Calculus (MATH11007): Sheet 13 — 2013

Coupled first order systems

1. For each of the following coupled first order systems, find the set of equilibrium points. (The dependent variables, x, y and v, are all real-valued.)

   (a) $\dot{x} = 3x + 2y,$
   $\dot{y} = x.$

   (b) $\dot{x} = 3x + 2y + 1,$
   $\dot{y} = x + 2y + 4.$

   (c) $\dot{x} = 3x + 6y,$
   $\dot{y} = 2x + 4y.$

   (d) $\dot{x} = x - xy,$
   $\dot{y} = x - 3y + 2xy.$

   (e) $\dot{x} = v,$
   $\dot{v} = x - x^5.$

   (f) $\dot{x} = v,$
   $\dot{v} = x + x^5.$

2. (a) Find the general solution of the following coupled first order system

$$\begin{aligned}\dot{x} &= -2x + 6y,\\ \dot{y} &= 6x + 7y.\end{aligned}$$

   (b) Sketch the trajectories in the phase plane.

   (c) Find the solution satisfying the initial condition $x(0) = 1$, $y(0) = 0$.

3. (a) Find the general solution of the following coupled first order system

$$\begin{aligned}\dot{x} &= 6x + 2y,\\ \dot{y} &= 2x + 9y.\end{aligned}$$

   (b) Sketch the trajectories in the phase plane.

   (c) Find the solution satisfying the initial condition $x(0) = 1$, $y(0) = 0$.

4. Consider the following pair of coupled equations:

$$\begin{aligned}\dot{x} &= x - y,\\ \dot{y} &= x + 3y.\end{aligned}$$

   (a) Show that there is only one independent solution of the form $\mathbf{v}e^{\lambda t}$ where $\mathbf{v}$ is a constant vector. Find $\mathbf{v}$ and λ.

   (b) Find a second solution by using a trial solution of the form

$$\mathbf{w}te^{\lambda t} + \mathbf{z}e^{\lambda t}$$

   where $\mathbf{w}$ and $\mathbf{z}$ are constant vectors and λ hass the value you found in part (a).