

Title: Finitely Additive Measures and Weak Convergence in $L_\infty(X, \mathcal{L}, \lambda)$

Speaker: John Toland (University of Bath)

Time: Thursday 5th October, 3.15 - 4.15pm

Abstract: A useful necessary and sufficient condition, for a bounded sequence in $L_\infty(X, \mathcal{L}, \lambda)$ that is pointwise convergent λ -almost-everywhere to be weakly convergent, will be developed following a crash course on the Yosida-Hewitt (1952) representation of $L_\infty(X, \mathcal{L}, \lambda)^*$ as a class of finitely additive measures.

When $(X, \mathcal{L}, \lambda)$ is the Borel measure space of a locally compact Hausdorff topological space X and X_∞ is its one-point-compactification, the resulting criterion can be localized to points of X_∞ .

The test will be illustrated by examples such as: do sequences such as

$$\sin(1/kx) \text{ in } L_\infty(0, \pi)?$$

converge weakly or have a weakly convergent subsequence, and what does weak convergence in ℓ_∞ mean?

Miscellaneous observations, paradoxes and pathologies that arise during the development will be discussed in passing.