KHADIM WAR EXERCISES

Exercise 1: Recall that $f : \mathbb{T}^2 \to \mathbb{T}^2$ is transitive if for every pair of nonempty open subsets U, V of $\mathbb{T}^2$, there exists $n > 0$ such that

$$f^{-n}U \cap V \neq \emptyset.$$

Prove that ergodicity with respect to volume implies transitivity.

Exercise 2:

a) Prove that transitivity is equivalent to the fact that there exists $x_0 \in \mathbb{T}^2$ such that

$$\overline{\{f^n(x_0), n \geq 1\}} = \mathbb{T}^2.$$

b) f is said to be mixing with respect to the volume measure μ if for every $A, B \in \mathcal{B}$ we have

$$\mu(A \cap f^{-n}B) \to \mu(A) \cdot \mu(B) \text{ as } n \to \infty.$$

Prove that if f is mixing then for every subsequence of integrers $\phi : \mathbb{N} \to \mathbb{N}$, there exists $x_0 \in \mathbb{T}^2$ such that

$$\overline{\{f^{\phi(n)}(x_0), n \geq 1\}} = \mathbb{T}^2.$$

Exercise 3: Let A be an $n \times n$ integer matrix with $det(A) = \pm 1$. Prove $f_A : \mathbb{T}^n \to \mathbb{T}^n$ is ergodic with respect to volume if and only if no eigenvalue of A is a root of unity.