Calculus (MATH11007): Sheet 11 — 2013

Discrete Dynamical Systems

1. For each of the following dynamical systems, find the equilibrium points, if any exist, and sketch a web diagram for starting points above and below the equilibrium point

   (a) $u(n+1) = \frac{1}{2}u(n) + \frac{3}{2}$

   (b) $u(n+1) = 2u(n) + 2$

   (c) $u(n+1) = -u(n) + 2$

   For each of the dynamical systems where there is an equilibrium point E, consider starting at a point $u(0) = E + \delta$ where δ is small; describe what happens to $u(n)$ as $n \to \infty$.

2. Find the general solution to

$$u(n+1) = 2u(n) + 3$$

   Hence find the solution with the initial condition $u(0) = 3$.

3. Find the general solution to

$$u(n+1) = -2u(n) + 3$$

   Hence find the solution with the initial condition $u(0) = 3$.

4. Suppose that a patient's body eliminates 40% of a medicine every day. A doctor determines that the optimal level of drug in the bloodstream is 40mg, which should be the equilibrium value.

   (a) How many mg of drug should the patient take every day so that the body reaches an equilibrium of 40mg [starting with no drug in the body]?

   (b) The patient accidently [and without realising it] takes 80mg on the first day. 100mg is a dangerous amount of the of the drug to have in the body. Will the patient reach the dangerous amount [assuming that the patient continues to take the daily amount found in (a)]?

   (c) Another patient accidently [and without realising it] takes 200mg on the first day. How many days will the patient have a dangerous amount of the drug in their body [assuming that the patient continues to take the daily amount found in (a)]?

5. (a) Does the following dynamical system have a non-zero equilibrium (fixed) point

$$u(n+1) = -u(n) \tag{1}$$

   (b) If we have a dynamical system $u(n+1) = F(u(n))$, we say that E_2 is a fixed point of order 2 if

$$E_2 = F(F(E_2)),$$

   Show that (1) has a non-zero fixed point of order 2.

   (c) Find a map that has a non-zero fixed point of order 3 but no non-zero fixed point of order 1 or 2 [Hint: you may wish to allow $u(n)$ to be complex]. Can you generalize your answer?