Calculus (MATH11007): Sheet 12	**2013**
Discrete Dynamical Systems II	

1. A discrete dynamical system is given by
$$u(n+2) = -u(n+1) - u(n). \tag{1}$$
   (a) Find the first six terms when (i) $u(0) = 1$ and $u(1) = 2$; and (ii) $u(0) = 0$ and $u(1) = -2$.
   (b) Find the general solution (1) and thus explain why the series is periodic with period 3.
   (c) Now suppose $u(n+2) = u(n+1) - u(n)$. Does this dynamical system admit periodic solutions? And if so, what is their period?

2. Find the general solution to the dynamical system
$$u(n+1) = \frac{1}{2}u(n) + n.$$
   What is the solution when $u(0) = 0$?

3. Consider the following dynamical system
$$u(n+1) = u(n)(2 - u(n)).$$
   (a) Sketch a web diagram for initial points (i) $u(0) < 0$, (ii) $0 < u(0) < 1$, (iii) $1 < u(0) < 2$.
   (b) Find the equilibrium points, $u(n) = E$, and for each, starting from a point $u(0) = E+\delta$ where δ is small, calculate what happens to the solution. *[Hint: You should allow δ to be both positive and negative]*.

4. Consider the logistic map of the form
$$u(n+1) = \frac{7}{2}u(n)(1 - u(n)). \tag{2}$$
   (a) Find the fixed points of order one.
   (b) Find all the fixed points of order two.
   (c) Examine the stability of each of the fixed points of order two.

5. Consider the two closely separated initial conditions: (i) $u(0) = 0.4$ and (ii) $u(0) = 0.4 + 1/128$. Use `Matlab` to plot the evolution for 40 time steps of these two initial points for the following values of a for the logistic map (2):

   (a) $a = 2.9$, (b) $a = 3.5$, (c) $a = 3.65$, (d) $a = 3.75$.

   You should produce 4 plots, one for each value of a, with the evolutions starting from both initial conditions show on the same plot.

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