ERGODICITY - EXERCISES

Exercise 1. Show that the doubling map, $T : \mathbb{R}/\mathbb{Z} \to \mathbb{R} /\mathbb{Z}$, $T(x) = 2x \mod 1$, is ergodic with respect to the Lebesgue measure. Try to show this in two different ways using various characterizations of ergodicity from lecture.

(1) Method 1. Show that invariant sets have measure 0 or 1. (**Hint:** Use approximation by dyadic intervals.)
(2) Method 2. Show using Fourier series that any T-invariant L^2 function is constant a.e. (**Hint:** Riemann-Lebesgue Lemma.)

Exercise 2. Show that the full shift is ergodic with respect to the invariant measure defined in lecture. (If I don't cover it in lecture I will also ask you to show that the measure is σ-invariant.)