RECAP OF MEASURE THEORY

Exercise 1

(1) Let $X = \mathbb{N}$ and let $\mathcal{A} = \{A \subset X : A \text{ or } A^c \text{ is finite}\}$. Define

$$\mu(A) = \begin{cases} 1 & \text{if } A^c \text{ is finite} \\ 0 & \text{if } A \text{ is finite} \end{cases}.$$

Is $\mathcal{A}$ an algebra? Is $\mathcal{A}$ a σ-algebra? Is this function additive? Is it countable additive?

Exercise 2

(1) Show that there are subsets of $\mathbb{R}$ which are not Lebesgue measurable (hint: consider an irrational circle rotation, choose a single point on each distinct orbit)
(2) Show that there are Lebesgue measurable sets which are not Borel measurable (hint: recall the Cantor function, modify it to make it invertible, consider some preimage)