Modelling the Rise of Volcanic Plumes to Estimate Mass Eruption Rates

Andrew J. Hogg and Mark J. Woodhouse

Abstract Explosive volcanic eruptions can generate plumes that carry ash high into the atmosphere posing hazards to aviation while airborne and to lives and infrastructure as the ash settles back to ground level. It is crucial to quantify the rate at which ash is produced if its subsequent dispersion is to be modelled accurately. Direct measurement of this mass eruption rate is impossible; however, observations of plume dynamics provide a means of determining this quantity.

1 Introduction

Explosive volcanic eruptions inject ash into the atmosphere and its subsequent dispersion and deposition causes major hazards. The very largest eruptions have transported such large volumes of ash into the stratosphere that global temperatures and the climate are impacted. However even smaller events pose hazards to health and infrastructure as the erupted ash settles back to ground level. Airborne ash also poses a danger to aircraft flights because modern jet engines are susceptible to damage, even from low concentrations. This hazard gained particular notoriety during the 2010 eruption of Eyjafjallajökull, Iceland, during which European airspace was closed for several weeks with the cancellation of over 100,000 flights and the direct economic loss across all sectors estimated as US$5bn.

Prior to the 2010 Eyjafjallajökull eruption, the International Civil Aviation Organization adopted a precautionary policy of ash avoidance, with no concentration of ash in the atmosphere considered safe for aircraft. However, the disruption to aviation during the first week of explosive activity at Eyjafjallajökull (14–18 April 2010) led

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to a relaxation of this policy in Europe, with the UK Civil Aviation Authority and Eurocontrol introducing ash concentration thresholds for commercial air traffic. Ash concentrations below 2mg m^{-3} are considered safe for flights, while flight operations at higher concentrations require a Safety Case to be accepted by national regulators. Typically, Safety Cases have been accepted for ash concentrations up to 4mg m^{-3}. The introduction of thresholds of ash concentration places increased demands on accurate modelling of atmospheric ash dispersion during volcanic crises.

The crucial component for forecasting ash dispersion is the estimate of the strength of the volcanic source. In other words, what is the rate of production of ash from the volcano? Direct measurements of this mass eruption rate (MER) are extremely difficult due in part to the explosive character of the volcanic dynamics at the vent, and so the MER is typically determined through interpretation of indirect measurements. In this paper we report on mathematical modelling that is used to interpret observations of volcanic plumes and to determine the MER, and we indicate how this model has been used in operational settings.

Volcanic plumes rise through the atmosphere because the erupted ash is hot and rapidly heats the surrounding gases to generate a buoyant release. The resultant plume ascends and mixes with the denser atmospheric air, increasing its bulk density. The atmosphere through which it rises is usually stably stratified (its density decreases with height), and so eventually the plume reaches altitudes at which it is no longer less dense than the surroundings and begins to spread horizontally. The height of rise, therefore, is related to the strength of the source (i.e. the MER), as well as other atmospheric properties such as the stratification and the wind field.

Data from historic eruptions for which there are observations of the height of rise through the atmosphere, H, and estimates of the MER, Q, indicate that there is approximately a $\frac{1}{4}$-power relationship between the two. For example, [1] propose the empirical fit, $H = 0.220Q^{0.259}$, where H is measured in kilometers and Q in kilograms per second. This relation has been used operationally to infer the MER from observations of the height of rise of the plume. Though simple, this approach is problematic because the historical data is bias towards the very largest eruptions for which it is easiest to estimate the erupted volume, and because the formula neglects the effects of atmospheric conditions, which will be shown to be important; critically for the relatively small-scale eruption of Eyjafjallajökull in 2010, the empirical relation takes no account of the effects on the plume of the atmospheric wind.

2 Mathematical Model of Volcanic Plumes

Integral models of turbulent plumes were first developed by [2]. The approach entails averaging over the fluctuating turbulent characteristics to produce models that capture the steady vertical variation of the plume properties, such as its velocity, density and cross-sectional area. The key effect of turbulence is to induce mixing between the rising buoyant fluid and the surroundings. This mixing is embodied into an entrainment velocity that captures that rate at which atmospheric fluid is engulfed into the plume [2]. Integral models have the distinct advantage over more complicated simulations of the fluid flow that they are very simple to compute (a vital feature for operational hazard assessment), and yet they have been shown to capture features such as the height of rise accurately. Our model extends the theory of turbulent plumes [2] to volcanic settings, in which there is non-negligible wind. The wind bends over the ascending plume (see Fig. 1) and modelling equations are developed as functions of the arc-length, s, such that the vertical position, $z(s)$ and the horizontal position, $x(s)$ (where the x-axis is aligned with the atmospheric wind), are given by

$$\frac{\mathrm{d}x}{\mathrm{d}s} = \cos\theta \qquad \text{and} \qquad \frac{\mathrm{d}z}{\mathrm{d}s} = \sin\theta, \tag{1}$$

where $\theta(s)$ is the angle between the axis of the plume and the horizontal.

The model mathematically embodies four physical principles to derive equations for temperature, $T(s)$ (and hence density $\rho(s)$), axial velocity, $U(s)$, inclination, $\theta(s)$ and cross-sectional area, assumed circular with radius, $R(s)$, given by

$$\frac{\mathrm{d}}{\mathrm{d}s}\left(\rho U R^2\right) = 2\rho_a U_e R, \tag{2}$$

$$\frac{\mathrm{d}}{\mathrm{d}s}\left(\rho U^2 R^2 \sin\theta\right) = (\rho_a - \rho)\, g R^2, \tag{3}$$

$$\frac{\mathrm{d}}{\mathrm{d}s}\left(\rho U^2 R^2 \cos\theta\right) = 2\rho_a U_e R V, \tag{4}$$

$$\frac{\mathrm{d}}{\mathrm{d}s}\left(\rho U R^2 \left(C_p T + \frac{U^2}{2} + gz\right)\right) = 2\rho_a R U_e \left(C_{pa} T_a + \frac{U_e^2}{2} + gz\right) \tag{5}$$

These equations express mass conservation, momentum balance in the vertical and horizontal and energy conservation, respectively. Quantities with subscript a refer to atmospheric variables, V is the wind speed, g gravitational acceleration and C_p the specific heat capacity. Volcanic ash is assumed to be conserved during plume ascent

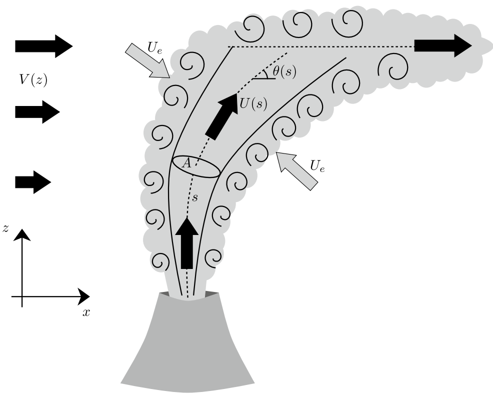

Fig. 1 Schematic of a volcanic plume in a crosswind and the variables used in the mathematical model of the motion

because its settling is much slower than the plume velocity. Gases are modelled using ideal gas laws, with properties that vary with the solids mass fraction (see [3]). The final important closure relates the entrainment velocity, U_e, to the axial speed and the wind speed. This has been determined empirically as

$$U_e = k_s|U - V\cos\theta| + k_w|V\sin\theta|, \tag{6}$$

where k_s and k_w are known constants [3]. Importantly this formula connects the enhanced rate of mixing to the wind speed.

The governing Eqs. (1)–(5) may be integrated numerically from prescribed conditions at the volcanic vent to determine the maximum height of rise. It is possible to have source conditions that do not generate a buoyant plume, because there is insufficient heat transfer from the erupted material to the entrained air to reduce the density of the mixture so that it becomes buoyant. In this case the volcano issues an initially vertically-aligned jet that reaches its maximum height quite close to the source. However if the plume becomes buoyant then it ascends into the high atmosphere, driven by density differences rather than its initial momentum.

The model (1)–(5) provides a quantitative link between the source conditions (and hence the MER) and the maximum height of rise, and depends upon the atmospheric properties such as the vertical variation of temperature T_a, and the wind speed, V. We discuss the implementation of the model below, but first highlight some results.

2.1 Results

The most important result is that wind increases mixing and that this leads to lower heights of rise for otherwise identical explosive eruptions. This is most easily demonstrated by applying the model to analyse the height of rise through a US-standard atmosphere in which the temperature decreases linearly with height within the troposphere, while the wind increases with height reaching a maximum V_1 at the tropopause (see [3]). We integrate (1)–(5) for various values of the wind speed, V_1 and varying source conditions (see Fig. 2). The model results are consistent with the data, although there are inevitably some differences because the atmospheric conditions vary for each eruption. Importantly, though, the predicted height of rise decreases as the wind speed, V_1, increases. This has crucial consequences.

We used our model to interpret observations of the 2010 eruption of Eyjafjallajökull. During the first explosive phase of the eruption, the plume was observed to rise to approximately 8km on 14 April 2010, before falling back to 5km on 15 April and then rising again to over 8km on 16–17 April. One interpretation of this variation is that the volcanic eruption is varying its intensity (and MER), and this is the only possible quantitative explanation if one employs the empirical relation between height of rise of MER. However the wind speed (measured at an elevation of 7km) also varied during this period from relatively low speed (25ms^{-1}) to a much higher speed (50ms^{-1}) before falling back to below 20ms^{-1}. Thus an alternative

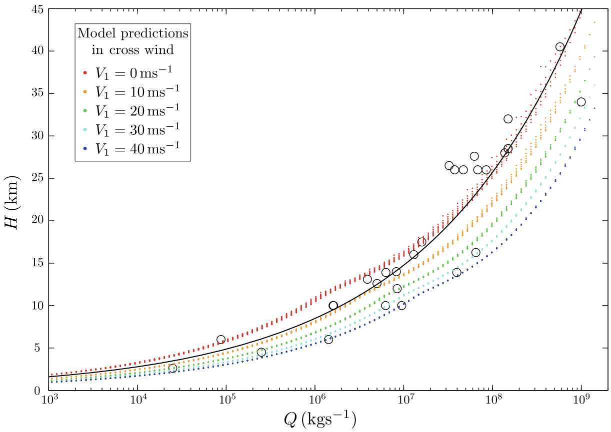

Fig. 2 The height of the plume rise, H, as a function of the mass eruption rate, Q, showing field data (○), the empirical curve of [1] (solid line), and the results from simulations with varying atmospheric wind speeds at the tropopause (coloured symbols)

explanation for the fluctuating plume rise is that the volcano is erupting at a constant rate throughout, but that the variation in wind speed is leading to the variation in plume height; this explanation is supported by our model analysis. Furthermore our calculations suggest that the MER is 6×10^6kg s^{-1}, far in excess of the estimate of 2×10^5kg s^{-1} that would be calculated using the Sparks-empirical relation when the plume had risen to it lowest height [3].

2.2 Implementation

The mathematical model is implemented within free-to-use software, PlumeRise, available at www.plumerise.bristol.ac.uk. This software integrates the model numerically using measured atmospheric conditions, as well as chosen values of the other parameters to determine the maximum height of rise. In addition it may be used as an inverse model to determine the MER given an observed height of rise. All computations are performed using the online server; the user has only to specify the source conditions, atmospheric profiles, and parameter values.

3 Impact

PlumeRise was launched in 2014 and is widely used by the community that has responsibilities for assessing the hazards to aviation. PlumeRise has over 500 registered users and over 100,000 separate calculations have been made in different eruptive scenarios from around the globe. These include over 170 practitioners and researchers in international and non-governmental agencies, including representatives from 8 of the 9 Volcanic Ash Advisory Centres (which have global responsibility for assessing the safety of aircraft flights).

The mathematical model of the rise of volcanic plumes and it implementation as PlumeRise have led to direct improvements in the operational assessment of environmental risk from volcanic ash during eruptive events and in contingency planning, which for the aviation industry, has the associated economic impact of avoiding unnecessary airspace closures while maintaining safe flying. Additionally, the research has impacted the use and development of quantitative plume models within several international agencies, including the quantification of uncertainty.

An example of its usage was during the long-lived Holuhran-Bardarbunga eruption, Iceland (2014–15). Although this event predominantly produced ground-based lava flows, the Icelandic Civil Protection and Emergency committee daily examined the possible consequences of an ash-producing eruption. The Icelandic Meteorological Office (IMO) held responsibility for monitoring and forecasting activities, both for Iceland, but also internationally. The IMO used PlumeRise operationally to compute source conditions for their dispersion models, and during the long-lived eruption, twice daily predictions of the potential heights of rise of the volcanic plume for a range of eruption rates were computed using the latest meteorological data; predictions that were shared with the committee for Icelandic Civil Protection.

Airborne ash additionally has consequences beyond aviation, affecting population health and livelihoods and the research has impacted international procedures for hazard assessment. In Mexico, for example, over 26 million people live within 100km of Popocatépeti Volcano and are affected by ash fallout from its frequent eruptions during the past three decades. Ash dispersion is modelled quantitatively by the Centro de Ciencias de la Atmósfera and the national civil protection agency of Mexico (Centro Nacional de Prevención de Desastres) to forecast the hazard to communities and the MER determined by [3] is a vital input to these computations.

Finally we remark that the 2010 eruption of Eyjafjallajökull challenged the international community to develop better forecasting tools and PlumeRise was part of the response. Best practice for plume modelling has been recently reviewed by International Association of Volcanology and Chemistry of Earth's Interior and its recommendations underpin hazard assessment worldwide. Continued development of models will be a critical component of strengthening mitigation measures.

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