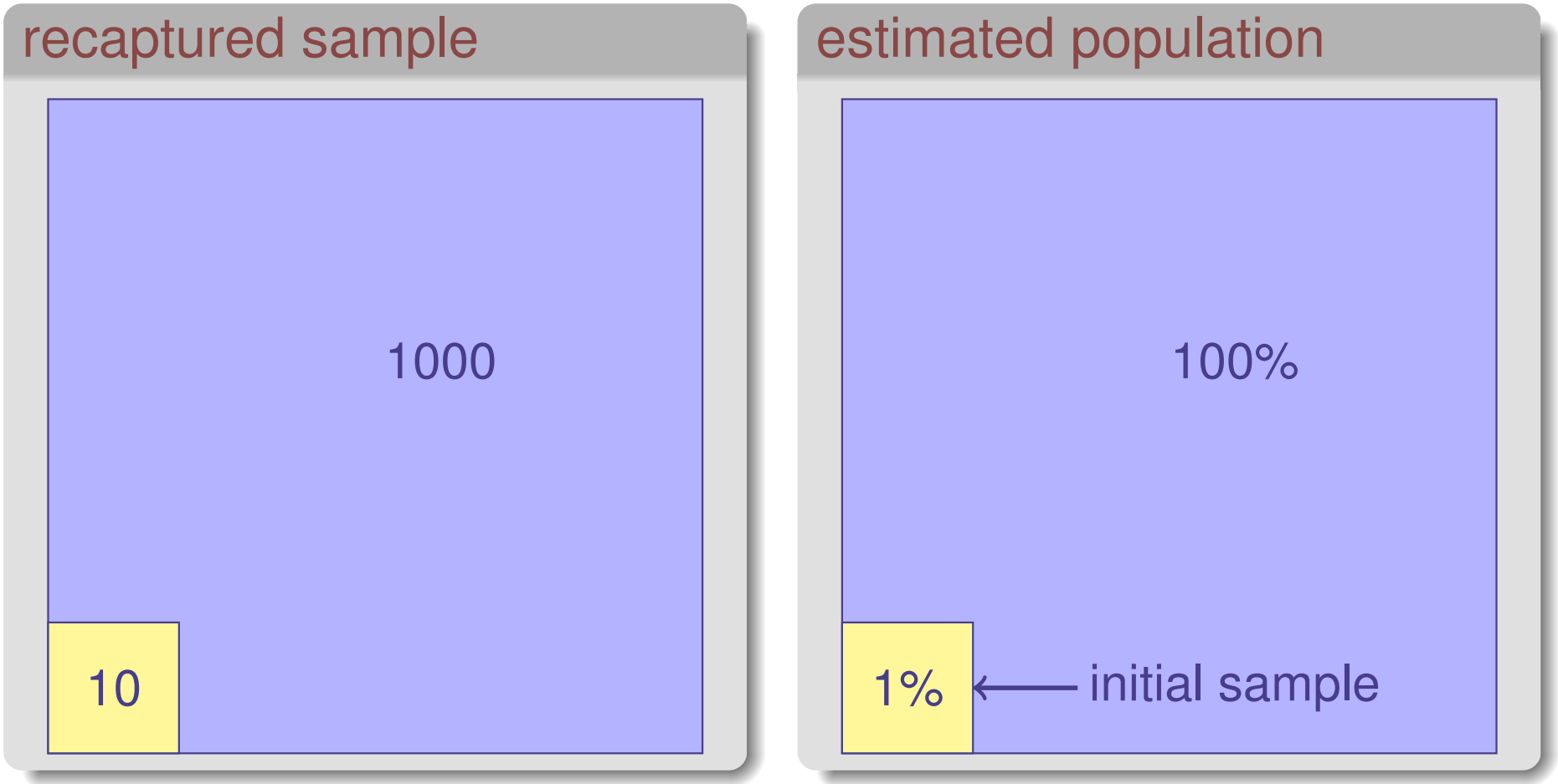
estimate



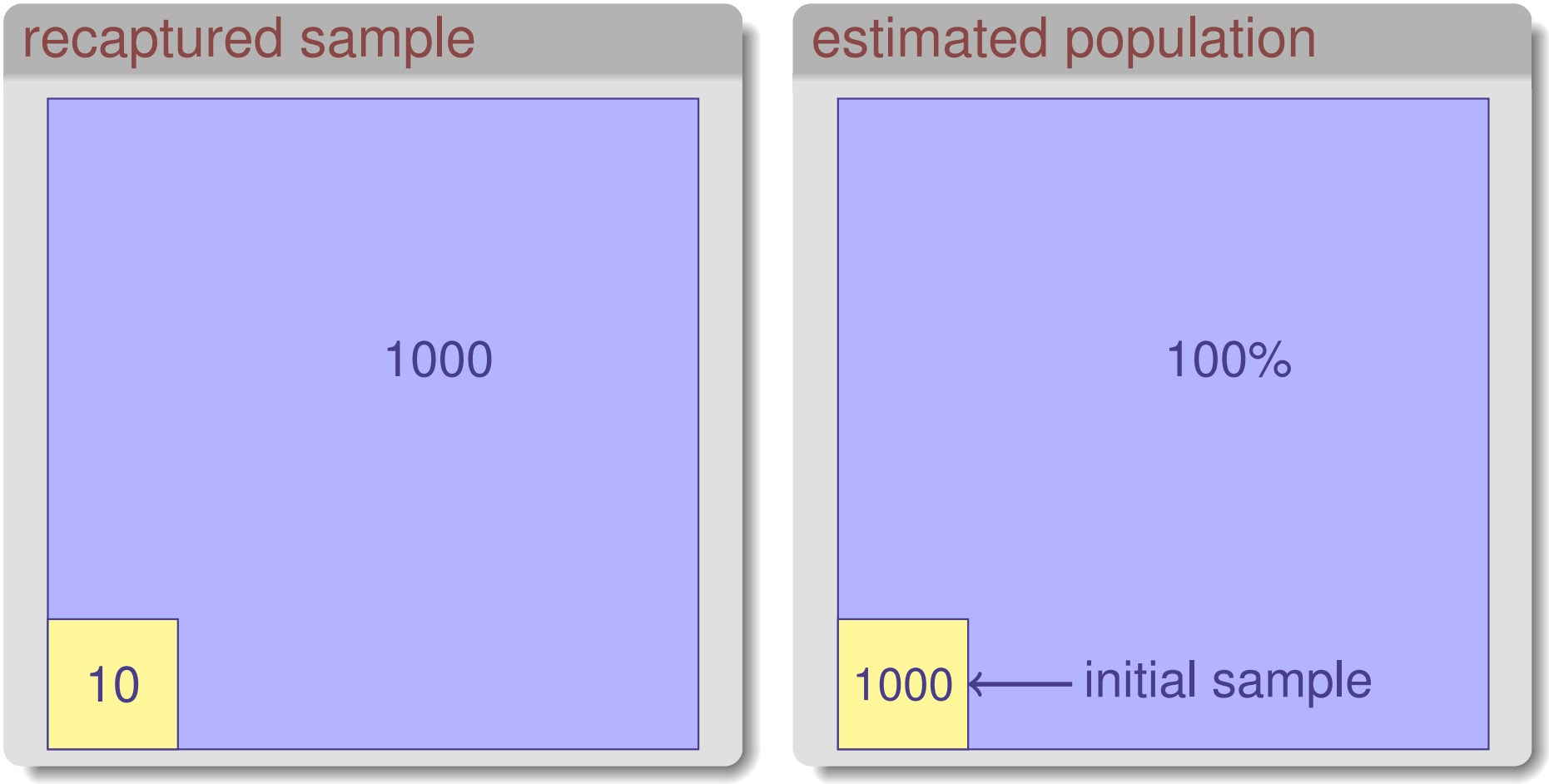
estimate



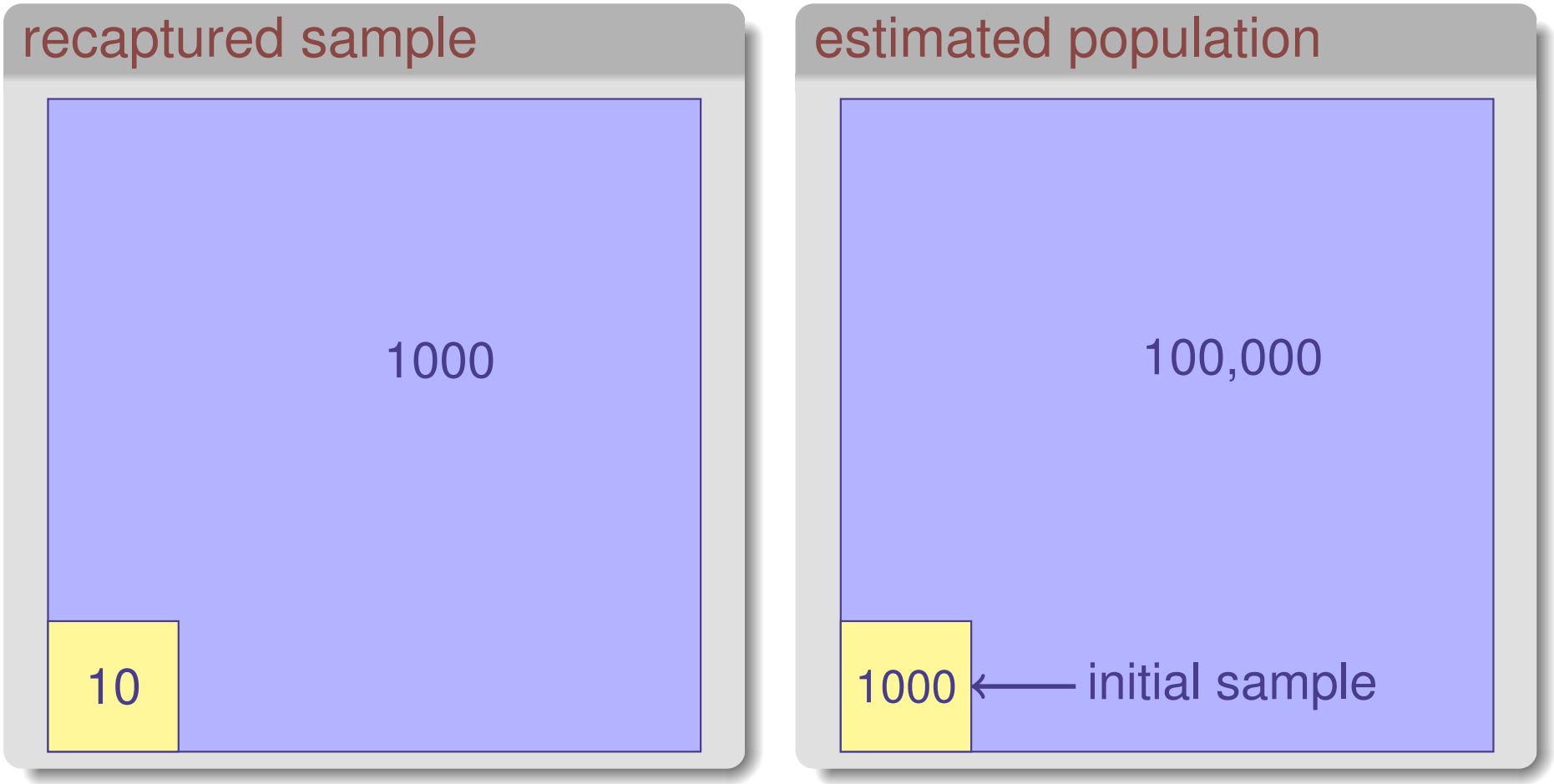
estimate


recaptured sample
1000
10
estimated population
100%
1%
initial sample

estimate


recaptured sample
1000
10
estimated population
100%
1000
initial sample

estimate


recaptured sample
1000
10
estimated population
100,000
1000
initial sample

assumptions

implicit hypotheses

- fish amount or measure is not altered

assumptions

implicit hypotheses

- fish amount or measure is not altered
- tagged fish move around the lake and come back

assumptions

implicit hypotheses

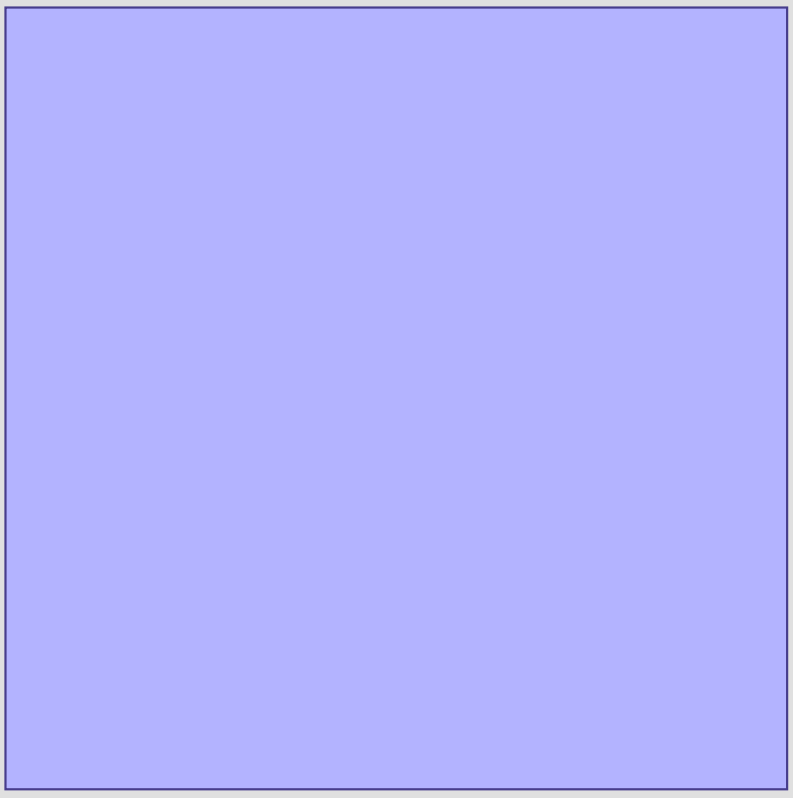
- fish amount or measure is not altered
- tagged fish move around the lake and come back
- tagged and non-tagged fish get well mixed

possible failure 1

possibility 1

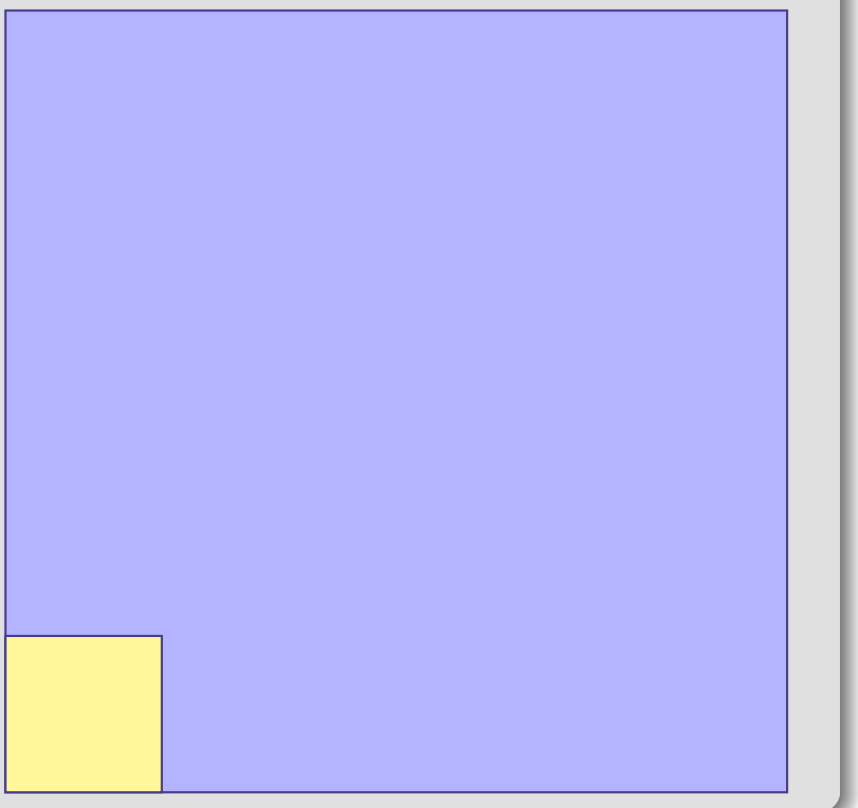
estimate

recaptured sample



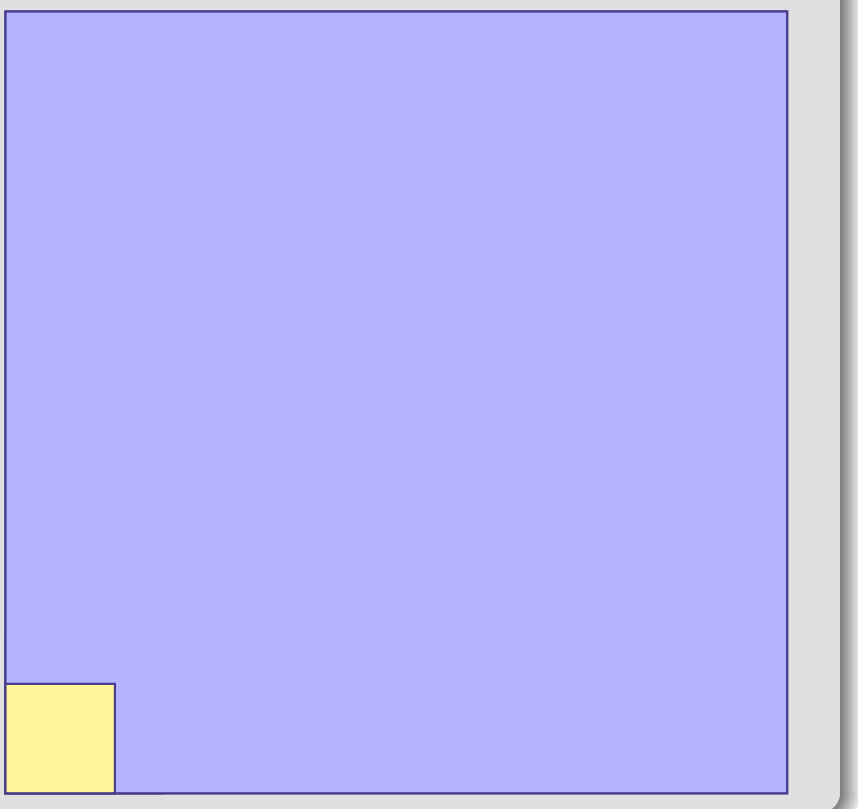
estimate

recaptured sample



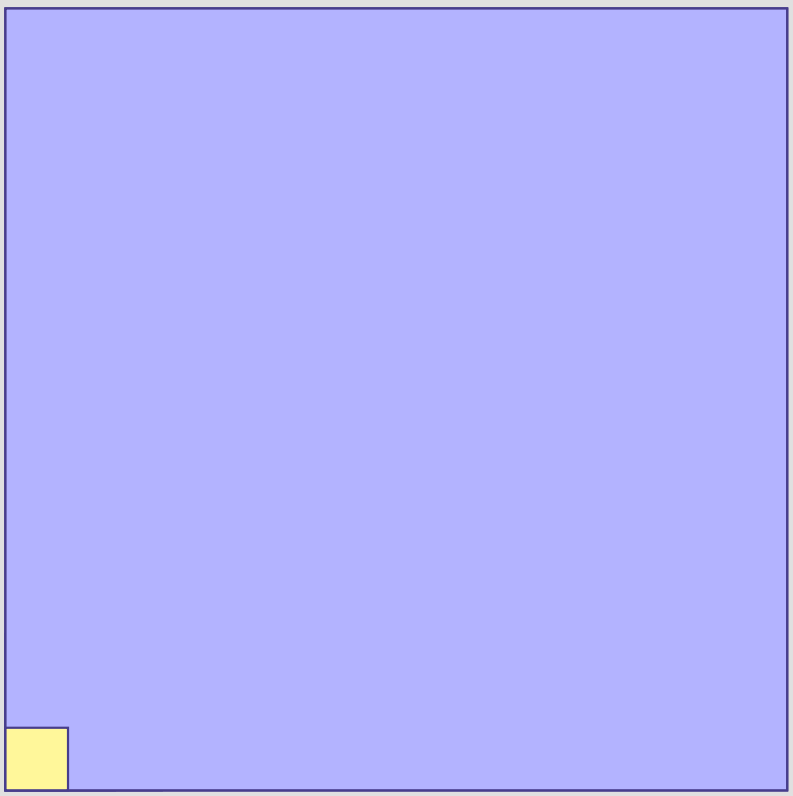
estimate

recaptured sample



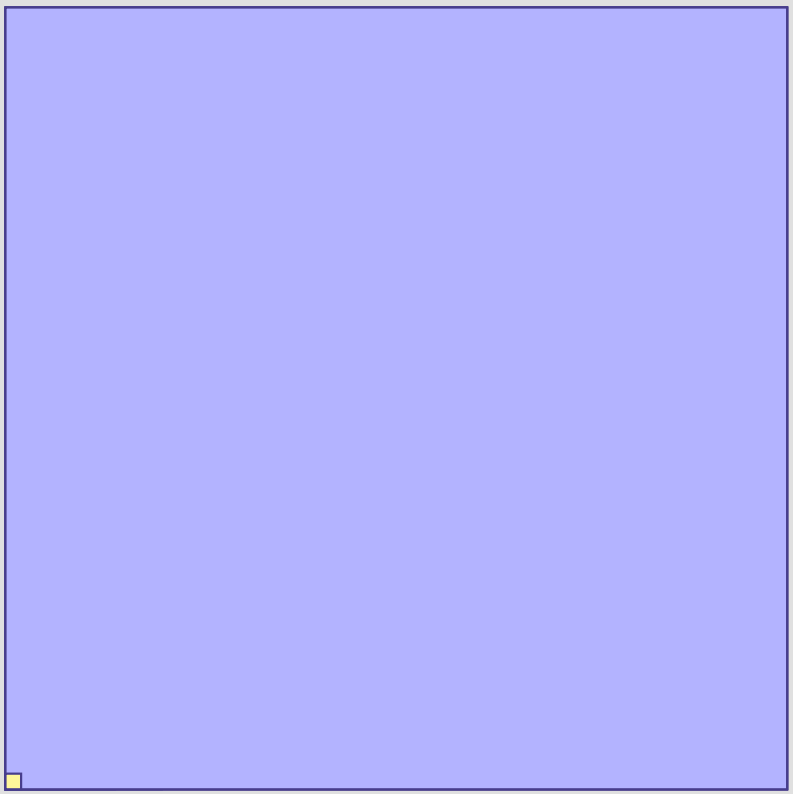
estimate

recaptured sample



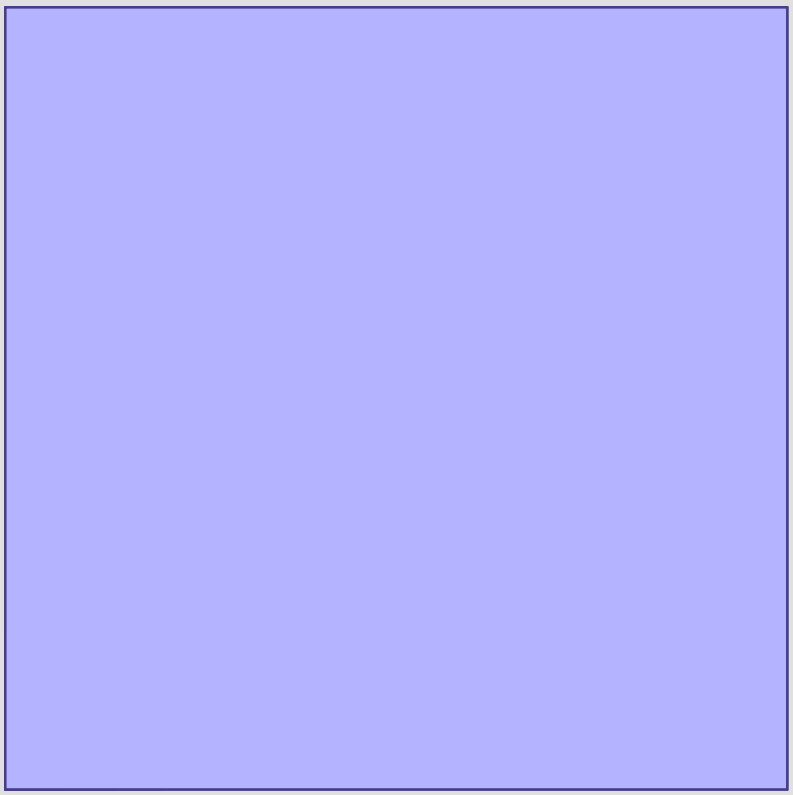
estimate

recaptured sample



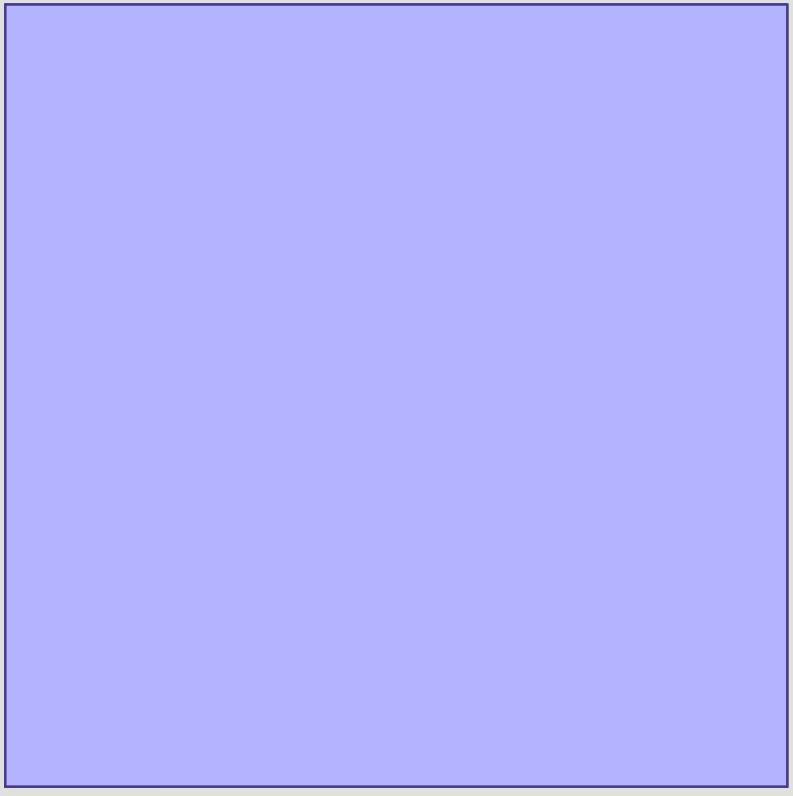
estimate

recaptured sample

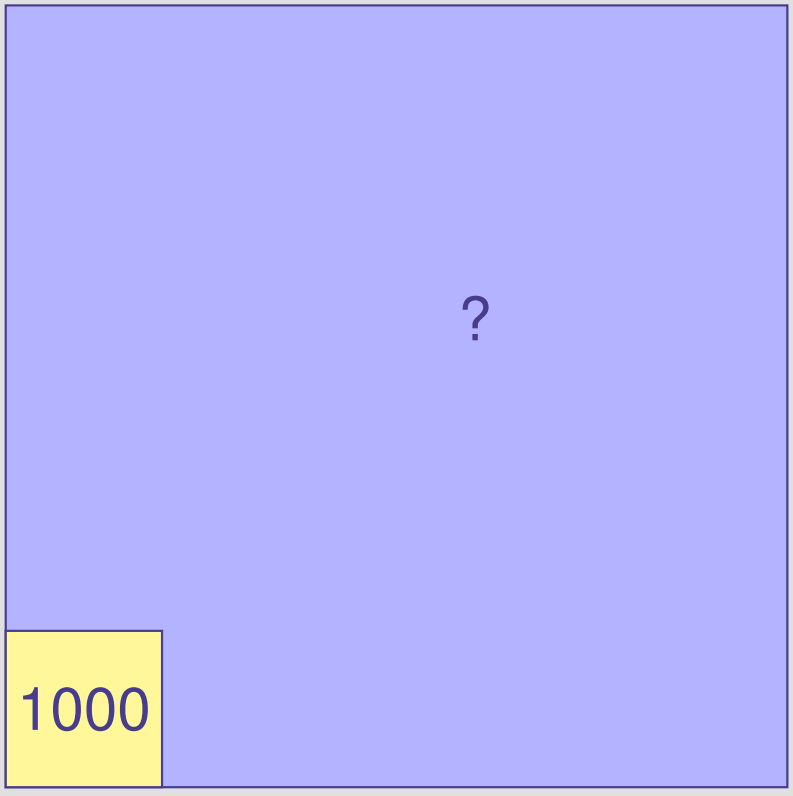


estimate

recaptured sample



estimated population?



preservation of the measure

measure preserving process

- $f : L \to L$ process where

preservation of the measure

measure preserving process

- $f : L \to L$ process where
- for any sample $A \subset L$

preservation of the measure

measure preserving process

- $f : L \to L$ process where
- for any sample $A \subset L$
- we have

$$\boxed{m(f(A)) = m(A)}$$

possible failure 2

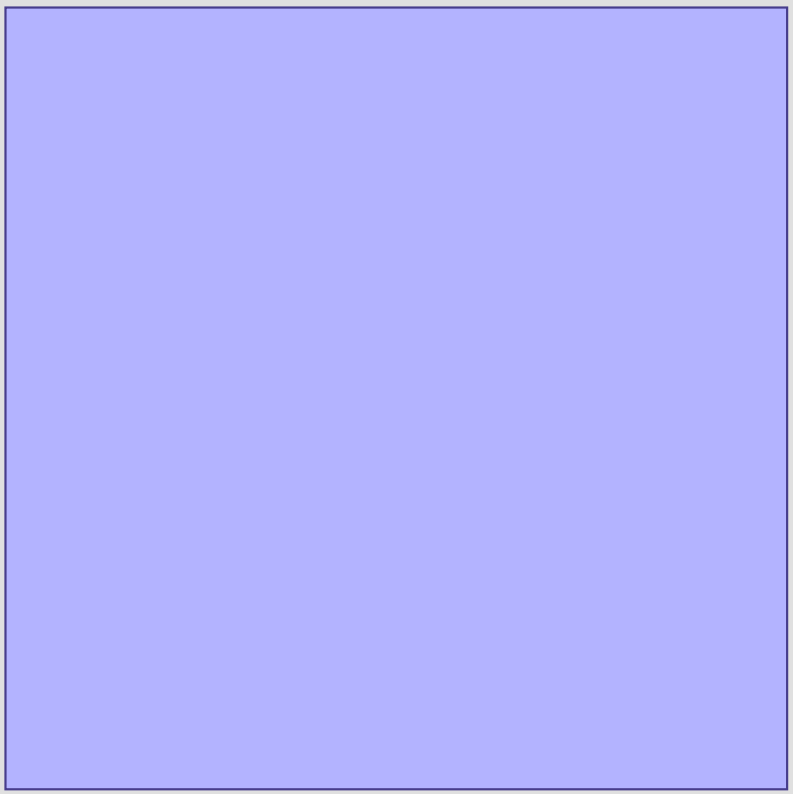
possibility 2

possible failure 2 - trace

possibility 2 - trace

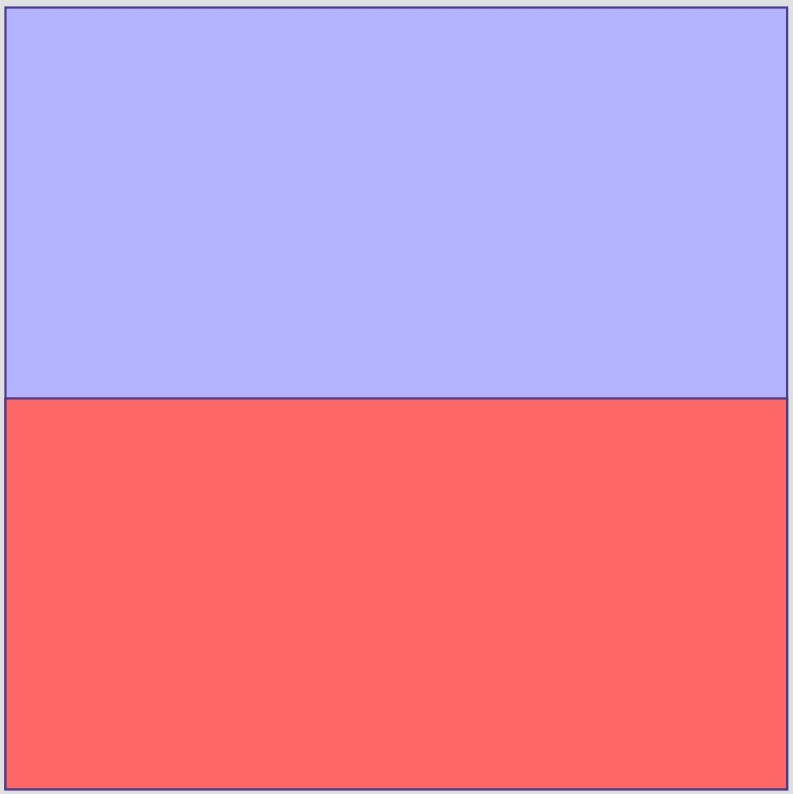
possibility 2

recaptured sample



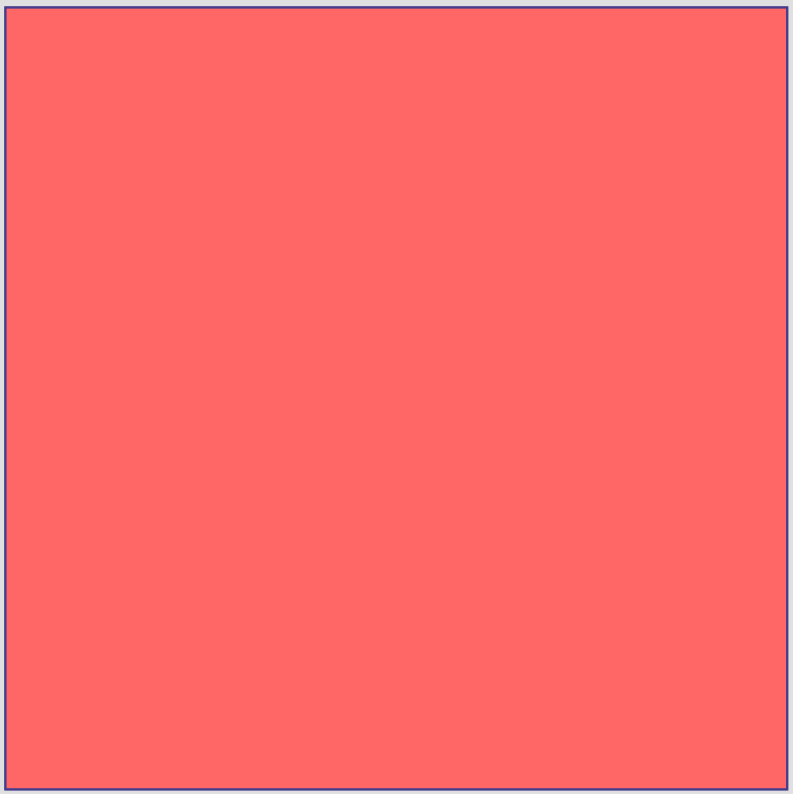
possibility 2

recaptured sample



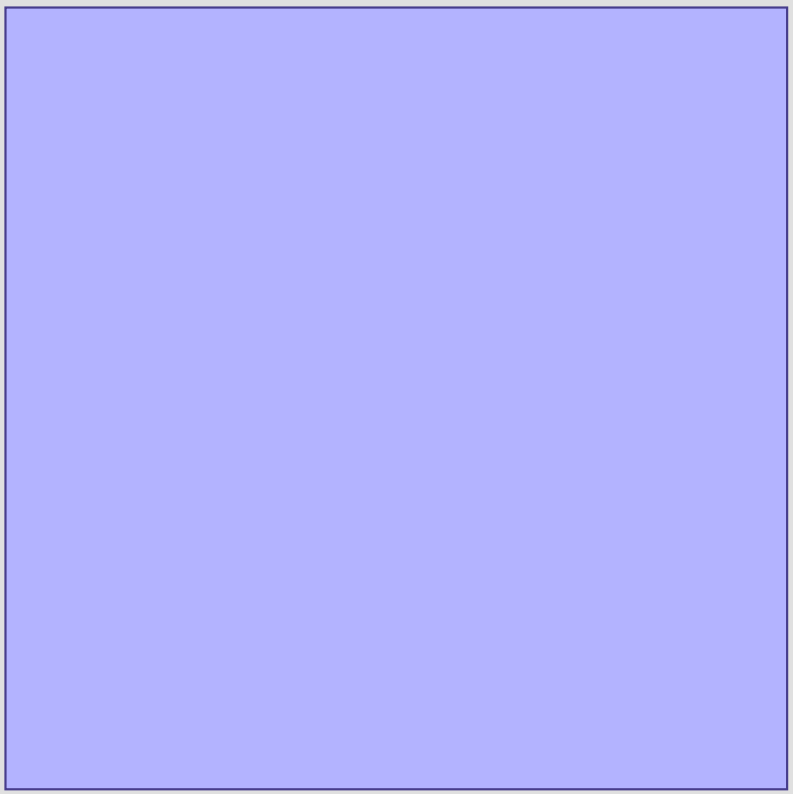
possibility 2

recaptured sample



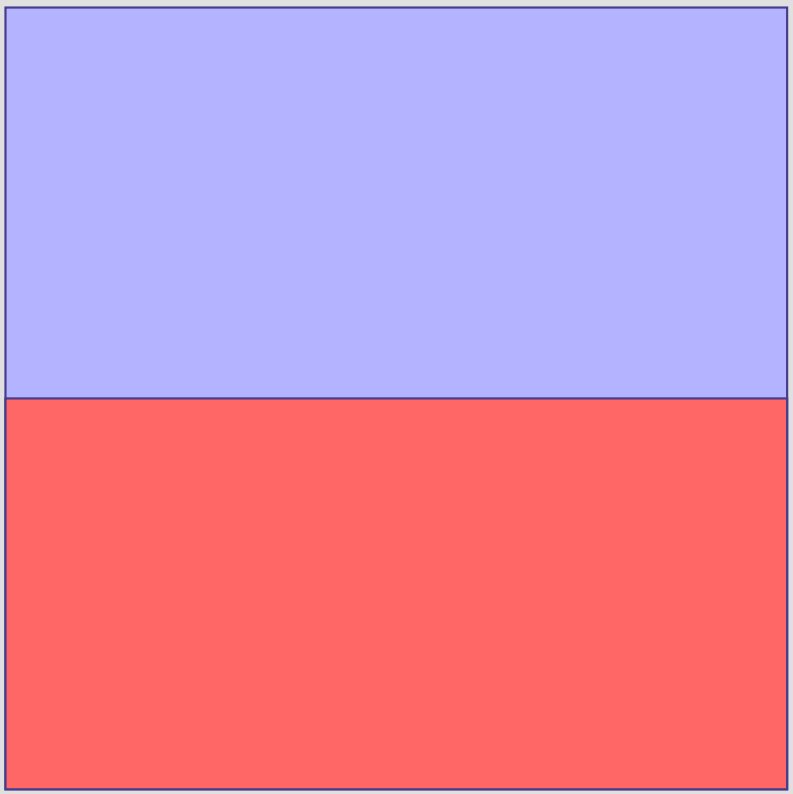
possibility 2

recaptured sample



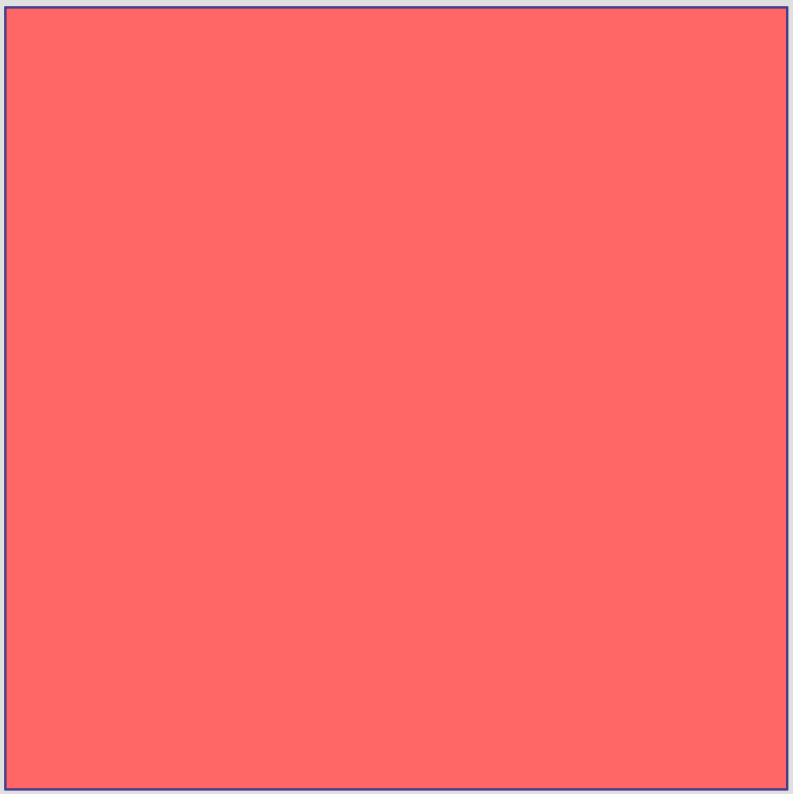
possibility 2

recaptured sample



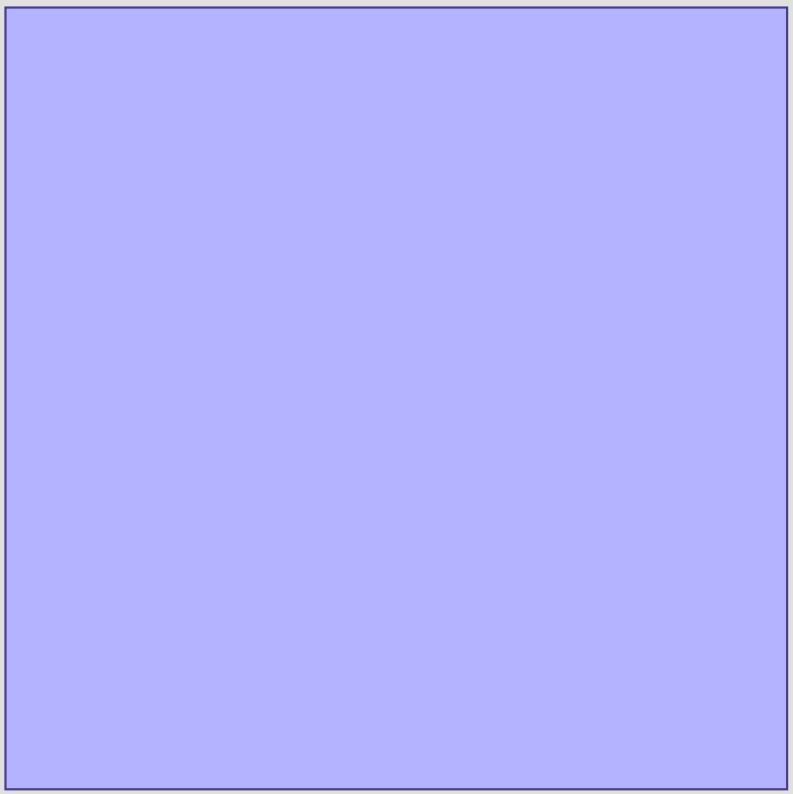
possibility 2

recaptured sample

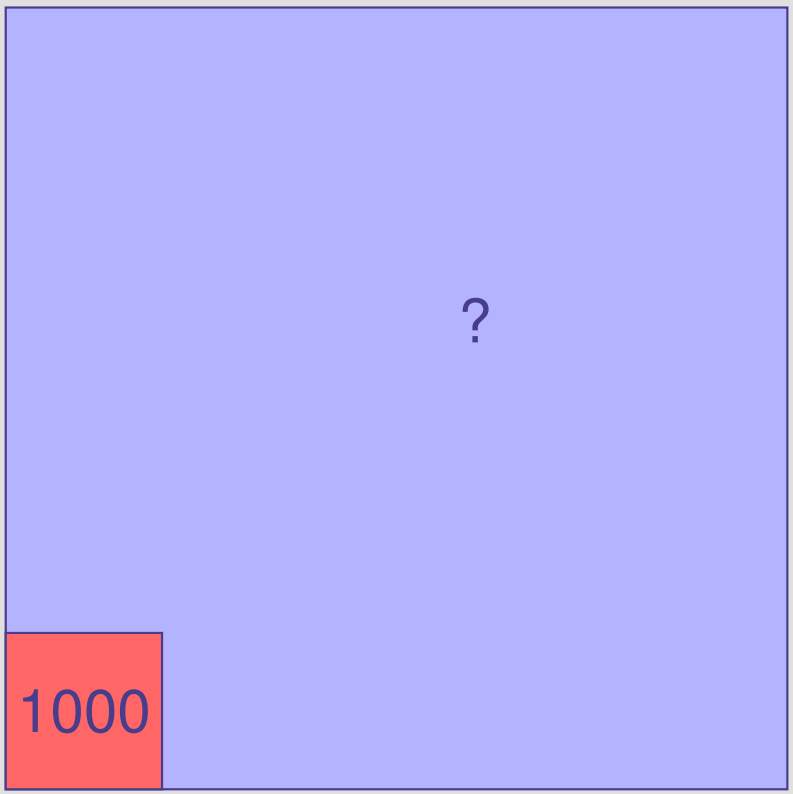


possibility 2

recaptured sample



estimated population



possibility 3

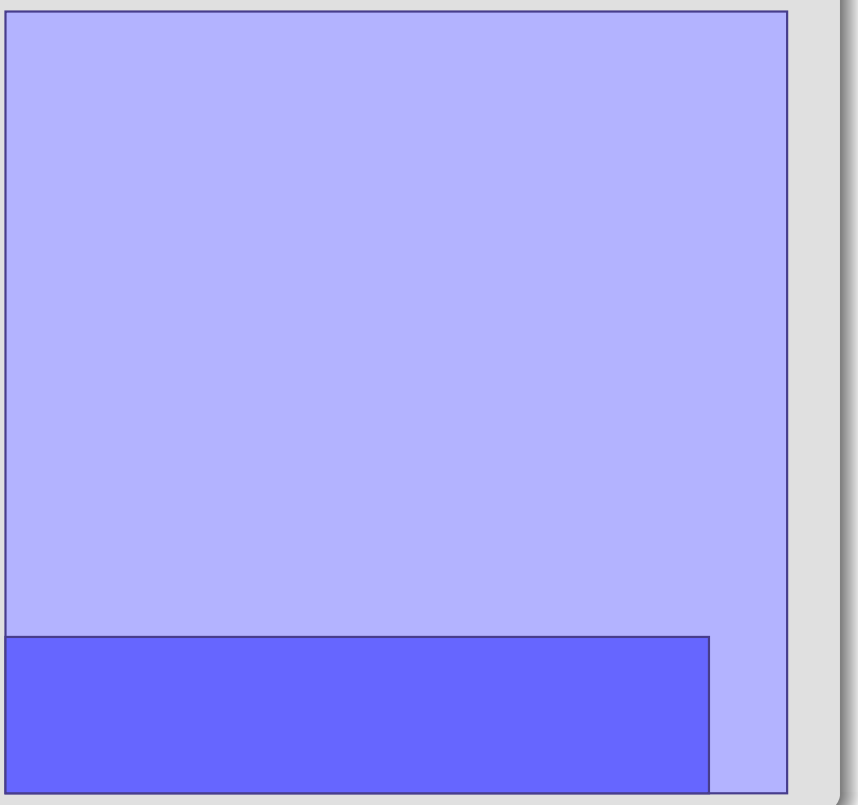
possibility 3

possibility 3

possibility 3 - trace

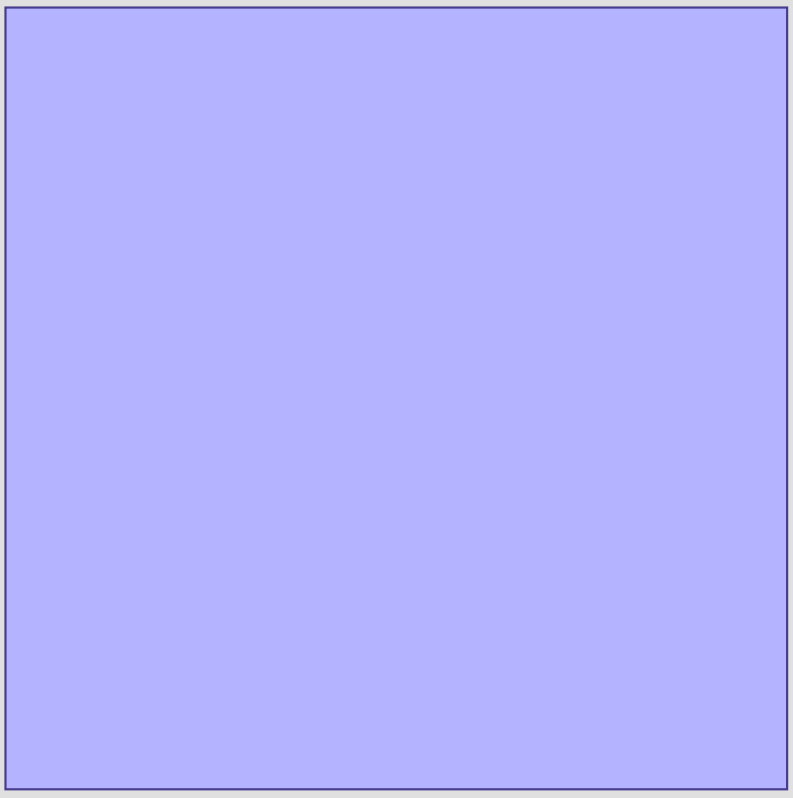
estimate

recaptured sample



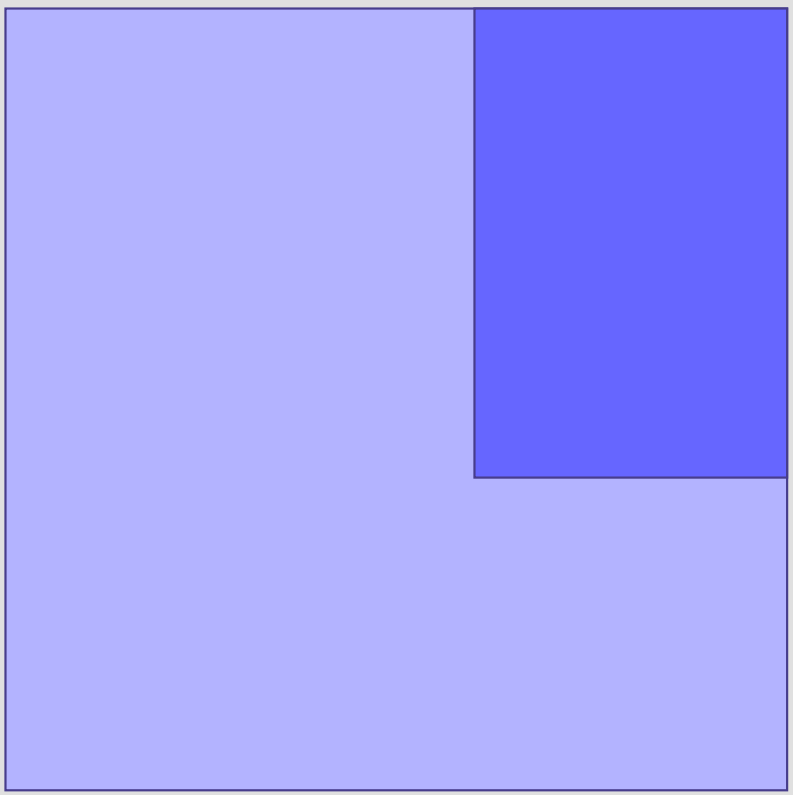
estimate

recaptured sample



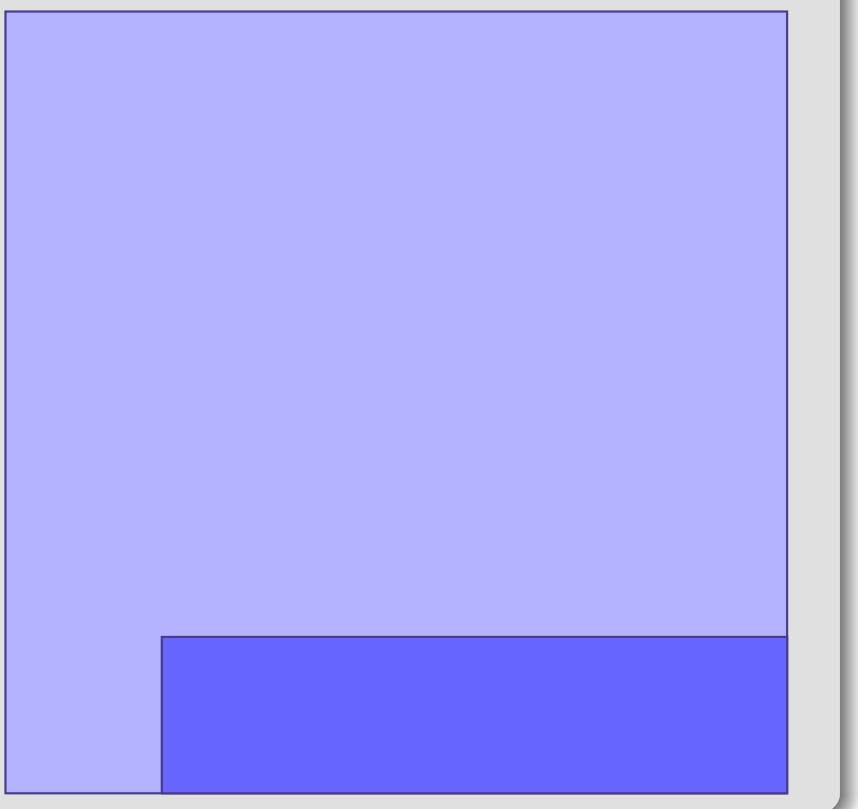
estimate

recaptured sample



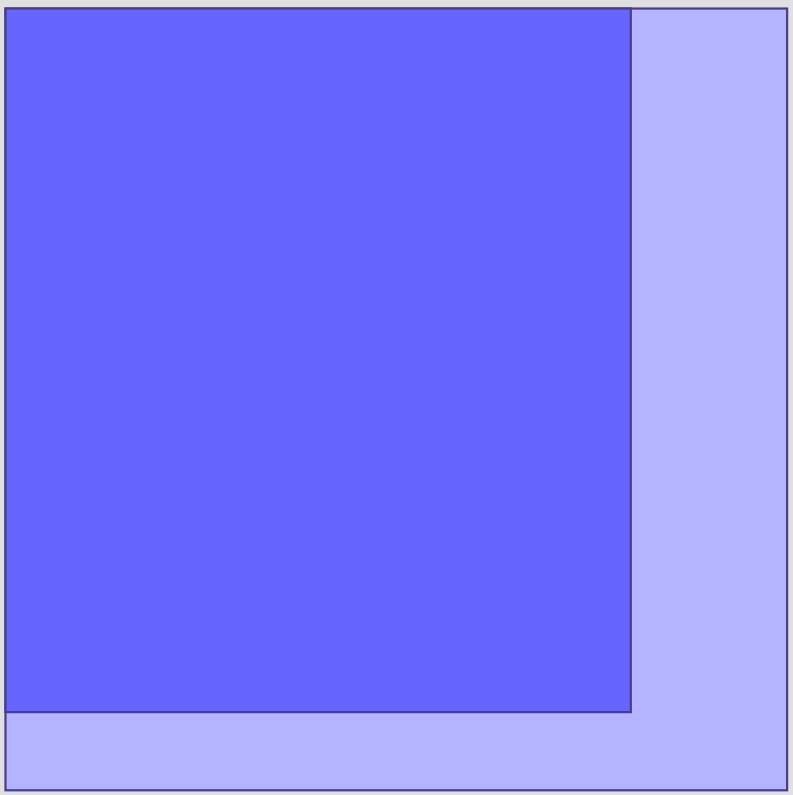
estimate

recaptured sample



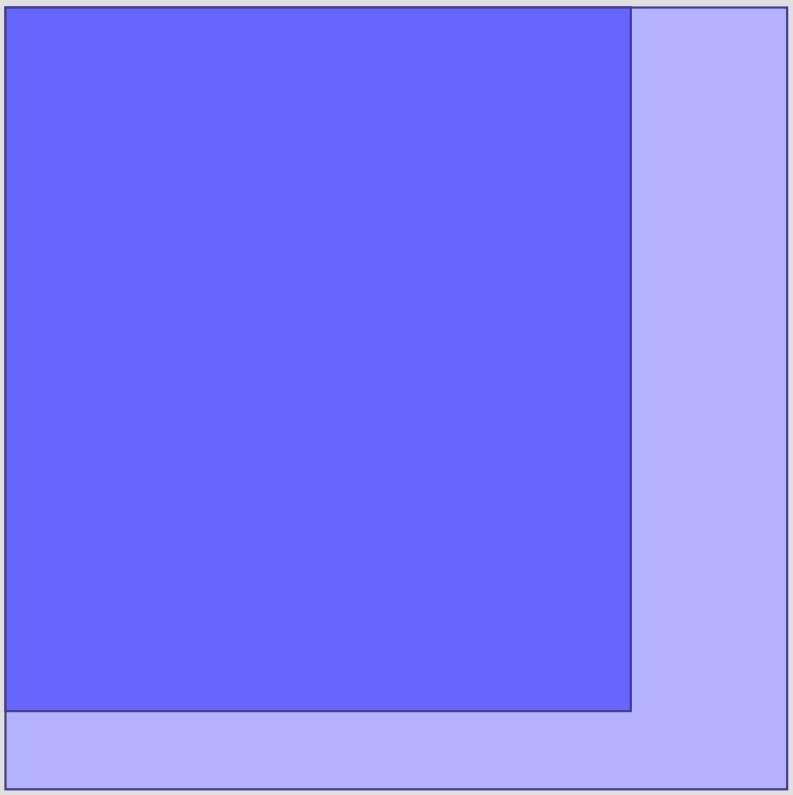
estimate

recaptured sample

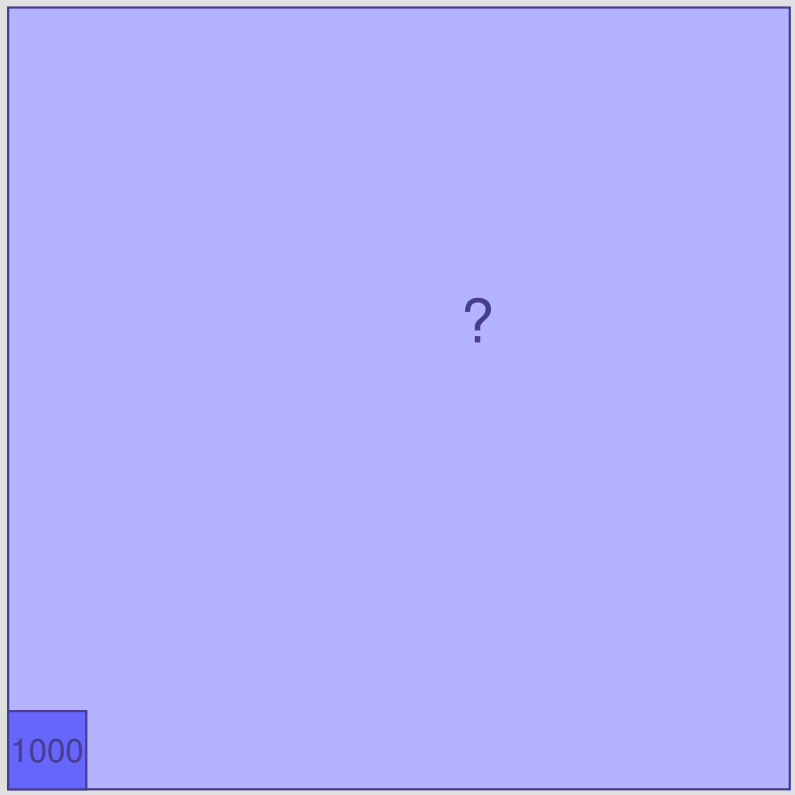


estimate

recaptured sample



estimated population

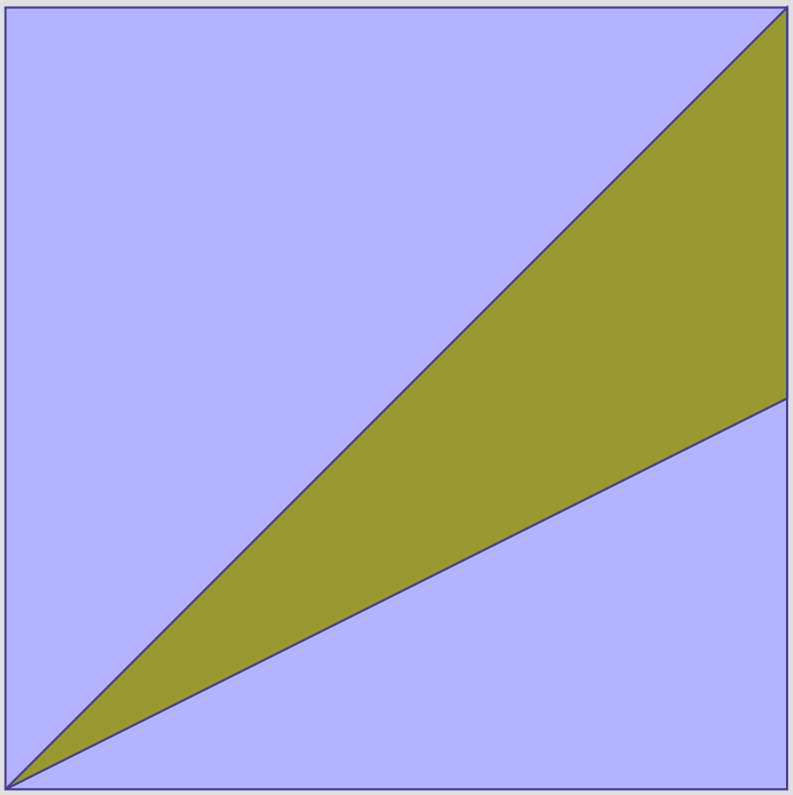


possibility 4

possibility 4

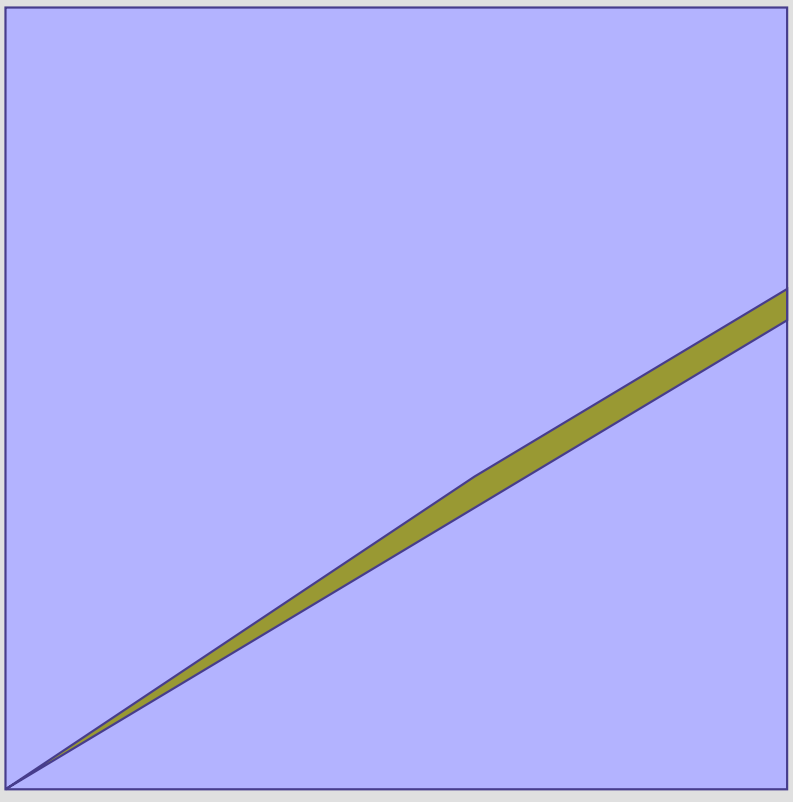
estimate - mixing situation

recaptured sample



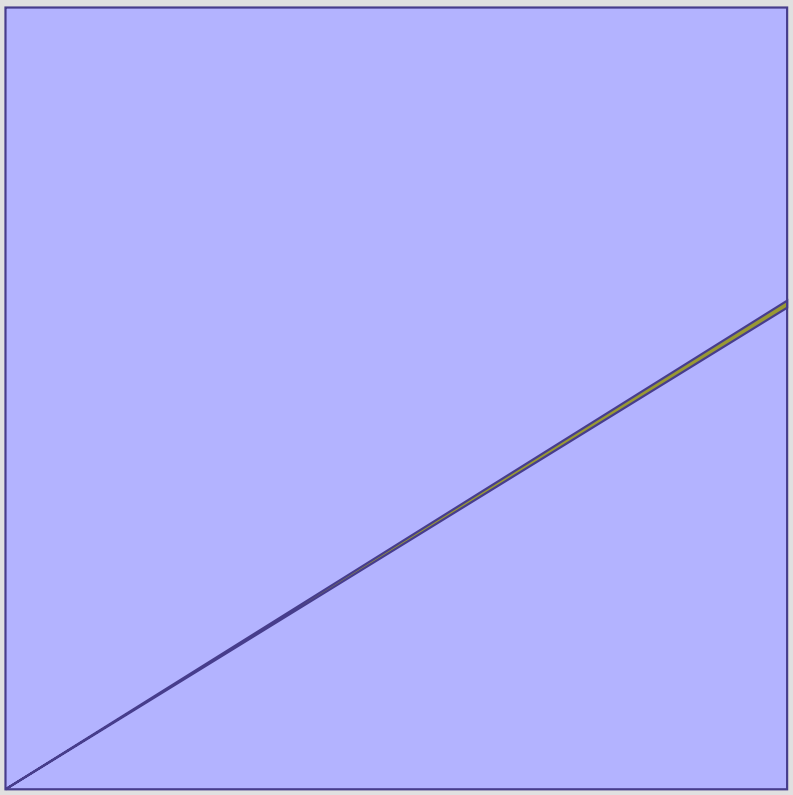
estimate - mixing situation

recaptured sample



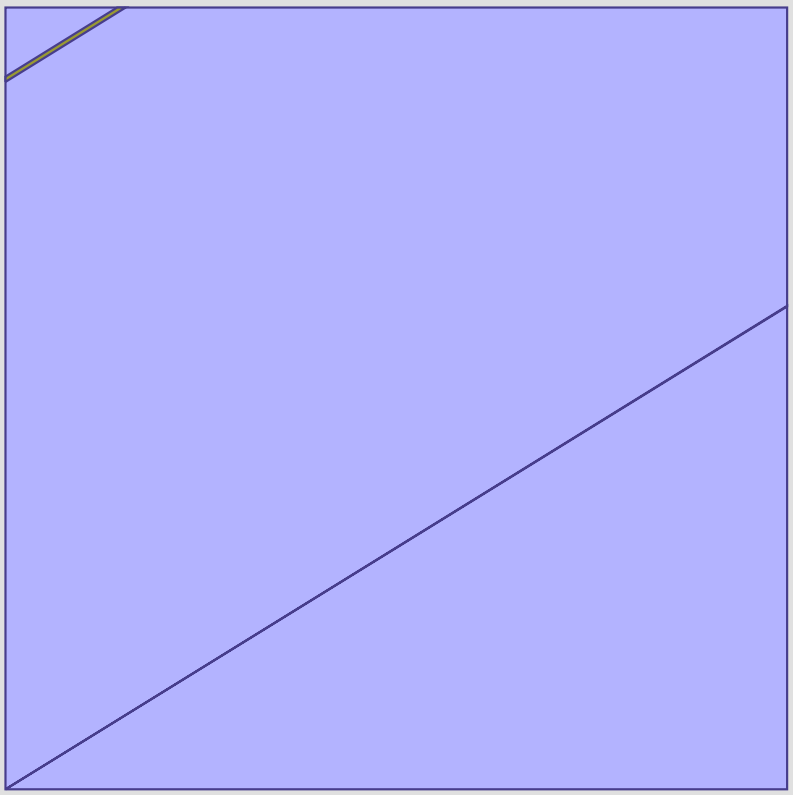
estimate - mixing situation

recaptured sample



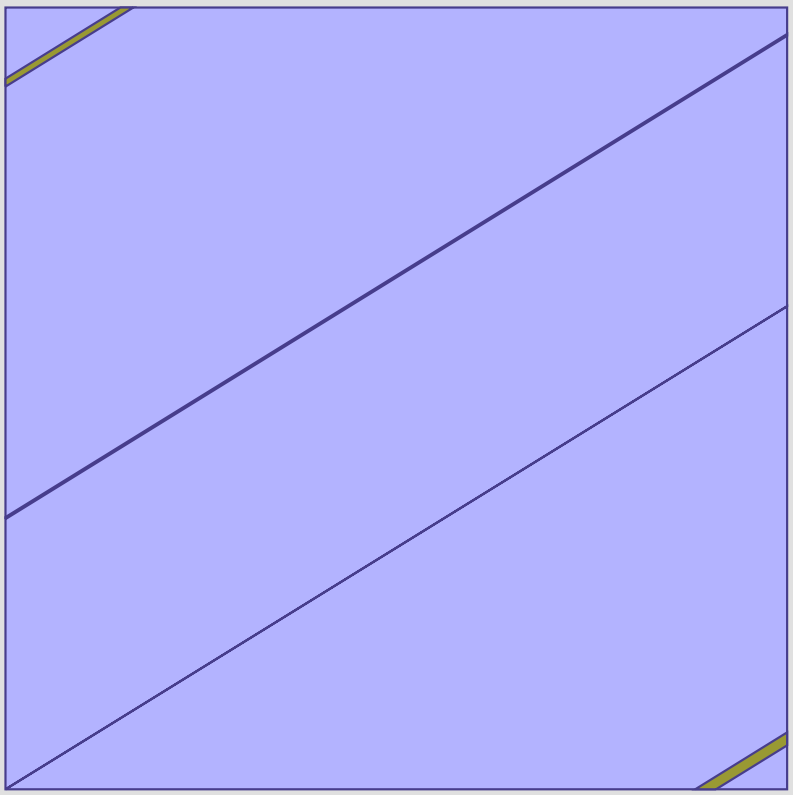
estimate - mixing situation

recaptured sample



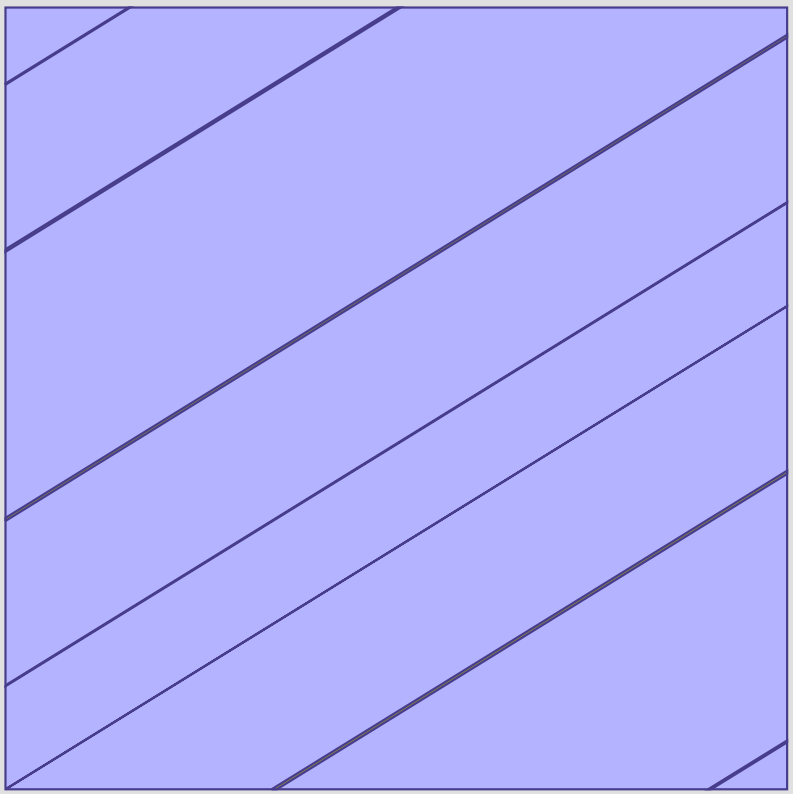
estimate - mixing situation

recaptured sample

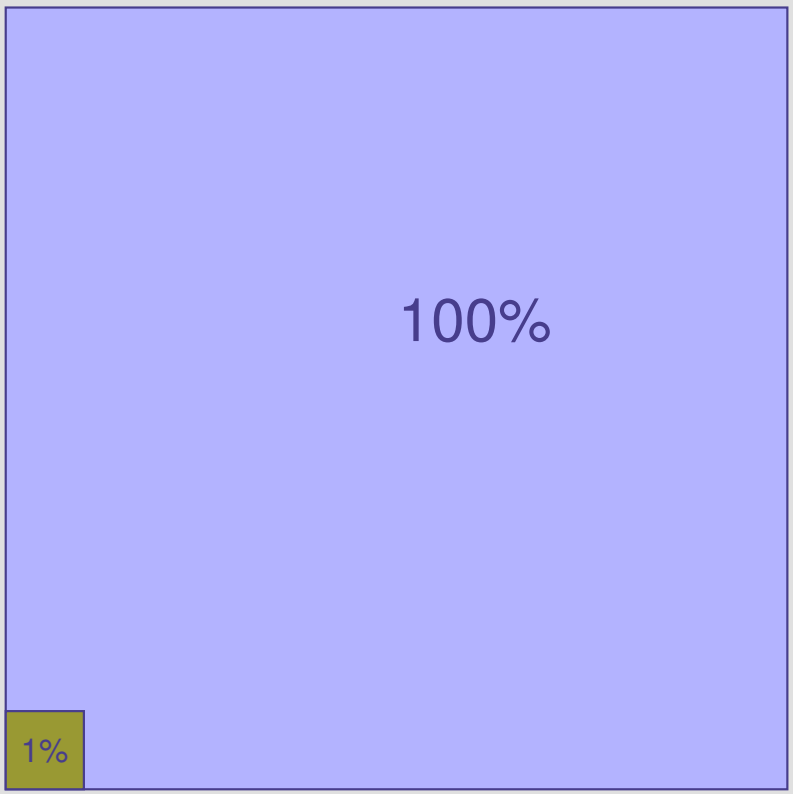


estimate - mixing situation

recaptured sample

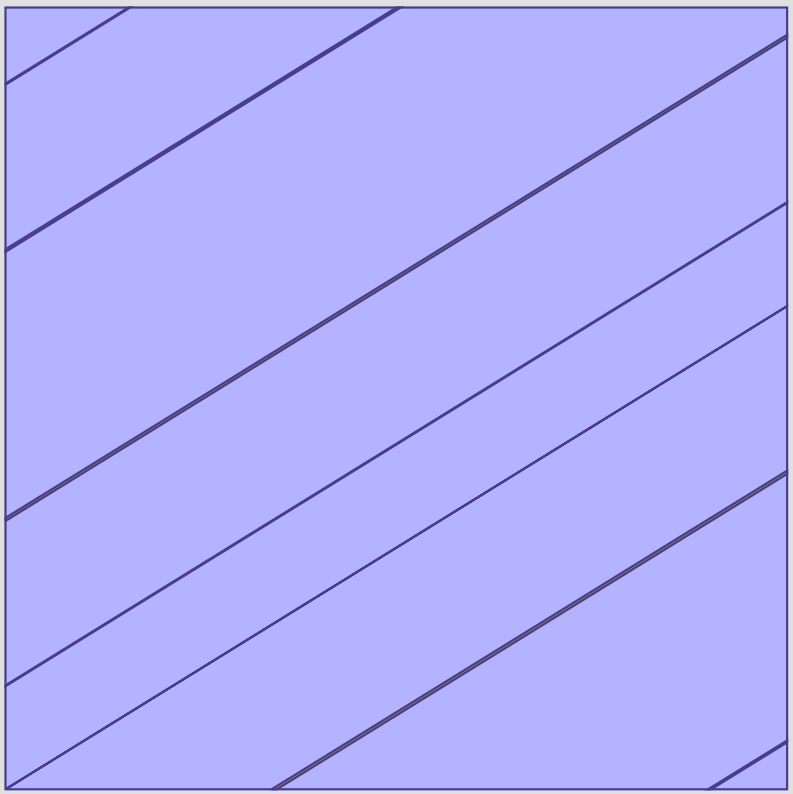


estimated population

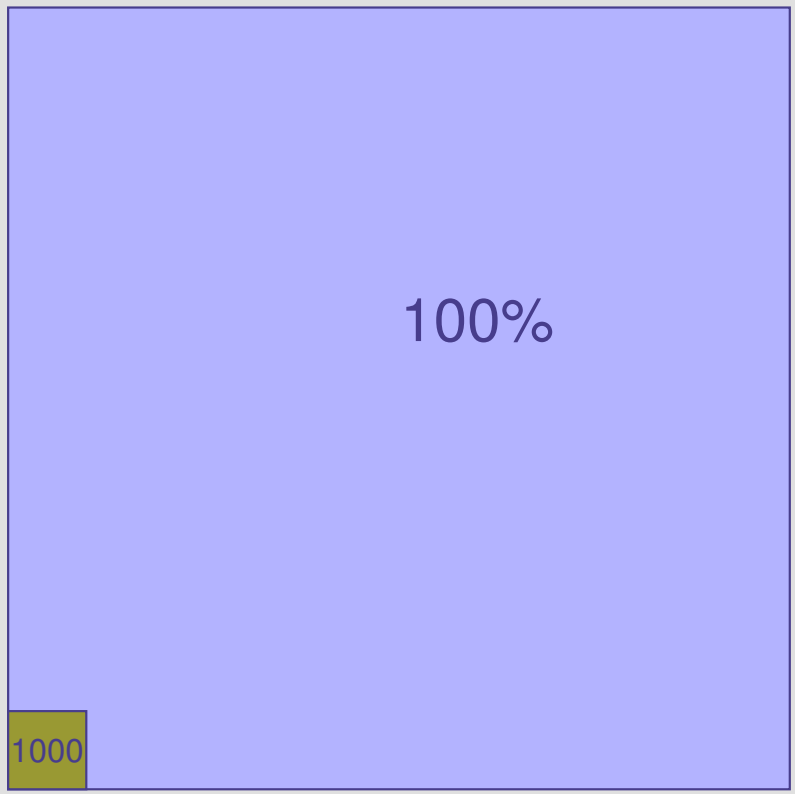


estimate - mixing situation

recaptured sample

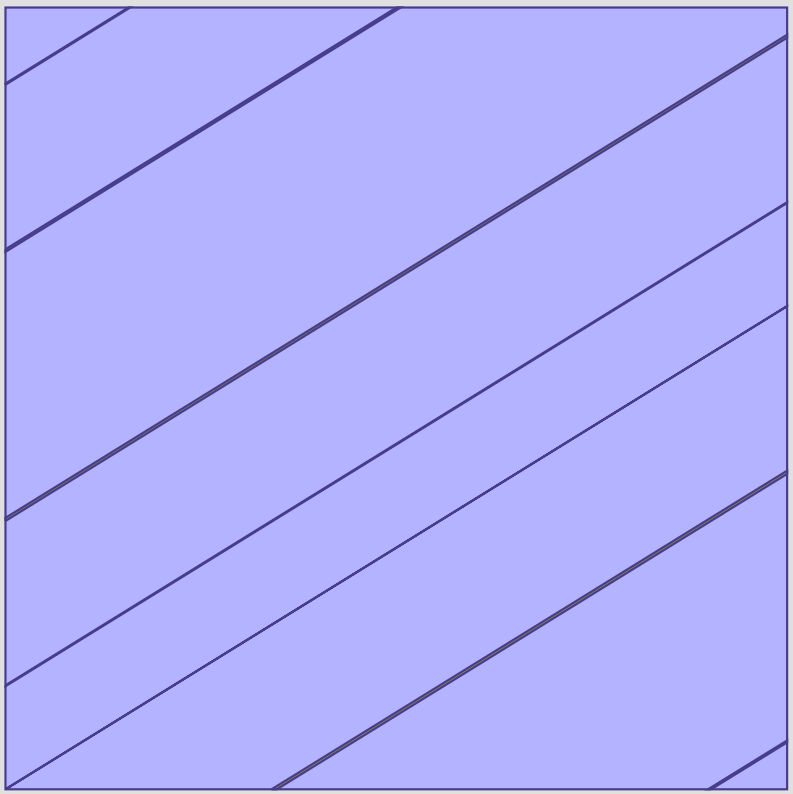


estimated population

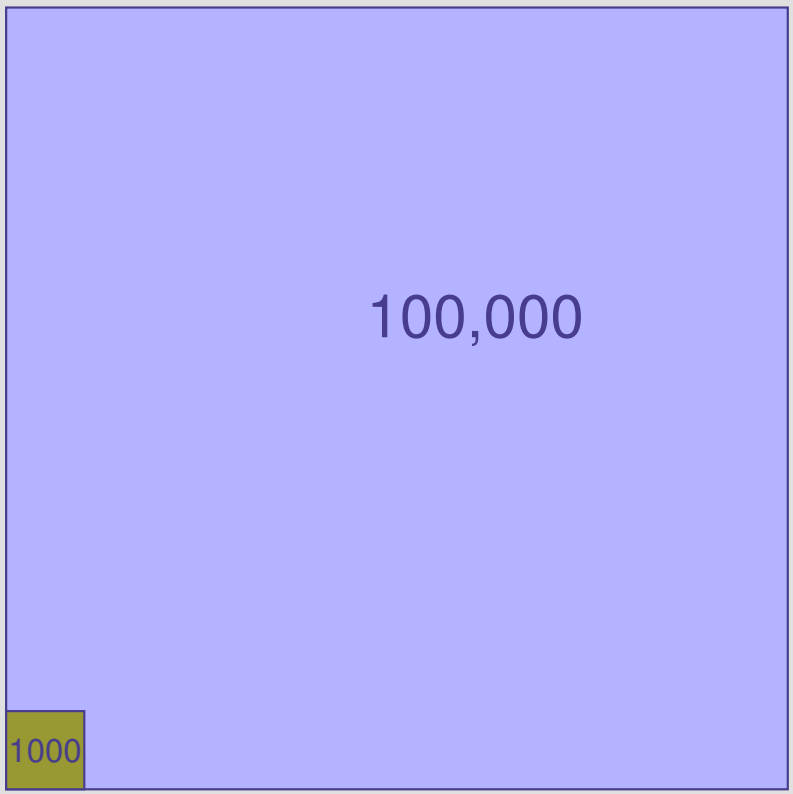


estimate - mixing situation

recaptured sample



estimated population



mixing property

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- $f : L \to L$ is mixing

mixing property

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- if every pair of samples $A, B \subset L$ satisfy

mixing property

mixing property

- $f : L \to L$ is mixing
- if every pair of samples $A, B \subset L$ satisfy
- $$m(A \cap f^n(B)) \to m(A)m(B)$$

mixing property

in our context

- if the lake population is mixing

mixing property

in our context

- if the lake population is mixing
- $A \subset L$ initial sample

mixing property

in our context

- if the lake population is mixing
- $A \subset L$ initial sample
- $$\frac{m(A \cap f^n(A))}{m(A)}$$

mixing property

in our context

- if the lake population is mixing
- $A \subset L$ initial sample
- $$\frac{m(A \cap f^n(A))}{m(A)} \to m(A)$$

mixing property

mixing **property**

mixing property

mixing **property**

$\Downarrow$

mark and recapture method works

mixing property

NO mixing property

?

mark and recapture method works

possibility 3

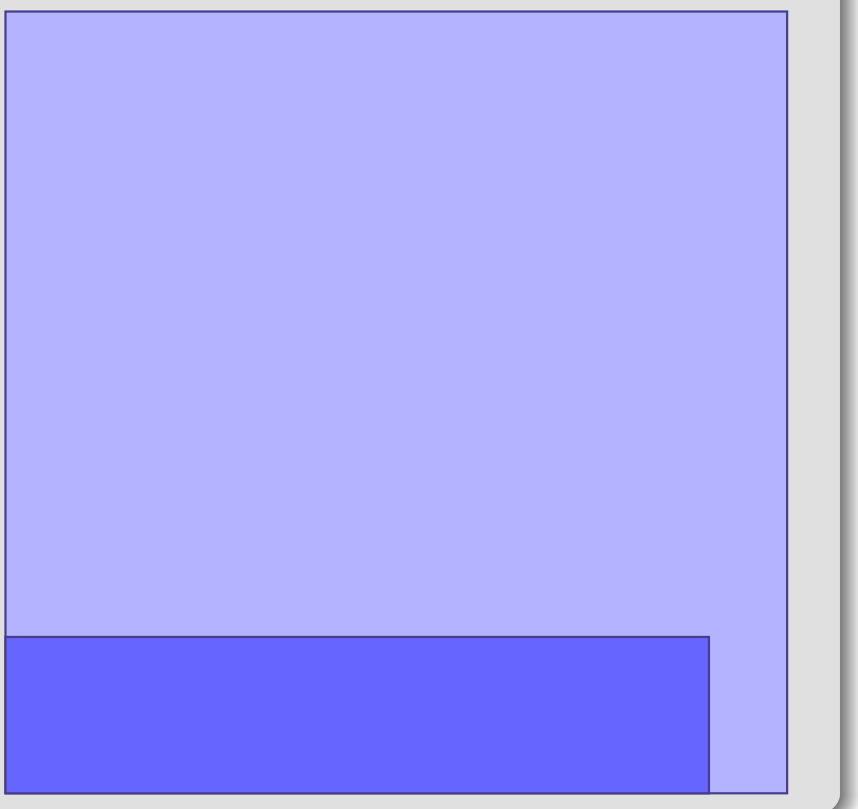
possibility 3

possibility 3

possibility 3 - trace

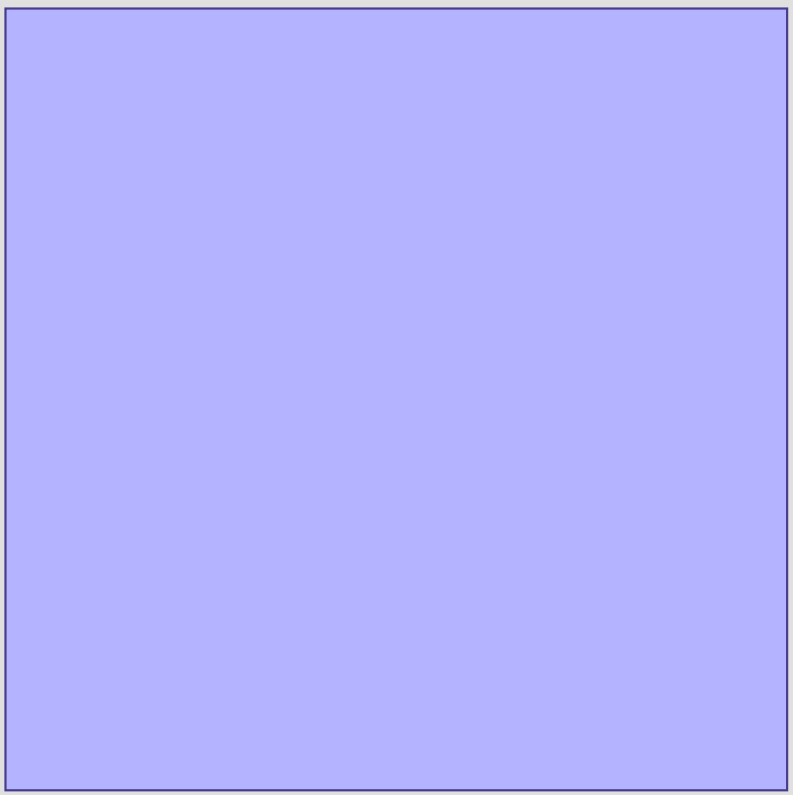
estimate

recaptured sample



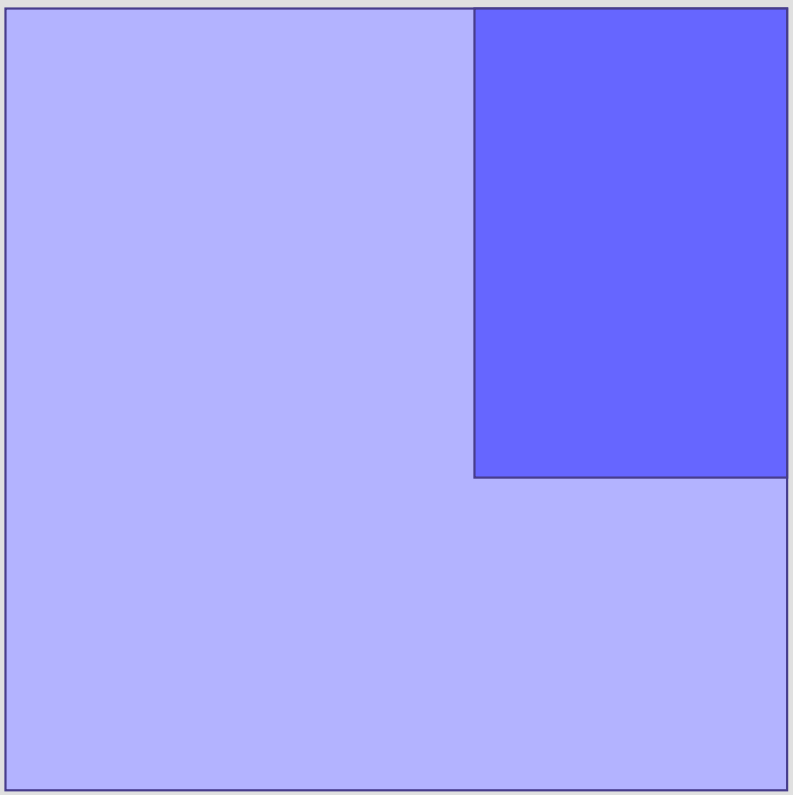
estimate

recaptured sample



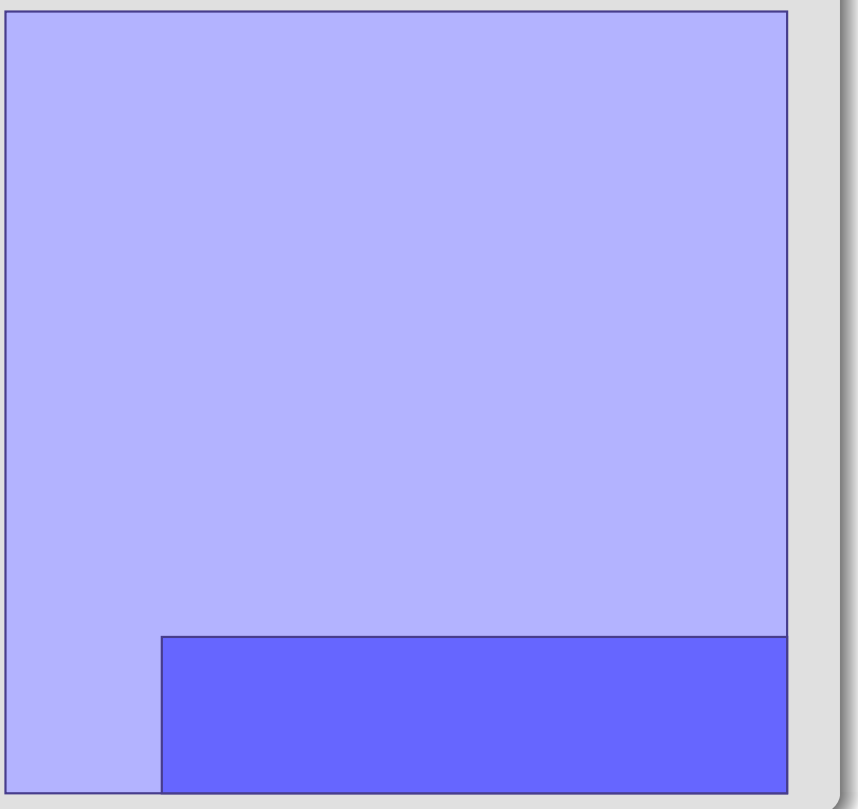
estimate

recaptured sample



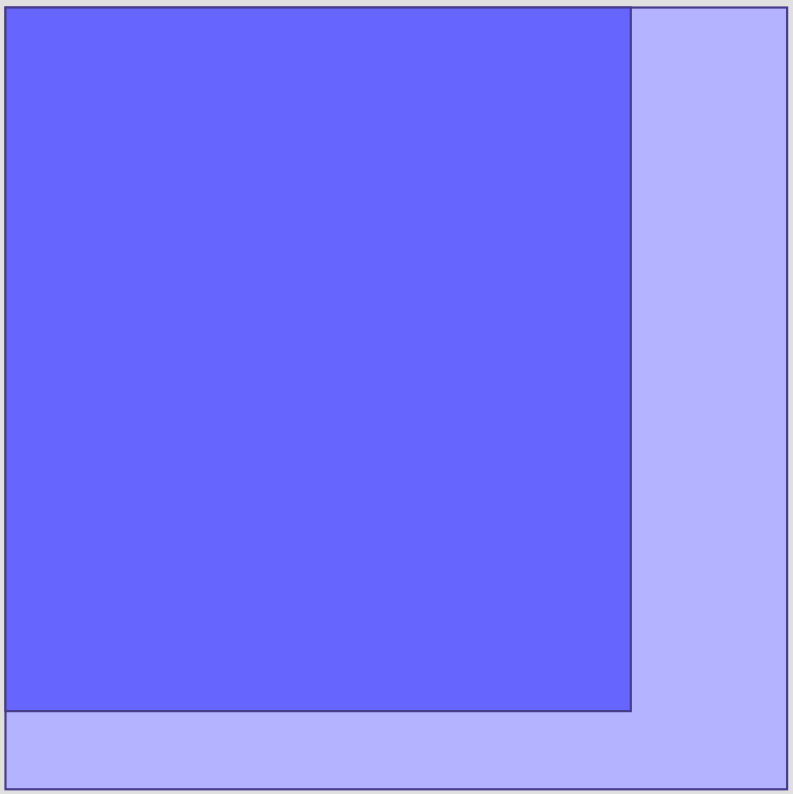
estimate

recaptured sample



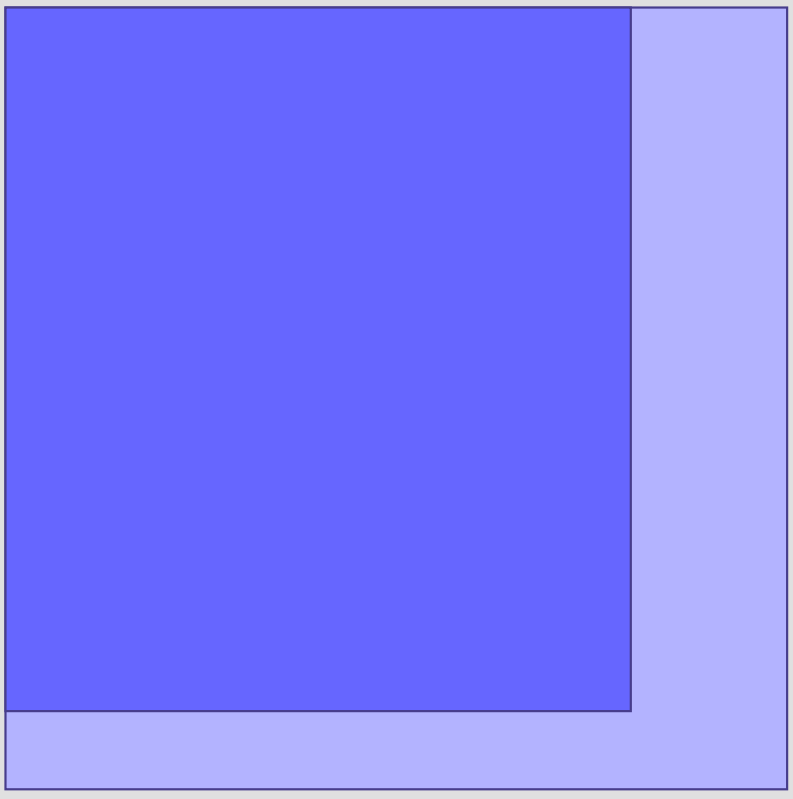
estimate

recaptured sample

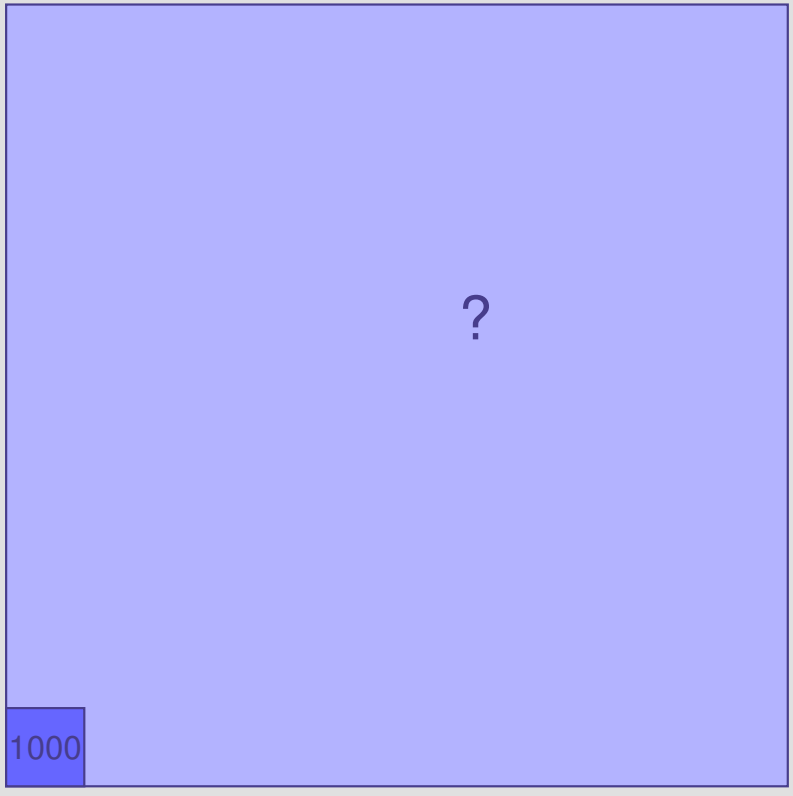


estimate

recaptured sample



estimated population



possibility 3

in this example

$$\nexists \lim \frac{m(A \cap f^n(A))}{m(A)}$$

possibility 3

in this example

$$\nexists \lim \frac{m(A \cap f^n(A))}{m(A)}$$

what about this:

$$\frac{1}{N} \sum_{n=0}^{N-1} \frac{m(A \cap f^n(A))}{m(A)} \to \text{?}$$

ergodicity

ergodicity (exercise)

- $f : L \to L$ is ergoodic

ergodicity

ergodicity (exercise)

- $f : L \to L$ is ergoodic
- if every pair of samples $A, B \subset L$ satisfy

ergodicity

ergodicity (exercise)

- $f : L \to L$ is ergoodic
- if every pair of samples $A, B \subset L$ satisfy
- $$\frac{1}{N} \sum_{n=0}^{N-1} m(A \cap f^n(B)) \to m(A)m(B)$$

ergodic property

in our context

- if the lake population is ergodic

ergodic property

in our context

- if the lake population is ergodic
- $A \subset L$ initial sample

ergodic property

in our context

- if the lake population is ergodic
- $A \subset L$ initial sample
- $$\frac{1}{N} \sum_{n=0}^{N-1} \frac{m(A \cap f^n(A))}{m(A)}$$

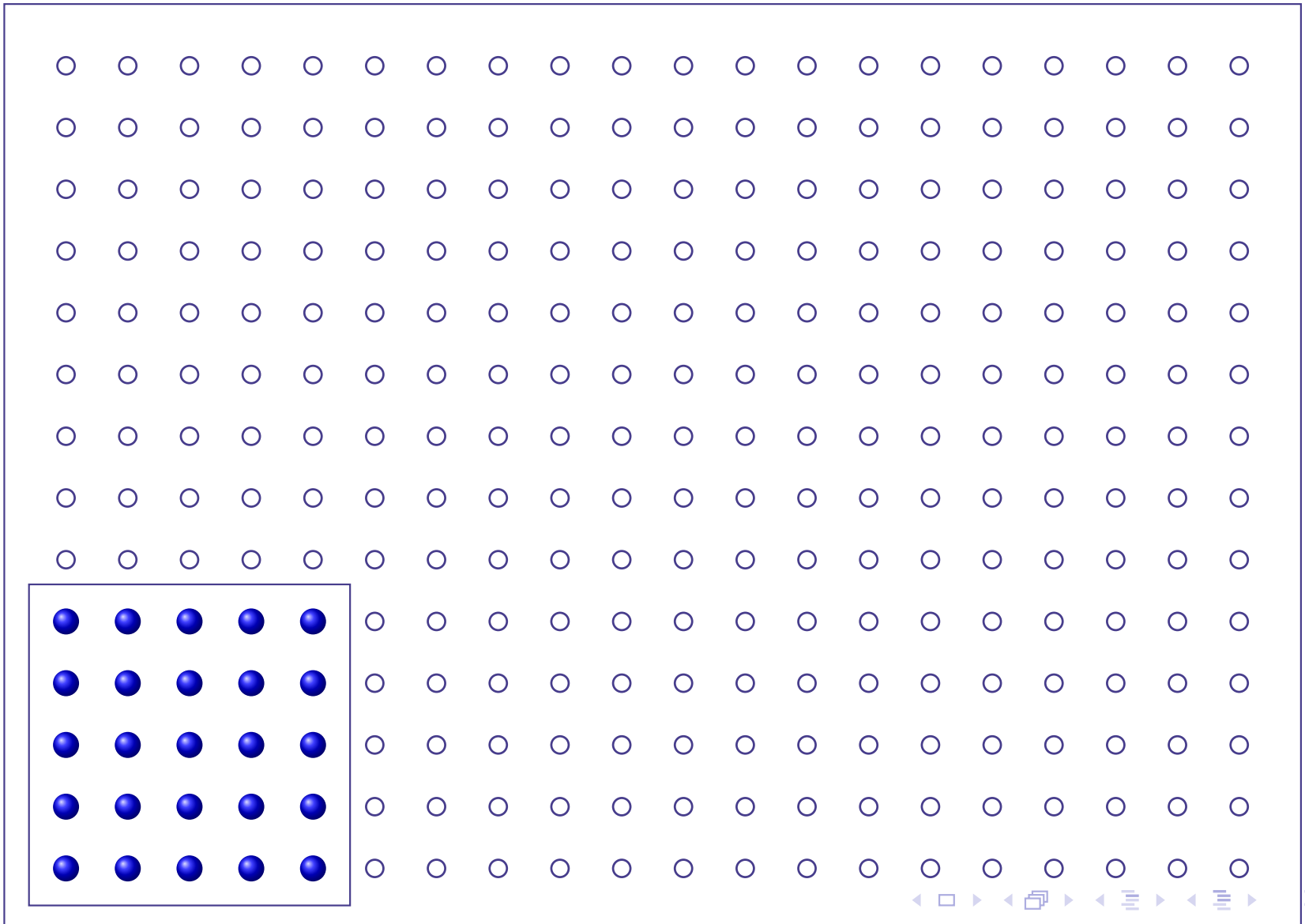
ergodic property

in our context

- if the lake population is ergodic
- $A \subset L$ initial sample
- $$\frac{1}{N}\sum_{n=0}^{N-1}\frac{m(A \cap f^n(A))}{m(A)} \to m(A)$$

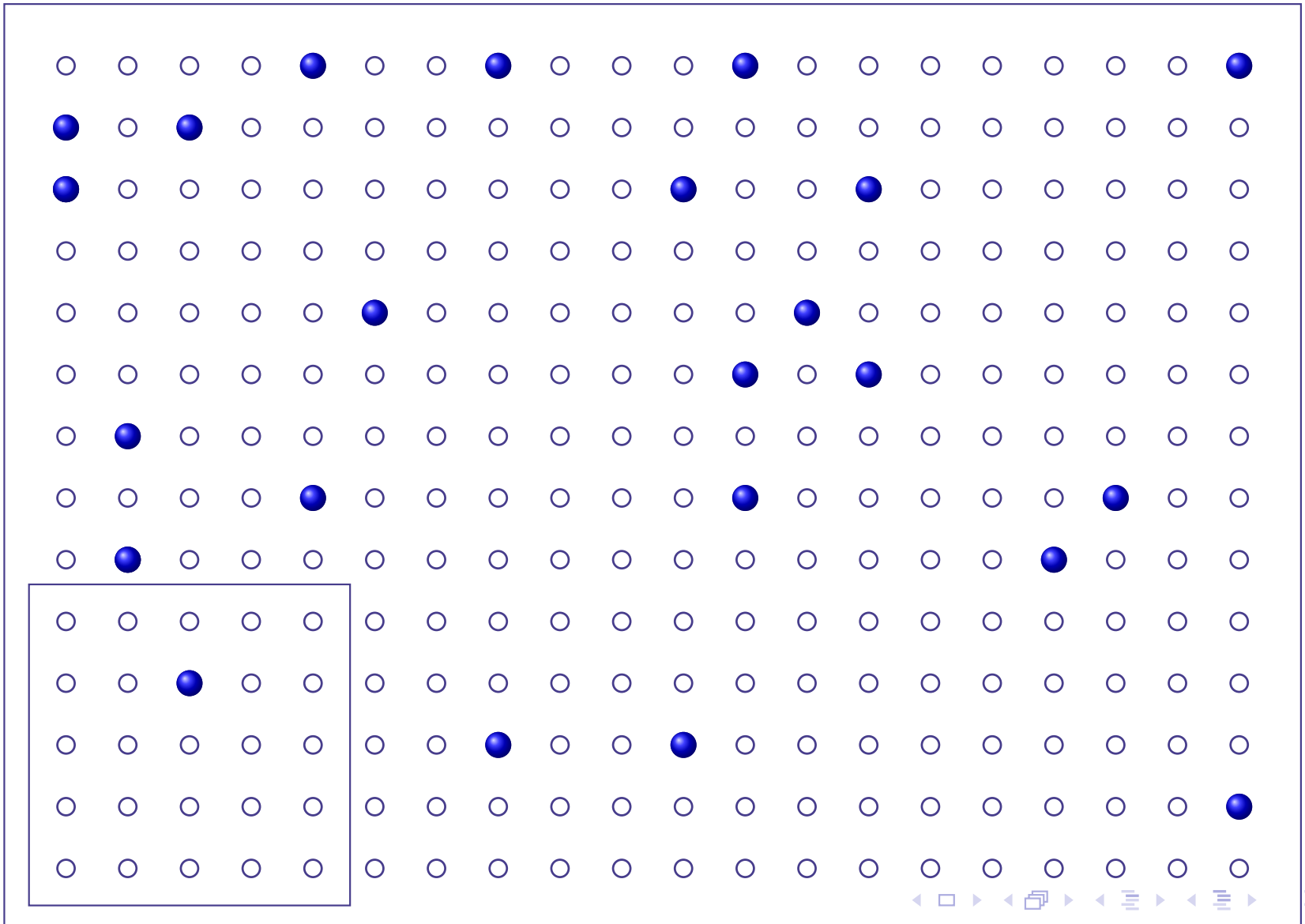
ergodicity

$$\frac{1}{N}\sum_{n=0}^{N-1}\frac{m(A\cap f^n(A))}{m(A)} \to m(A)$$

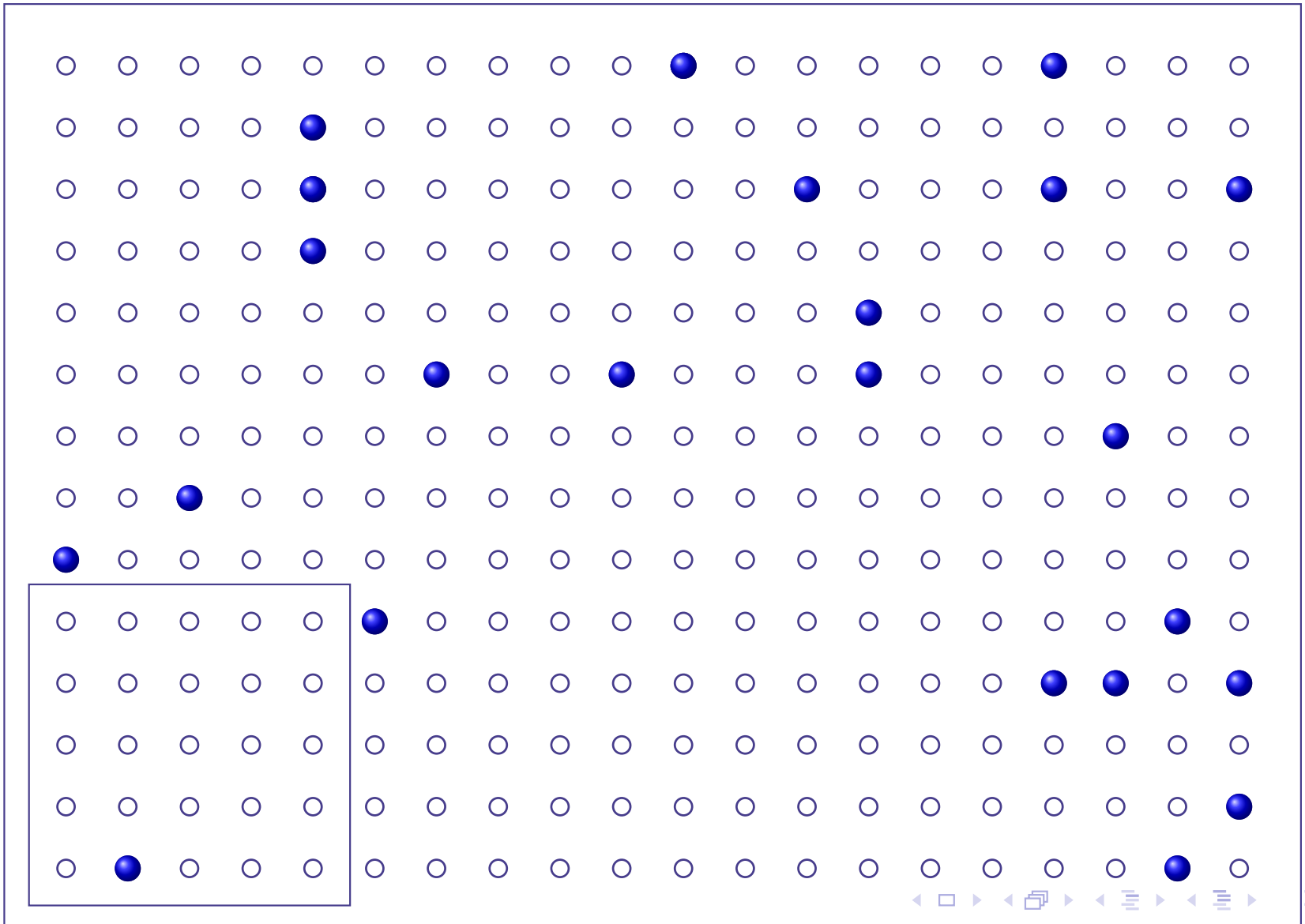


ergodicity

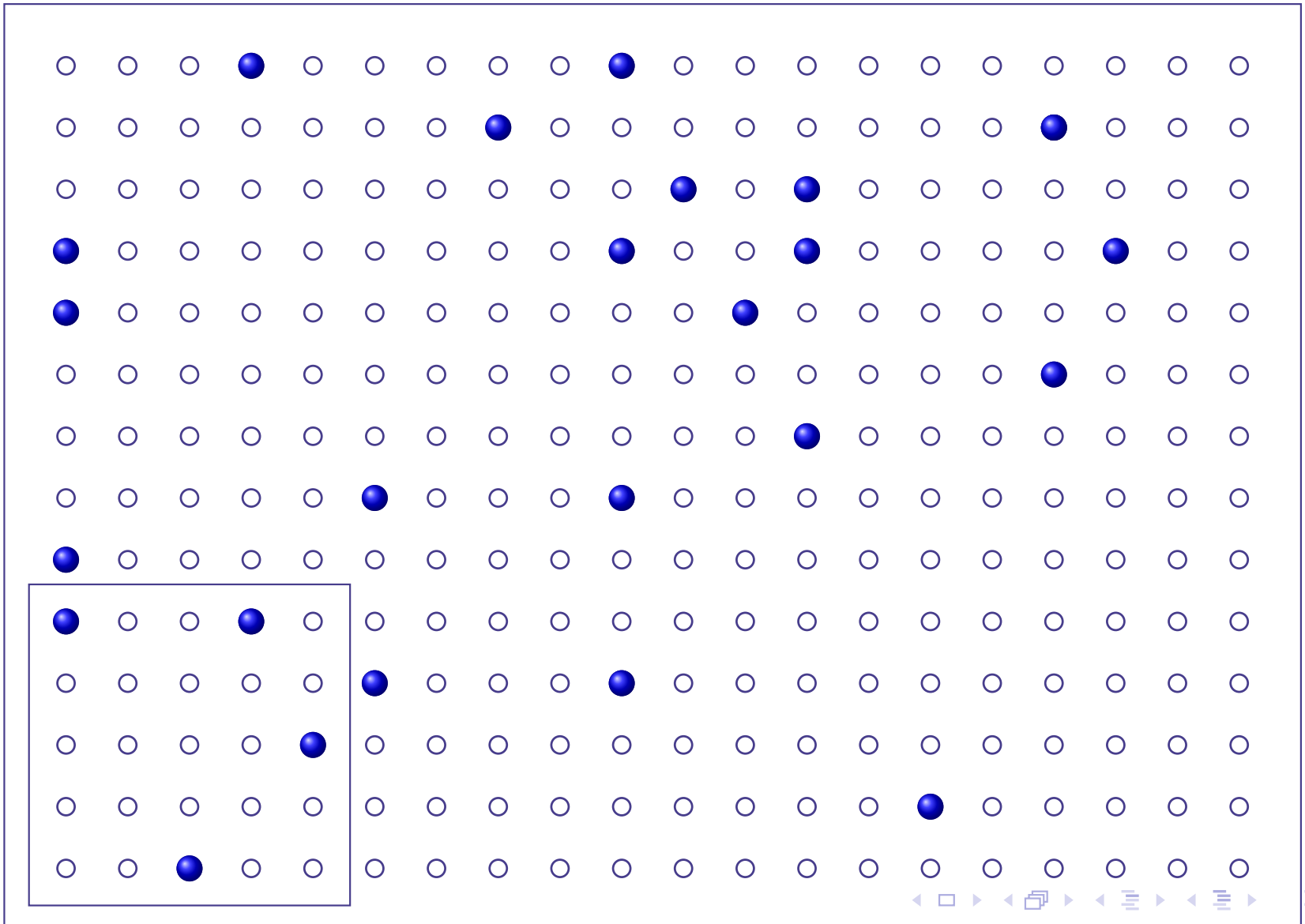
$$\frac{1}{N}\sum_{n=0}^{N-1}\frac{m(A\cap f^n(A))}{m(A)} \to m(A)$$



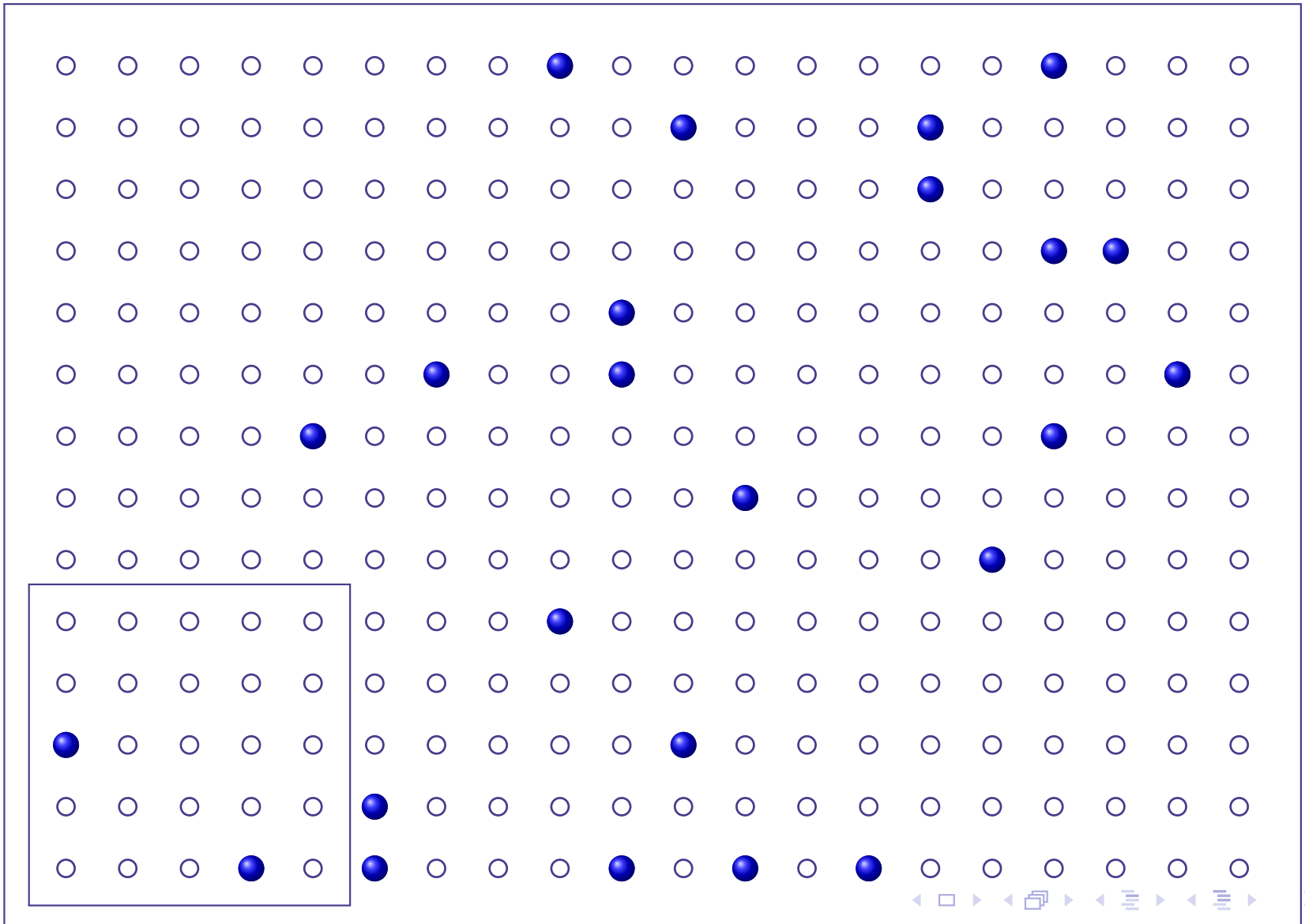
$$\frac{1}{N}\sum_{n=0}^{N-1}\frac{m(A\cap f^n(A))}{m(A)} \to m(A)$$



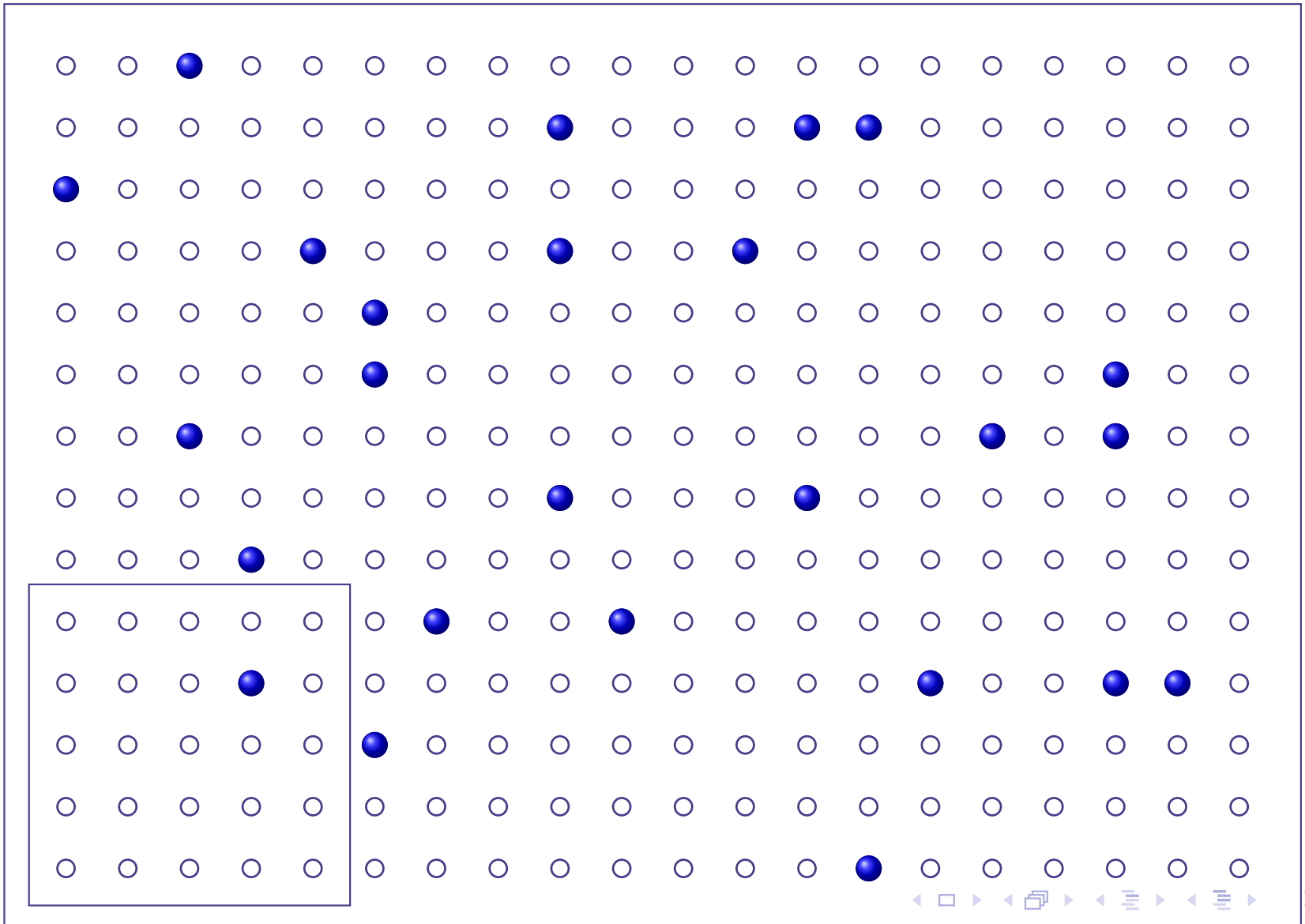
$$\frac{1}{N}\sum_{n=0}^{N-1}\frac{m(A\cap f^n(A))}{m(A)} \to m(A)$$



$$\frac{1}{N}\sum_{n=0}^{N-1}\frac{m(A\cap f^n(A))}{m(A)} \to m(A)$$

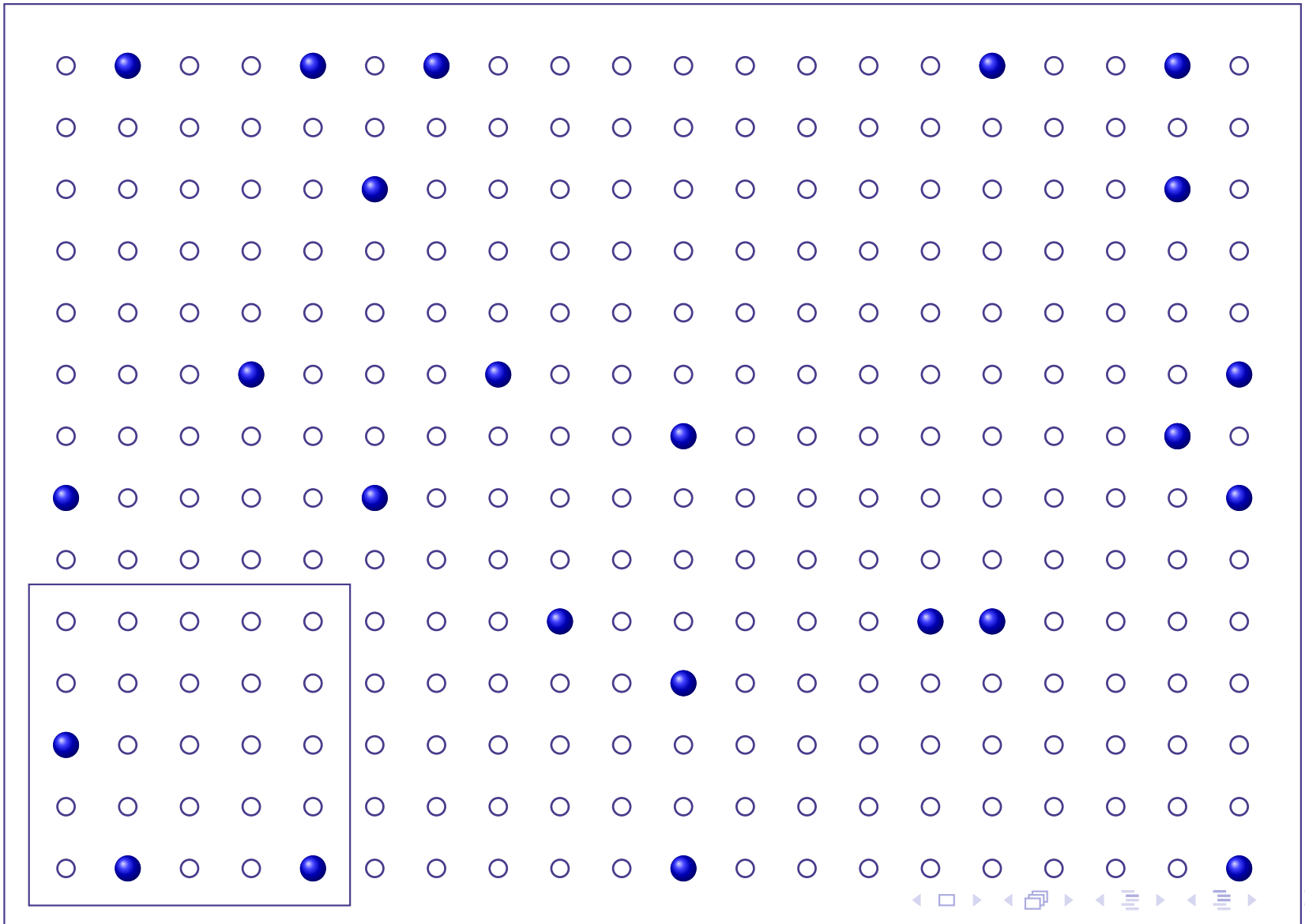


$$\frac{1}{N}\sum_{n=0}^{N-1}\frac{m(A\cap f^n(A))}{m(A)} \to m(A)$$



ergodicity

$$\frac{1}{N}\sum_{n=0}^{N-1}\frac{m(A\cap f^n(A))}{m(A)} \to m(A)$$



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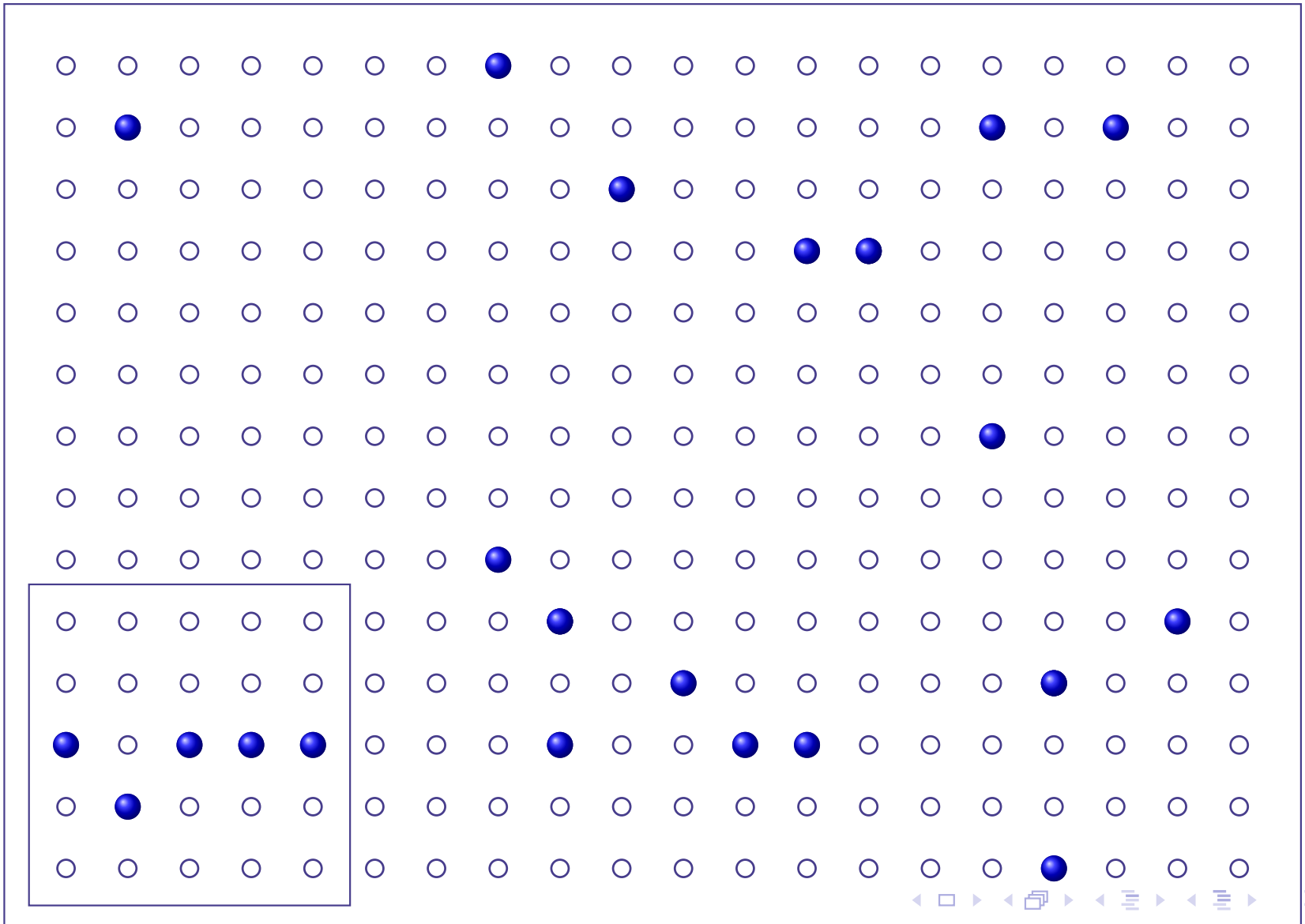
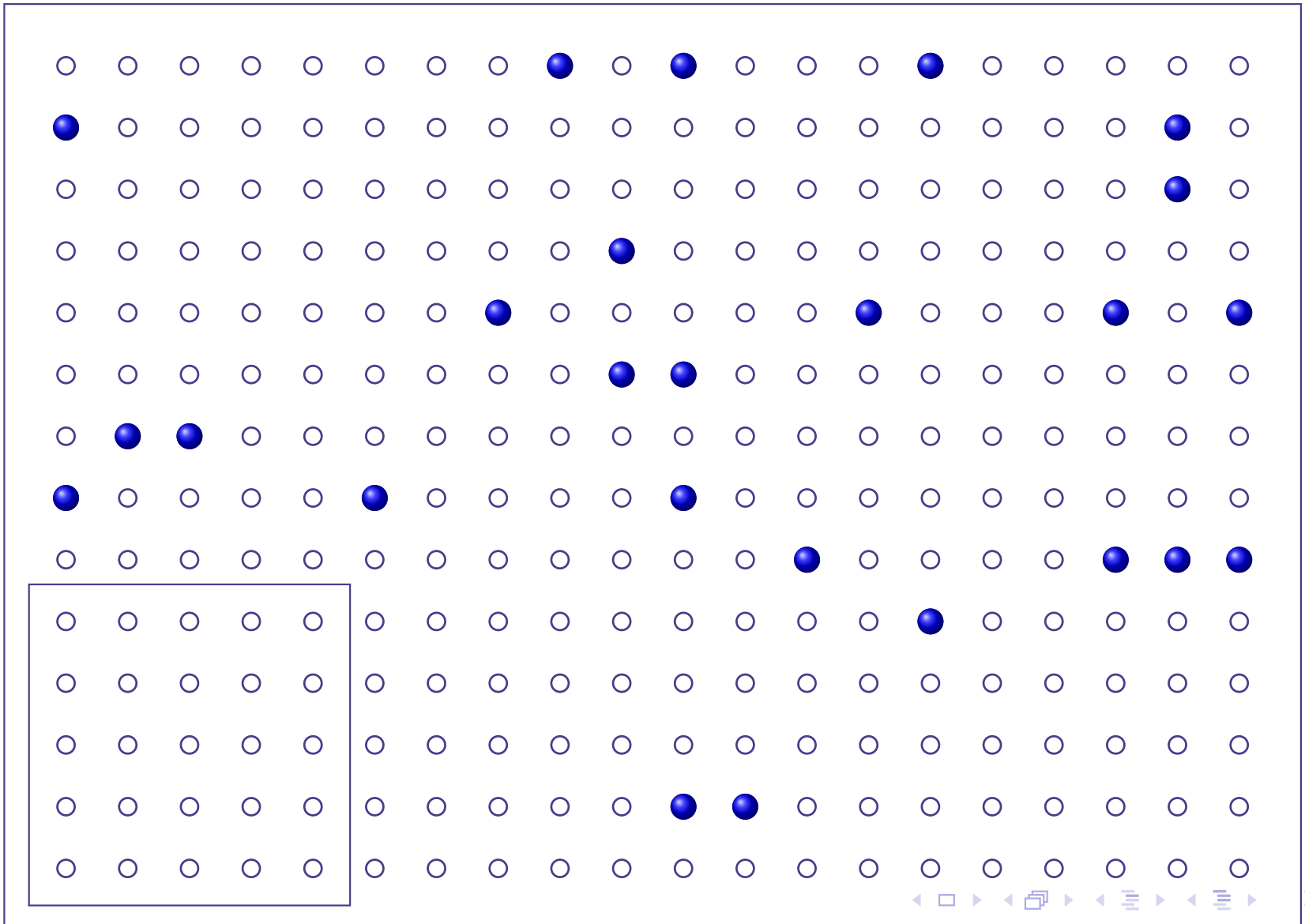
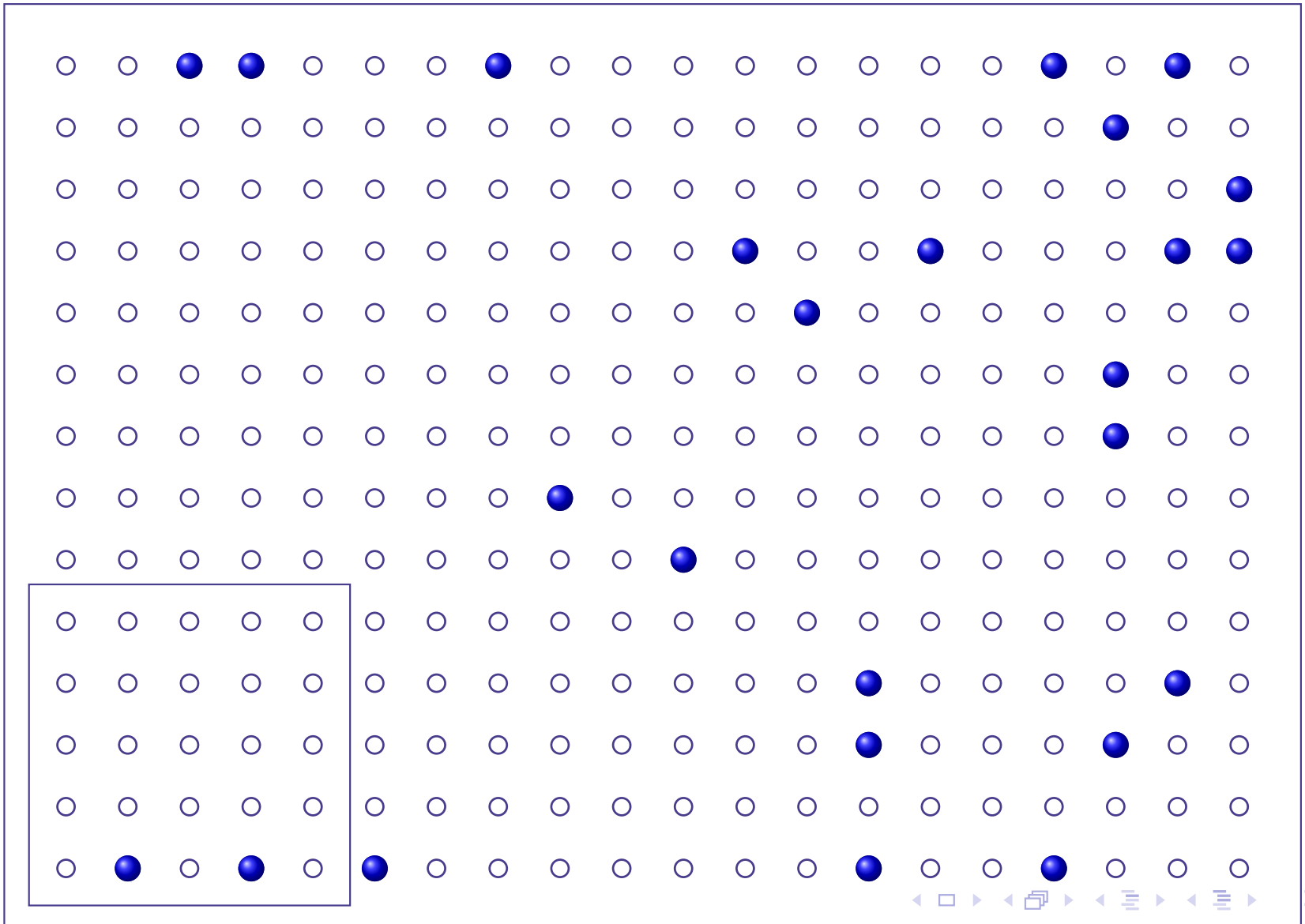


$$\frac{1}{N}\sum_{n=0}^{N-1}\frac{m(A\cap f^n(A))}{m(A)} \to m(A)$$



ergodicity

$$\frac{1}{N}\sum_{n=0}^{N-1}\frac{m(A\cap f^n(A))}{m(A)} \to m(A)\bigstar$$



exercise

exercise

exercise

exercise

- show that f ergodic $\Leftrightarrow$

exercise

exercise

- show that f ergodic $\Leftrightarrow$
- $$m\left(\bigcup_{n\geq 0} f^n(A)\right) = 1 \qquad \forall A \text{ s.t. } m(A) > 0$$

fragility of ergodicity

irrational traslation on $\mathbb{T}^2$

fragility of ergodicity

rational traslation on $\mathbb{T}^2$

stable ergodicity

stable ergodicity

stable ergodicity

stable ergodicity

- f is stably ergodic

stable ergodicity

stable ergodicity

- f is stably ergodic
- if

$$g \sim f \quad \Rightarrow \quad g \text{ ergodic}$$

open problem

open problem

stably ergodic $\Rightarrow$ mixing ?

thank you

thank you

thank you!