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Defending against lava flows: theory, experiments and field confirmation

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Abstract. The consequences of lava flowing downhill around and over topography and interacting with human-made constructions is modelled by considering the flow of a Newtonian fluid. Small obstacles can be overtopped by the flow, but topography of sufficient height will deflect the flow around it and form dry regions in the wake. Both numerical solutions and the results of laboratory experiments are discussed. We provide numerous pictures of flow patterns and evaluate the force they exert. The experimental results, focusing on flows past circular cylinders, are in good agreement with our numerical evaluations. Flows over depressions, which act to concentrate the flow, are also discussed.

Keywords: lava flows, topographic forcings, gravity currents, flow-structure interactions

1 Introduction

Volcanic eruptions and the subsequent flow of lava lead to deaths of both humans and animals, as well as resulting in destruction of many properties and dire financial problems. Approximately 2000 people have been killed by lava flows in the last 20 years. On average, tens of millions of cubic meters of lava are erupted each year onto the Earth's surface, either into the atmosphere or under the oceans, travelling along the ground at speeds of up to 100km/hr. Can we predict how lava flows are diverted by natural topography and by buildings? Where and in what orientation should constructions be placed to maximise the 'dry spots', free from lava? What is the anticipated force on a defending wall and to what height and length need it be built to play a useful role? These are some of the questions addressed in this review-like paper, which summarises material spelt out in greater detail in Hinton et al. [1,2,3].

2 The model

Consider a time independent two-dimensional flow of flux Q per unit width of thin, viscous, Newtonian liquid, of kinematic viscosity ν, to model a lava flow down an inclined plane at angle β to the horizontal. The thickness of the flow is then given by [4]

$$H_\infty = (3\nu\, Q/g \sin\beta)^{\frac{1}{3}}. \tag{2.1}$$

To this (vertical) lengthscale can be added horizontal and vertical lengthscales L and D dependent on the topography or building on the slope encountered by the lava flow. Introducing downslope and cross-slope dimensionless variables x and y, and a dimensionless axis perpendicular to the slope z by

$$(x, y) = (X, Y)/L\,, \qquad z = Z/H_\infty, \tag{2.2}$$

we find that the dimensionless depth $h(x, y)$ of the lava satistfies [1]

$$(\partial h^3)/\partial x = \nabla[h^3 \underset{\sim}{\nabla}(\mathcal{F}(h + \mathcal{M}m)], \tag{2.3}$$

where $m(x, y)$ is a dimensionless expression for the underlying topography, along with the governing non-dimensional parameters

$$\mathcal{F} = H_\infty/L\tan\beta \;=\; (3\nu Q/g \sin\beta)^{1/3}/(L\tan\beta). \tag{2.4}$$

and

$$\mathcal{M} = D/L\tan\beta. \tag{2.5}$$

3 Flow patterns

3.1 One-dimensional mounds

Consider, to start and to illustrate some of the fundamental aspects of the flows, a one-dimensional situation (independent of the cross-flows co-ordinate, y), with the mound given by $m(x)$. (2.3) can then be integrated once, using the boundary condition $h \to 1$ as $x \to -\infty$, to obtain

$$h^3\left(1 - \mathcal{M}\frac{dm}{dx}\right) = 1 + \mathcal{F}h^3\,\frac{dh}{dx}. \tag{3.1}$$

Because it is one-dimensional, all the flow must go over the mound. The most important consequence, determined from numerical solution of (3.1) for a variety of $m(x)$, $\mathcal{F}$ and $\mathcal{M}$ is that for small $\mathcal{M}$, $\mathcal{M} < \mathcal{M}_c$, where $\mathcal{M}_c$ is a critical value, dependent on the details of $m(x)$ and the value of $\mathcal{F}$, the flow progresses uniformly over the mound, with a downward sloping upper surface everywhere. However, for $\mathcal{M} > \mathcal{M}_c$ a pond develops upstream of the obstacle, the surface of which is horizontal. The value $\mathcal{M}_c$ is the smallest value of $\mathcal{M}$ so that $1 - \mathcal{M}m'(x)$ is somewhere negative. As an example, for $m = \exp(-x^2)$, $\mathcal{M}_c = (e/2)^{1/2} \approx 1.16$.

3.2 Two-dimensional mounds

For mounds described by $m(x, y)$, the flow can: go over the mound; around the mound; not reach the top of the mound (if higher than some critical value); not completely cover the ground, that is, develop 'dry patches' - relatively safe places to be during a lava flow. Figure 1-4 display numerically determined flow fields for a variety of $\mathcal{F}$, $\mathcal{M}$ and $m(x, y)$. An interesting series of examples is provided by an elliptical mound given by

$$m(x, y) = \exp\{-[x^2 + (y/b)^2]\}, \tag{3.2}$$

which tends to a long barrier as $b \to \infty$. Figure 5 shows the expected flow thickness for two values of b. What is the force exerted on such a topographic feature, envisaged as a defending wall to an oncoming lava flow? In the limit $b \to \infty$, for a barrier just sufficiently high to stop the oncoming flow (which climbs up the barrier) the maximum force $\sim \rho g (L \tan \beta)^2$, which for the illustrative values $L = 50m$ and $\tan \beta = 0.25$, leads to a maximum force of the order $10^7 Nm^{-1}$.

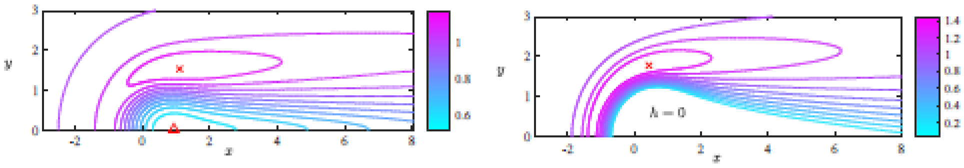

Figure 1: Contour plots of the thickness of flow over topography specified by $m = exp(-r^2)$ for $\mathcal{F} = 0.1$. a) $\mathcal{M} = 0.5$ and b) $\mathcal{M} = 1.5$. Red crosses mark the points of maximum thickness. Note the dry zone in b)

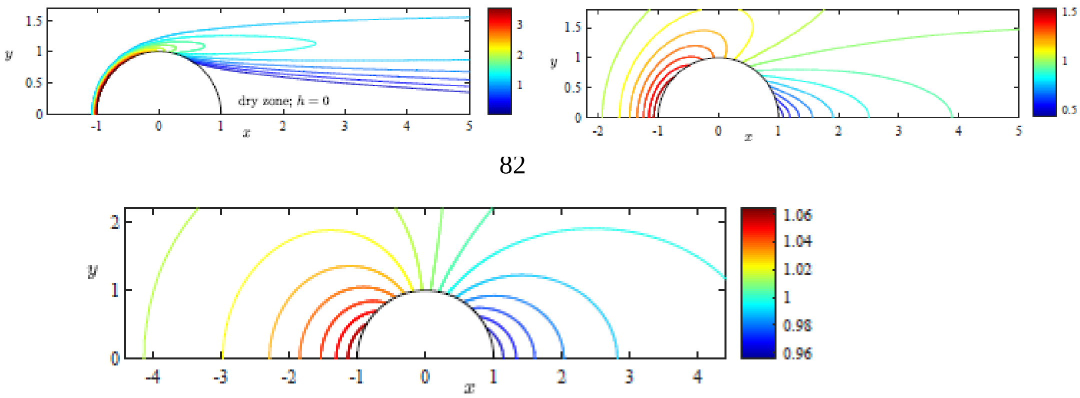

Figure 2: Contour plots of the thickness of flow past a circular cylinder under the condition of no normal flow at the boundry, for $\mathcal{F} = 20, 1$ and 0.025.

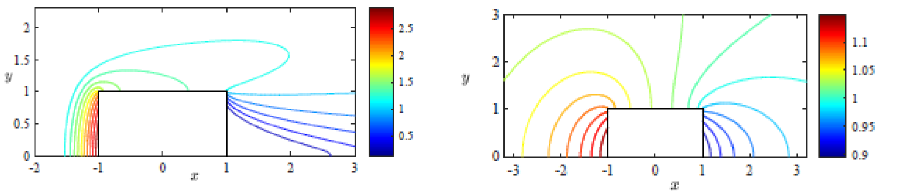

Figure 3: Contour plots of the thickness of flow around a square-on square obstacle for $\mathcal{F} = 10$ and 0.25. Note that the flow remains attached to the square in both cases and there is no dry region for these values of $\mathcal{F}$.

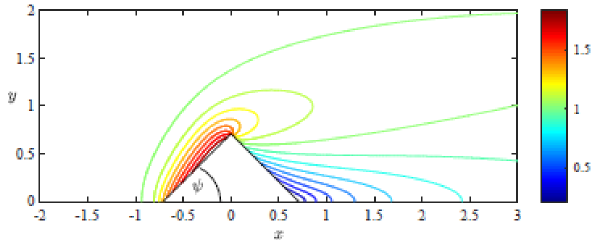

Figure 4: Contour plot of the thickness of flow around a square rotated 45° to the oncoming flow for $\mathcal{F} = 0.25$.

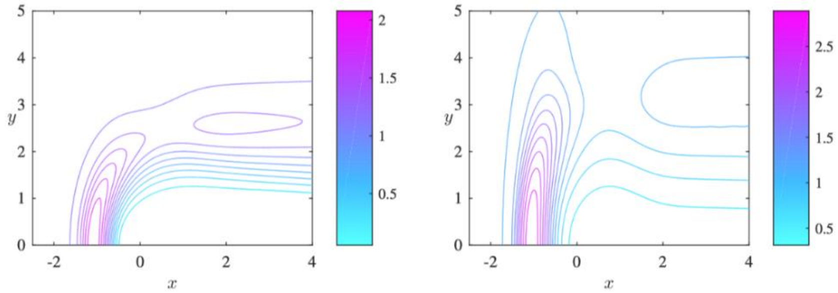

Figure 5: Contour plots of the thickness of flow over an elliptical mound with $\mathcal{F} = 0.05$ and $\mathcal{M} = 1.4$ for a) $b = 0.2$ and b) $b = 4$.

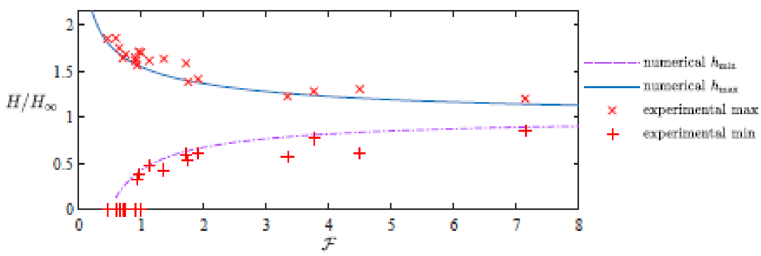

Figure 6: Calculated and experimental results for the maximum and minimum flow thickness as a function of $\mathcal{F}$ for flow past a cylinder. A zero flow thickness indicates the existence of a dry zone downstream of the cylinder.

3.3 Experimental verification

We carried out a series of experiments on a slope of width 30cm, length 120cm inclined at angles between 3.5 and 23 degrees on which we affixed (tall) cylinders of radius between 2.4 and 4.8cm [2]. The upstream flow thickness varied between 0.5 and 1.5cm, leading to values of $\mathcal{F}$ between 0.5 and 7.1 (the cylinders were all tall and so $\mathcal{M}$ is not a relevant parameter.) Figure 6 displays a compendium of the results for the maximum and minimum flow thickness, with good agreement between theoretical predictions, obtained by numerically solving (2.3), and the experimental results.

3.4 Depressions

Real topography includes not only mounds and hills, but also depressions; and both together. An initial analysis of some effects due solely to depressions is contained in [3] and the flow thickness for two cases make up figure 7. For smallish depressions the flow thickness is but slightly perturbed. For deeper depressions large ponds of fluid accumulate and have a significant effect on the flow downstream.

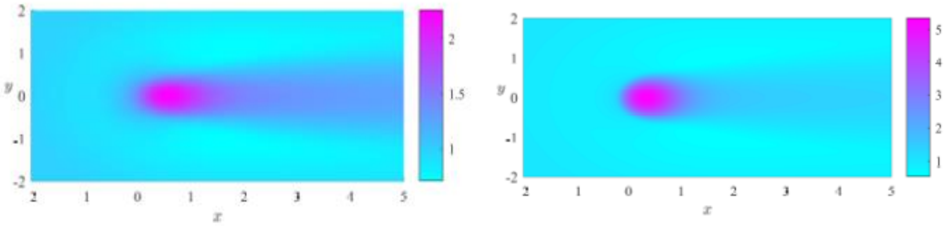

Figure 7: Flow thickness over a circular Gaussian depression for $\mathcal{F} = 0.1$ and a) $\mathcal{M} = -0.8$ and b) $\mathcal{M} = -1.6$.

Depressions are significantly different from hills because a sufficiently high hill, not touched by the flow at its higher points, does not come into contact with the flow; and hence the higher parts of the hill don't influence the flow. No matter how deep the depression it will influence the flow and there will be some flow (though maybe small) right to the bottom. In principle this resembles the influence of Moffatt eddies, slow motions in a sharp corner, well away from the forcing flow [5].

Of considerable interest and novelty are flows over topography containing both hills and depressions. We plan to publish on this topic in the future.

3.5 Field observations

Here is not the best place to compare our model results with real data taken in the field. However, numerous opportunities present themselves as outlined on Hawaii [6], Santorini [7] and elsewhere. This, too, will be reported elsewhere (Hinton et al. 2022).

4 Conclusions

Lava flows are frequent on the Earth and can cause much damage. Defending people and property in such situations is a very worthwhile endeavor. Our work has begun to lay down some of the foundations and principles that might be employed. Many further questions remain, including what shape of cross-sectional area A (of a building) maximises the area of the dry zone. How sensitive is the result to the input parameters? How will the concepts we have developed be used in any way usefully during forthcoming volcanic eruptions?

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