

1. For a message encoded with the 2×2 parity check, write down an example of
 - (a) a double error that will not be miscorrected.
 - (b) a double error which may be misinterpreted as a single error, and hence miscorrected.
 (You could draw the message in a 2-dimensional grid, but if you prefer a message explicitly encoded with a generator matrix, that would work as well.)
2. If a linear code has length 1024 and rank 1000,
 - (a) What are the dimensions of a generator matrix?
 - (b) What are the dimensions of a check matrix (with a minimal number of rows)?
3.
 - (a) Write down a generator matrix for the 3×3 parity check code.
 - (b) Write down a check matrix for this code.
 - (c) Is the string $v = 111101111111111$ a code word? (The answer should be 'no'.)
 - (d) What is the error syndrome for v ?
 - (e) What codeword is closest to v ?
 - (f) Modify the check matrix to make it slightly bigger and increase the minimum distance by 1.
4. (a) Prove the following statement.

If H is a check matrix for a binary code with minimal distance d , where d is odd, then the matrix

$$H' = \begin{pmatrix} & & & & 0 \\ & & & & 0 \\ & & H & & \vdots \\ & & & & 0 \\ 1 & 1 & \cdots & 1 & 1 \end{pmatrix},$$

is a check matrix for a code with minimum distance $d + 1$.

- (b) Can you modify this construction for a code over a general finite field \mathbb{F}_q ?

The following two exercises involve a bit a counting, or at least the first one does and the second one can. We haven't done this sort of counting in lectures, but we will do a bit in week two, so it's good to be able to do these problems.

5. How large a table would have to be used to do syndrome-table based error correction for up to 5 errors for a linear code with length 200?
6. Prove that the $(2^k - 1, 2^k - k - 1)$ binary Hamming code has the property that every vector in $v \in \mathbb{F}_2^{2^k - 1}$ is either a codeword or else is distance exactly one away from a codeword. (Find the exercise in the course notes if you need some hints, but I suggest that you try without any hints first.)