Calculus (MATH11007): Sheet 9 **2012**

Second-order differential equations

1. (a) Find the general solution of

$$y'' + 5y' + 6y = 0\,.$$

   (b) Find the solution of the above differential equation subject to $y(0) = 1$, $y'(0) = 0$. Sketch the solution.

2. Find the solution of

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - 4\frac{\mathrm{d}x}{\mathrm{d}t} + x = 0; \quad x(0) = 0,\ \frac{\mathrm{d}x}{\mathrm{d}t}(0) = 1.$$

   Sketch the solution.

3. Find the solution of

$$y'' + 9y = 0$$

   subject to (a) $y(0) = 0$, $y'(0) = 1$; and (b) $y(0) = 1$, $y'(0) = 0$.

4. Find the general solution of

$$y'' + 4y' + 3y = f(x)$$

   where (a) $f(x) = \mathrm{e}^{2x}$; (b) $f(x) = x^2$; (c) $f(x) = \sin(5x)$; (d) $f(x) = \mathrm{e}^{-x}$.

5. Find the most general function that satisfies

$$u'' - 4u' + 8u = 0$$

   subject to

   (a) $u(0) = 0$, $u(\frac{\pi}{2}) = 0$;
   (b) $u(0) = 0$, $u(1) = 0$.

6. Find the general solution of

$$z'' - 6z' + 9z = \mathrm{e}^{3x}.$$

7. For what values of L are there non-zero solutions to

$$y'' + 9y = 0; \qquad y(0) = 0, \quad y(L) = 0.$$

8. For what values of ω are there non-zero solutions to

$$y'' + 2y' + (1 + \omega^2)y = 0, \qquad y(0) = 0, \qquad y(\pi) = 0.$$

9. The function $y(x)$ satisfies the following differential equation and boundary conditions

$$y'' - (2 + \epsilon)y' + (1 + \epsilon)y = 0; \qquad y(0) = 0 \qquad y'(0) = 1.$$

   (a) Find the solution when $\epsilon = 0$.

   (b) Find the solution when $\epsilon > 0$ and show that the solution to (a) is recovered as $\epsilon \to 0$.