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The transport of sediment over a sloping breakwater

A.J. Hogg[1] and D. Pritchard[2]

[1]Centre for Environmental & Geophysical Flows, School of Mathematics, University of Bristol, Bristol BS8 1TW, UK

[2]BP Institute for Multiphase Flow, University of Cambridge, Madingley Road, Cambridge CB3 0EZ, UK

Abstract

We analyse the transport of suspended sediment by a single swash event on a sloping breakwater and investigate the effects of overtopping on the motion and the redistribution of particles. By deploying a Lagrangian frame of reference, we calculate the net transport by these flows and demonstrate that overtopping promotes the landwards transport of sediment, primarily because the backwash of the swash is significantly weakened. Furthemore we quantify the flux of sediment that is transported over the crest to be deposited behind the breakwater.

Introduction

Wave breaking and collapse on a sloping beach or breakwater drives a rapid shallow flow under which the shoreline moves back and forwards across the 'swash zone'. Such events may transport significant volumes of sediment by their erosive action or by the advection and deposition of pre-mobilised particulate. They therefore play a significant role in determining the morphology of a beach or a soft breakwater and in the accumulation of sediment in the region the behind the breakwater. Studies of sediment movement by these intense, transient events are in their infancy. Programmes of field measurements face significant challenges in determining the properties of these shallow flows, while mathematical models are limited due to incomplete understanding of the physical mechanisms that control the flows and of the subtle, yet highly nonlinear interactions that occur between the fluid and sediment phases.

The mechanics of the swash zone are determined by a number of physical processes including unsteady, gravitational-driven fluid motion, the development of turbulent structures within advancing flow and the percolation of the flow into the unsaturated, porous substrate (see the reviews of Butt & Russell[1] and Elfrink & Baldock[2]). It is generally thought, however, that the motion is primarily driven by the upslope, gravitational collapse of water to form a relatively shallow, transient flow (figure 1). Thus a leading-order description of the hydrodynamics may be based upon the use of shallow water equations in which hydraulic resistance is neglected. Shen & Meyer[8] derived a solution to these equations to model the flow up a planar beach following the collapse of a bore in which they showed that the shoreline undergoes a ballistic motion with a constant downslope acceleration due to gravity. This approach was recently extended by Peregrine & Williams[3] to model the flow that overtops a sloping breakwater on the assumption that at the crest of the breakwater the Froude number of the flow is equal to, or exceeds, unity.

In this paper we study theoretically the ability of these flows to transport sedimentary particles both as bed- and suspended load and we examine the net transport that occurs over a swash event. Throughout this paper we draw the distinction between 'total load' models of sediment transport in which the instantaneous flux of particles adjusts immediately to the local conditions and suspended load models for which advection by the suspending fluid is of considerable importance, so that there is a lag between changes in the hydrodynamics and the sediment transport. Naturally for steady flows, these two approaches are identical. Our analysis couples sediment transport to the hydraulic model of Peregrine & Williams[3] and by treating the equations of motion in a Lagrangian frame of reference, we are able to make considerable analytical progress that obviates the need for lengthy numerical calculations and the associated difficulties of tracking the motion of the shoreline. Furthermore this analytical technique permits the robustness of our results to be examined by investigating the changes in the patterns of transport for particles with different physical properties, or by varying the empirical relationship between the flux of suspended particles and the flow speed. This Lagrangian technique has been recently applied to reveal the role of 'settling lag' for sediment transport by tidal currents and infra-gravity waves over inter-tidal regions [4],[5].

The paper is structured as follows. First, we formulate the hydraulic model, identify the relevant dimensionless parameters and review the overtopping solution of Peregrine & Williams[3]. We then calculate the associated patterns of sediment transport predicted by total load models and by the suspended load. Finally we summarise our results and draw some brief conclusions from this study.

Shallow water model

We assume that the swash-flow is sufficiently shallow so that vertical fluid accelerations are negligible and the pressure adopts a hydrostatic distribution. Thence we employ the shallow water equations to model the conservation of fluid mass and momentum and the transport of suspended sediment, which is assumed to be sufficiently dilute so that it does not significantly supplement the density of the suspending fluid. Denoting the depth of the fluid by h, the depth-averaged velocity and mass concentration by u and c, respectively, and aligning the x-axis to be parallel to the planar surface of the breakwater which is inclined at an angle θ to the horizontal (see figure 1), the governing equations are given by

$$\frac{\partial h}{\partial t}+\frac{\partial}{\partial x}(uh) = 0, \qquad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+g\cos\theta\frac{\partial h}{\partial x} = -g\sin\theta, \qquad (2)$$

$$\frac{\partial c}{\partial t}+u\frac{\partial c}{\partial x} = \frac{m_e q_e - w_s c}{h}, \qquad (3)$$

where g denotes gravitational acceleration, w_s is the settling velocity of the particles and $m_e q_e$ is the rate of erosion from the bed per unit area. In this expression of the erosive flux, m_e is a dimensional constant and q_e is an empirically-determined function of the flow. (Pritchard & Hogg[6] provide a more complete derivation of these equations and discuss the approximations that underlie them). In this model we have neglected the feedback between the sediment movement and the fluid motion because morphological changes occur on a much slower timescale than the swash events[7].

Sediment is not eroded until the shear stress exceeds a threshold value and then the erosive flux is often modelled as a function of

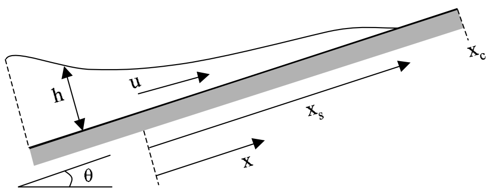

Figure 1: The flow configuration.

the excess shear stress. Although within the framework developed below, we could study any dependence of q_e upon the velocity and height of the flow, here, for brevity, we assume that q_e is proportional to u^2 for the suspended load and we neglect the existence of a threshold for erosion. This follows Pritchard & Hogg[7], who demonstrated that the general patterns of erosion are robust to a wide range of parameterisations of the erosive flux. Furthermore these swash flows are relatively intense with peak velocities of approximately $2\,\mathrm{ms}^{-1}$, whereas the threshold velocity for the erosion of fine sand is approximately $0.25\,\mathrm{ms}^{-1}$. Thus apart from regions close to flow reversal, the neglect of the threshold for erosion does not appreciably affect the pattern of net sediment transport. However we stress that the calculations below could have included an erosion threshold, but this does not introduce any significant qualitative differences to the results. In the discussion that follows we define the instantaneous and net sediment fluxes by

$$q(x,t) = uhc \qquad \text{and} \qquad Q(x) = \int_{t_{in}(x)}^{t_{de}(x)} q\,\mathrm{d}t, \tag{4}$$

where $t_{in}(x)$ and $t_{de}(x)$ are the times at which location x is inundated and denudated, respectively.

Following [3] we non-dimensionalise this system of-equations using the lengthscale $A/\sin\theta$ and the velocity scale $(gA)^{1/2}$, where $2A$ is the vertical excursion of the swash event. Furthermore we nondimensionalise the height of the flowing layer by $A/\cos\theta$ and the concentration field by m_e/w_s. Replacing the variables by their dimensionless counterparts, we find that the governing equations are given by

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(uh) = 0, \tag{5}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{\partial h}{\partial x} = -1, \tag{6}$$

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} = E\,\frac{q_e(u) - c}{h}. \tag{7}$$

The residual dimensionless parameter in (5)-(7), $E = w_s \cot\theta (gA)^{-1/2}$, measures the rate at which the sediment concentration responds to the hydraulic conditions and will henceforth be termed the exchange rate. If $E \gg 1$ then the concentration rapidly adjusts to the local conditions and $c = q_e(u)$ to leading order. This regime corresponds to 'total load' models of sediment transport. Conversely if $E \ll 1$ then the concentration field lags the local conditions and the sediment is mainly advected by the flow with little exchange with the bed. Typical values of E may be estimated as follows: for a steep breakwater, $\tan\theta = 0.1$, maximum velocities of a swash event may range from 0.5 to $4\,\mathrm{ms}^{-1}$. Thus, as we show below, the velocity scale $(gA)^{1/2}$ ranges from 0.25 to $2\,\mathrm{ms}^{-1}$. If the surface of the breakwater is composed of sand particles then the typical settling velocity is approximately $10^{-2}\,\mathrm{ms}^{-1}$. Combining these dimensional parameters, we find that the exchange rate parameter E is $0.05 - 0.4$.

A final important parameter is the dimensionless extent of the breakwater, x_c. This is the distance up the slope from where the bore is initiated to the crest of the breakwater. In terms of these dimensionless units, the flow overtops the breakwater if $0 \le x_c < 2$.

We rewrite (7) in a Lagrangian frame of reference. Thus following fluid elements with position $x_L(t;\xi)$, where ξ labels the initial position, we find that

$$\frac{\mathrm{d}c_L}{\mathrm{d}t} = E\,\frac{q_e(u_L) - c_L}{h_L}, \qquad \text{with} \qquad \frac{\mathrm{d}x_L}{\mathrm{d}t} = u_L. \tag{8}$$

In this expression $u_L = u(x_L,t)$ and $h_L = h(x_L,t)$. This may be integrated to give

$$c_L = c_0 \exp\left(-\int_0^t \frac{E}{h_L}\,\mathrm{d}t'\right) + \int_0^t \frac{Eq_e}{h_L} \exp\left(\int_{t'}^t \frac{E}{h_L}\,\mathrm{d}t''\right)\,\mathrm{d}t', \tag{9}$$

where c_0 is the initial concentration. Thus in a Lagrangian frame of reference the concentration field may be represented as $c_L \equiv c^{pr} + c^{en}$, where c^{pr} equals the first term of the right-hand-side of (9) and denotes the concentration that is initially suspended within the collapsing bore and subsequently redistributed over the breakwater; and c^{en} corresponds to the second term of the right-hand-side of (9) and denotes the contribution from material eroded from the bed. In what follows we describe the evolution of c^{pr} and c^{en} separately.

Overtopping flow

An overtopping solution to the shallow water equations was derived by Peregrine & Williams[3]. It models the advance of the shoreline up the breakwater, overtopping with Froude number of unity, or higher, at the crest of the breakwater and subsequent shoreline retreat. It is given by

$$u(x,t) = \begin{cases} -t & x < -t - \frac{1}{2}t^2 \\ \dfrac{2(t - t^2 + x)}{3t} & -t - \frac{1}{2}t^2 \le x < x_{ch}(t) \\ \dfrac{2 - t - 2\sqrt{2(x_c - x)}}{3} & x_{ch}(t) \le x \end{cases}, \tag{10}$$

$$\text{where} \qquad x_{ch}(t) = \begin{cases} x_c & t < \sqrt{2x_c} \\ \sqrt{2x_c}t - \frac{1}{2}t^2 & t > \sqrt{2x_c} \end{cases}. \tag{11}$$

The height field is given by $h = (2 - t - u)^2/4$ and the position of the shoreline, $x_{sh}(t)$, which is determined by $h(x_{sh},t) = 0$, may be evaluated

$$x_{sh}(t) = \begin{cases} 2t - \frac{1}{2}t^2 & 0 \ge t < 2 - \sqrt{4 - 2x_c} \\ x_c & 2 - \sqrt{4 - 2x - c} \ge t < 2 \\ x_c - \frac{1}{2}(t-2)^2 & 2 \ge t \end{cases}. \tag{12}$$

In figure 2 we plot Lagrangian trajectories (particle paths) under this flow, noting that the curve $x = x_{ch}(t)$ corresponds to a contour of constant height, namely $h = \frac{1}{9}(2 - \sqrt{2x_c})^2$.

Sediment transport: total load

We first consider sediment transport under a total load model. Figure 3 shows the net flux $Q(x)$ across the truncated swash zone for various values of x_c and two total load models: equilibrium suspended sediment transport with $q_e = u^2$ and the Bailard model in which total load is independent of depth ($q = |u|^3 u$).

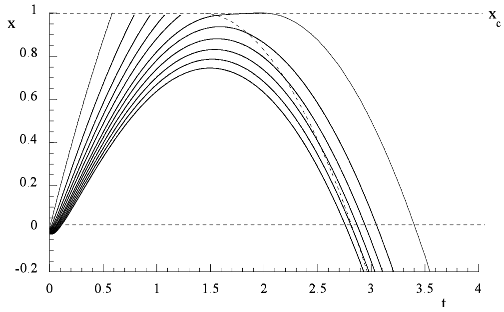

Figure 2: Selected Lagrangian trajectories, $x_L(t)$, under the overtopping flow (solid lines) when $x_c = 1.0$. Also plotted is the curve $x = x_{ch}(t)$ across which the acceleration is discontinuous (dashed line).

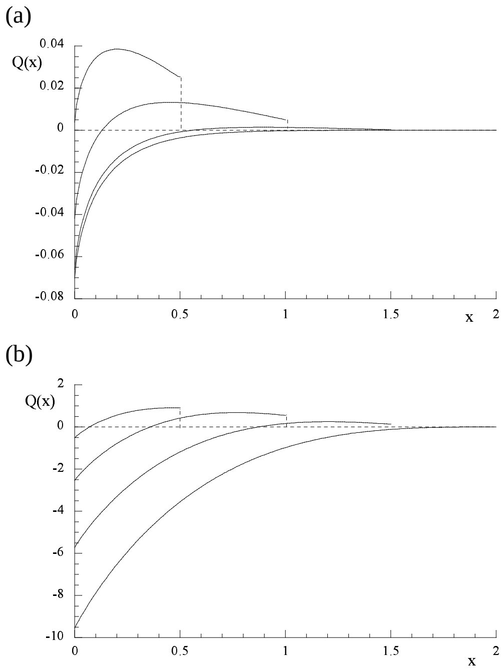

Figure 3: The net flux, $Q(x)$, across a breakwater with crest at $x_c = 0.5, 1.0, 1.5$ and 2.0 for (a) $q = u^3 h$; and (b) $q = |u^3|u$ (Bailard model).

Although the magnitudes of the net sediment fluxes are different, largely as a result of the choice of dimensionless variables, the patterns of transport are broadly similar.

First note that when there is no overtopping ($x_c \geq 2$), then $Q(x)$ increases monotonically to zero at $x = 2$. This implies that the flux is always offshore and that the breakwater/beach will be eroded all along its surface since $\mathrm{d}Q/\mathrm{d}x > 0$. The origin of this offshore transport is the asymmetry in the underlying hydrodynamics. Although the ballistic speed of the shoreline varies identically during the on- and off-shore phases of the motion, this is not the case behind the shoreline, where the backwash is prolonged. Thus this leads to greater sediment movement on the backwash and net offshore transport. It can be readily shown that this result is robust provided that the erosive flux is a monotonic function of the velocity [7].

When there is overtopping the net pattern of the sediment transport is somewhat different and there is now the possibility of onshore transport because the backwash is diminished (see figure 3). When the swash zone is severely truncated, the effect of the backwash on total load is negligible, and most of the sediment movement is landward, with some deposition towards the crest of the breakwater. As the degree of truncation is reduced, the effect of the backwash reasserts itself, and net seaward transport is restored.

Sediment transport: suspended load

We now consider suspended sediment transport: figure 4 shows the net fluxes for the representative case $q_e = u^2$ and $E = 0.1$, for four values of the cut-off point x_c.

Several points are evident in these figures. First note that $Q^{en} > 0$ for locations close to the crest of the breakwater ($|x_c - x| \ll 1$). In contrast to the total load models above, this landwards flux occurs even for a flow without overtopping ($x_c = 2$). The mechanism for this landwards flux on the upper part of the breakwater is settling lag [4]: sedimentary material is eroded during the initial part of the swash event and deposited progressively as the flow reverses. On the backwash the flow also erodes material; however the concentration of suspended sediment does not adjust immediately to the local velocity and so even though the backwash lasts longer than the uprush, the landwards flux exceeds the seawards flux on the upper part of the breakwater. Pritchard & Hogg[7] show that the existence of a zone within which there is landwards transport remains robust to variations in the empirical erosion function (q_e).

It is also noteworthy that the presence of the cut-off makes barely any difference to the landwards transport of internally mobilised sediment (figure 4 a) or externally supplied sediment (figure 4 b), because unless x_c is very close to the origin of the swash event, most of the flow has reversed before it is affected by the overtopping. The difference which the overtopping does make is to the flow and sediment transport on the backwash, both of which are substantially reduced (although the spatial pattern is very similar to that under swash on an untruncated beach: compare the results for $x_c = 0.5, 1.0$ and 1.5 with $x_c = 2.0$).

As a result, there are substantial net fluxes in the landwards direction (figures 4 a and b). Some material passes over the top of the breakwater and is lost; for values of x_c around 1, however, a substantial quantity is deposited just seaward of x_c and then not re-entrained on the weak backwash; these intermediate values of x_c may cause a significant quantity of sediment to be deposited just below the crest of the breakwater, although the net effect is still generally erosional. The effect of truncation on the fate of externally supplied sediment (figure 4 b) is fairly insignificant, since most of this material has settled out before the backwash in any case.

The fluxes of sediment over the crest of the breakwater for the entrained material, $Q^{en}(x_c)$, and the pre-mobilised material, $Q^{pr}(x_c)$, exhibit rather different behaviours as E is varied. The material in suspension when the swash is initiated gradually settles out along the breakwater. As E increases, the rate of deposition also increases and so fewer particles remain in suspension at the crest of the breakwater. Thus as shown in figure 5, the flux of sediment lost over the breakwater diminishes monotonically with E. However, the amount of material mobilised within the swash zone which can be carried over the breakwater, $Q^{en}(x_c)$, shows a more complicated variation, because there are two competing effects. When the exchange rate parameter is sufficiently small, there is little erosion during the swash event ($Q^{en} = O(E)$). Thus $Q^{en}(x_c)$ increases with E for $E \ll 1$. Conversely when E is sufficiently large then the concentration of suspended sediment is in equilibrium with the local flow con-

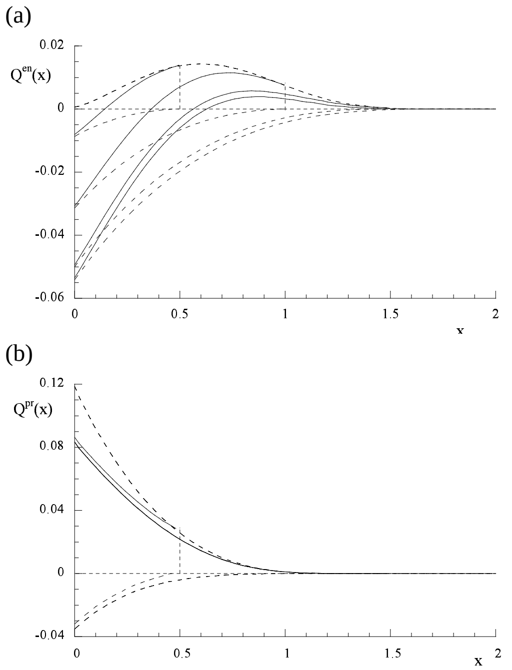

Figure 4: The net flux, $Q(x)$, across a breakwater with crest at $x_c = 0.5, 1.0, 1.5$ and 2.0 for $q_e = u^2$ and $E = 0.1$ as a function of position: (a) Internally mobilised sediment, Q^{en}; and (b) Pre-suspended sediment, Q^{pr}. The solid lines represent the net fluxes over the overtopping event; the dashed lines represent the contributions from the uprush and backwash components. (Note that in (b) the curves for $x_c = 1.0, 1.5$ and 2.0 are indistinguishable.)

ditions and attains a constant value determined solely by the velocity and height at the crest ($Q^{en} = O(1)$, when $E \gg 1$). Decreasing the exchange rate parameter in this regime increases the flux of sediment because material is advected over the crest that was mobilised lower down the breakwater where the velocity and rate of erosion are higher. These observations imply that there is a value of E for which $Q^{en}(x_c)$ is maximised (figure 5). We note that such a maximum may act as a mechanism for segregating by size of the sediment that is carried over the breakwater, because transport is maximised for a particular value of E (and thus of w_s).

The sediment flux at the crest also exhibits a complicated behaviour with the length of the breakwater (x_c): longer breakwaters provide a longer-lived uprush and thus more time to entrain sediment, but they also reduce the velocity of the flow at the time when it reaches the crest. Overall, the latter effect dominates, and there is quite a strong decrease in fluxes at the crest as x_c increases. Increasing the length of the breakwater, x_c, also decreases the value of the exchange parameter, E at which the sediment flux is maximised.

Conclusions

We have developed an analytical description of the transport of suspended sediment by a single swash event following the collapse and overtopping of a bore onto a sloping breakwater. Our analysis couples the hydrodynamic model of overtopping flow of Peregrine & Williams[3] to a sediment transport model and treats the equations in a Lagrangian frame of reference to determine the concentration of suspended sediment within the entire flow. It was particularly insightful to separate the transported sediment into contributions entrained during the swash event

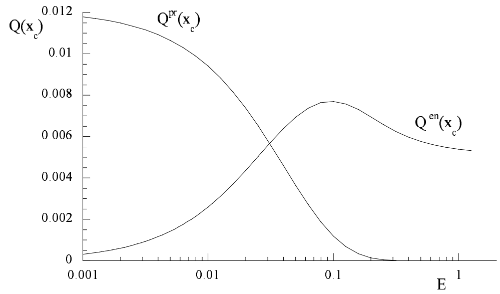

Figure 5: The net flux of sediment at the crest of the breakwater, $Q(x_c)$, as a function of E for $q_e = u^2$ and $x_c = 1.0$. The internally mobilised, $Q^{en}(x_c)$ and the presuspended components, $Q^{pr}(x_c)$ are plotted separately.

and pre-suspended within the collapsing bore and to analyse their evolutions separately. We demonstrated that the effects of overtopping were to increase significantly the landwards transport of suspended sediment. This occurs because sediment is carried over the crest of the breakwater and because the backwash is weakened and becomes unable to modilise sediment. Thus overtopping is likely to erode the face of the breakwater towards the seawards end of the swash zone, but there may be appreciable deposition close to the crest. Since the long-term effect of this erosion and deposition is to steepen the break water and make it less effective at dissipating wave energy, paradoxically the landwards movement of sediment may ultimately contribute to the degradation of the defence structure.

Acknowledgements

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