Gromov hyperbolic groups and their boundaries – TCC course Spring 2012, updated 2nd Feb.

The following exercises are collected from the lectures.

Question 1. Show that the relation of being quasi-isometric $(X \stackrel{q.i.}{\simeq} Y)$ is an equivalence relation.

Question 2. Show that if G is a group with two finite generating sets S and S', then the corresponding Cayley graphs $\Gamma(G, S)$ and $\Gamma(G, S')$ are quasi-isometric.

Question 3. Check the details of the proof of Corollary 3.4: the boundary of a proper Gromov hyperbolic geodesic metric space can be defined considering only geodesic rays starting from a fixed basepoint.

Question 4. Check the distance estimate for inversions in spheres, Remark 4.1 (b).

Question 5. Show that all geodesics in \mathbb{H}^n , considered in the Poincaré ball model, lie on circles or lines that meet the boundary sphere orthogonally.