Exercises Jana

(1a) Let $\varphi \in C^0(\mathbb{T}^2)$, let G_φ be the set of "good points"

$$G_\varphi = \{ x \in \mathbb{T}^2 ; \ \varphi^+(x) = \varphi^-(x) \}$$

Prove that $m(G_\varphi) = 1$

(1b) Let f be ergodic. Prove that

$$E(\varphi \mid \mathcal{B}_f)(x) = \int \varphi \, d\mu$$

$$\forall \ \varphi \in C^0(X, \mathcal{B}, \mu)$$

(2) Let $\varphi \in C^0(\mathbb{T}^2)$. Prove that

$$\varphi^+(x) = \varphi^+(y) \quad \forall y \in W^s(x)$$

$$\varphi^-(x) = \varphi^-(y) \quad \forall y \in W^u(x)$$

Exercises Jana (cont)

3 If $f: \mathbb{T}^2 \to \mathbb{T}^2$ ergodic, then $\forall \varphi \in C^0(\mathbb{T}^2)$ $\exists G_\varphi$ s.t. $m(G_\varphi) = 1$ and

$$\varphi^+(x) = \int \varphi \, d\mu \quad \forall x \in G_\varphi .$$

(this follows from Birkhoff and 1b)

Prove that there exists G with $m(G) = 1$ such that $\forall x \in G$ $\forall \varphi \in C^0(\mathbb{T}^2)$

$$\varphi^+(x) = \int \varphi \, d\mu$$

Hint: Remember there exists a countable set of functions $\varphi_n \in C^0(\mathbb{T}^2)$ that are dense in $C^0(\mathbb{T}^2)$ (Stone - Weierstrass)