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The effects of gas flow on granular currents

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Currents of particles have been quite successfully modelled using techniques developed for fluid gravity currents. These models require the rheology of the currents to be specified, which is determined by the interaction between particles. For relatively small slow currents, this is determined primarily through friction, which can be controlled and reduced by fluidizing the particles, so that they may become much more mobile. Recent results cannot be predicted using many of the proposed models, and may be defined by the interaction between the particles and the fluid through which they are passing. However, in addition, particles that are only initially fluidized also form currents that are also mobile, but otherwise are different from continuously fluidized currents. The mobility of these currents appears not to be connected to the time taken for them to degas. This suggests that defining the continuous stresses on the particle current may not be sufficient to understand its motion and that a challenge for the future is to understand the structure of these flows and how this affects their motion.

Keywords: fluidization; granular currents; particles; friction

1. Introduction

The transport of solid materials very frequently requires the material to be in granular form and transported as a current. The size of the flow can vary from relatively small-scale flows of tens of centimetres in length and depth in industrial circumstances to large-scale flows of several hundred metres in the environment. It is often important to understand the extent and speed of these currents, and their mechanics have been the subject of a large body of work. A mathematical framework for modelling dense particulate flows has been developed (e.g. Savage & Hutter 1989), and it shares many features with models of shallow gravity currents of dense fluids. In this approach, continuum mass and momentum conservation equations are written down and depth averaged, and are analogous to nonlinear shallow water equations, subject to a basal shear stress. However, the description of the physical internal mechanics within the currents that need to be applied to this framework are not clear and are represented only by the parametrization of the basal shear stress and an anisotropic normal stress: the interactions between the particles themselves, the particles and the surface over which they are moving, and the particles and the interstitial fluid.

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One contribution of 11 to a Theme Issue ‘New perspectives on dispersed multiphase flows’.

The important physical processes are those that determine the stresses within the current and at its interface with the surface over which it is passing. Most of the work on particle currents does not consider the interaction of the particles with the surrounding gas. For these analyses the dominant processes depend on the length of time of the contact between the particles: for instantaneous collisions the kinetic theory (Lun *et al.* 1984) can be used to describe the resultant stresses (Johnson & Jackson 1987; Campbell 1990), and for quasi-static contacts, frictional contacts (Savage 1984; Savage & Hutter 1989; Pouliquen 1999*a*,*b*). The two mechanisms were simply combined in a model by Johnson *et al.* (1990) who found that collisional stresses were dominant at the top of the current and frictional stresses at the bottom.

Both inter-particle collisions and friction are likely to be important in many currents, but their universal application is difficult. Inter-particle collisions will generate only significant stresses when particle currents are sufficiently large and vigorous. With respect to friction, Anderson & Jackson (1992) showed that frictional contacts must be included in the description of a granular flow down a slope to be realistic. Although models have been formulated that depend on friction, Lube *et al.* (2004) showed that the rapid radial flow of particles from released columns appeared to be independent of friction coefficient until it stops suddenly right at the end of the flow.

Most studies of granular flows neglect the influence of the interstitial gas on the motion of the current. Nott & Jackson (1992) studied flows through which gas was passed at a rate not sufficient to fluidize the particles, and found that collisional stresses become more important and frictional stresses less so. However, controlling the gas flow through the particles not only forces the effects of the support of the particles by the gas to be addressed, but also allows the effects of frictional forces within the granular flow to be controlled according to the proportion of the weight of the particles supported. In this paper, some of the effects of fluidization on currents of particles are described. When particles are continuously fluidized, it is possible to test the different rheological models that can be applied to particle currents. Of further interest is the effect of only fluidizing particles initially before the current is formed, and the effect that this appears to have in increasing the mobility of the particle current.

The effects of inter-particle collisions generated by fluctuations will not be considered in this paper. Measurements of fluidized beds away from bubbles have shown that particle temperature is negligible (Menon & Durian 1997; Rahman & Campbell 2002). The currents that are examined are relatively small scale (of the order of tens of centimetres) and slow, so their motion is not dominated by inter-particle collisions. A scaling analysis of the generation of granular temperature (appendix A of Eames & Gilbertson 2000) indicates that the effects of collision between the particles, and between the particles and the solid boundaries are negligible compared with the effects of the gas flow through them.

2. The effects of gas flow on particles

When particles are allowed to fall in a stream onto a horizontal surface, a triangular pile results. The particles within this pile are dominated by frictional contacts with their neighbours, so the interior of the pile remains static and

(*a*)

(*b*)

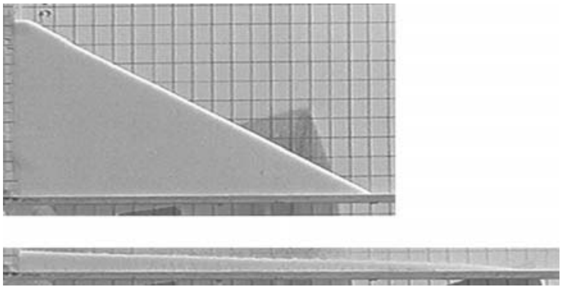

Figure 1. The effect of gas flow through glass ballotini on the flow of a current. The particles have been introduced from a funnel above the top, left-hand corner. (*a*) When there is no gas flow, the current is dominated by friction; (*b*) the particles are fluidized and the friction is greatly reduced. The squares in the background have sides of length 1 cm, and the scale for the two diagrams is the same. The pictures are of the experiments described in Eames & Gilbertson (2000).

particles that have recently arrived at the pile run down its surface in a series of lamina, resulting in a slow growth. When the particles are poured onto a surface through which a gas is passed at a sufficiently high rate, the behaviour of the current changes greatly, as shown in figure 1: the current moves as a bulk, is long and thin and fairly uniform, and travels much more quickly. The reason is that when the gas flow rate is sufficiently quick, the weight of the particles is supported so that the static pressure gradient through the current is given by

$$\frac{\Delta p}{h} = (\rho_{\rm p} - \rho_{\rm g})(1 - \epsilon)g, \tag{2.1}$$

where Δp is the pressure drop through the current; h is its depth; $\rho_{\rm p}$ and $\rho_{\rm g}$ are the densities of the particles and gas, respectively; and ϵ is the gas volume fraction or voidage within the current. As a result of this, friction within the flow is largely eliminated resulting in the difference of behaviour observed. The conventional expression for estimating the pressure drop through a fluidized bed is by use of the semi-empirical Ergun equation (Ergun 1952)

$$\frac{\Delta p}{h} = \frac{150(1-\epsilon)^2 \mu_{\rm g} U_{\rm g}}{\epsilon^3 d_{\rm p}^2} + \frac{1.75(1-\epsilon)\rho_{\rm g} U_{\rm g}^2}{\epsilon^3 d_{\rm p}}, \tag{2.2}$$

where $d_{\rm p}$ is the particle diameter and $\mu_{\rm g}$ the gas viscosity. The expression is based on a derivation for packed beds and so becomes inaccurate should a bed be expanded or heterogeneous. $U_{\rm g} = u_{\rm g}/\epsilon$ is the superficial gas velocity (i.e. the volume flux). The first term of the expression is for laminar flow (particle Reynolds number $Re_{\rm p} = \rho_{\rm g} u_{\rm g} d_{\rm p}/\mu_{\rm g} < 20$) and the second term for turbulent flow; for gas particles in air with a diameter of the order of 100 μm, typically, the second term can be neglected. Combining (2.1) and the linear part of (2.2) gives an expression for the minimum velocity of fluidization,

$$u_{\rm mf} = \frac{d_{\rm p}^2 \epsilon_{\rm mf}^3 (\rho_{\rm p} - \rho_{\rm g}) g}{150 \mu_{\rm g} (1 - \epsilon_{\rm mf})}, \tag{2.3}$$

where, in theory, the weight of the particles is balanced by the drag of the gas flow through them. This is physically an important parameter as most of the particles in the bed effectively become weightless and so friction is nearly eliminated and the particles behave in a fluid-like manner. This affects not only the internal mechanics of a powder, but also how it interacts with walls as if friction is negligible; there can be slip at the boundaries, and the effects of their presence on the shear of the particles will not be transmitted into the bulk of the bed.

The details of what happens at the point of fluidization depends upon the size of the powder and can be described by using the Geldart classification (Geldart 1972), which is based on the bubbling behaviour of a powder when it is fluidized. A bed of the finest particles, group A particles, expands at velocities immediately above U_{mf} without the presence of bubbles, and at a higher critical velocity this breaks down and bubbles form; group B particle beds bubble immediately at velocities greater than U_{mf}, but each of the bubbles is centred around a self-contained ball of gas. There has been a debate about the causes of this behaviour, but it is probably caused by mild cohesion (e.g. Tsinontides & Jackson 1993).

The concept of the point of minimum fluidization is powerful and well established. The Ergun equation can produce good estimates of the size of the fluid flow necessary to fluidize a powder. However, the behaviour of real beds, especially when shallow, can be different. While it is easy to generate the pressure–flow rate curve that agrees with the classical view of a fluidized bed with a sharp transition from a linear increase in pressure with gas flow rate to a constant pressure drop, equation (2.3) predicts that U_{mf} is a property of a powder and should not depend on the bed of interest, while in fact U_{mf} decreases with bed height and the proportion of the weight of the particles supported (figure 2). The reason for this non-standard behaviour is probably the strong effect that voidage has on pressure drop, and that the beds are not homogeneous. For example, it is possible to observe distinct channelling behaviour in shallow beds, which may account for the observations. In addition, the model underlying the Ergun equation is that the bed is similar to a packed bed; in areas where there is significant dilation this physical model will break down.

3. The continuous fluidization of granular currents

When particles form a current over a porous, horizontal surface through which there is a gas flow sufficient to fluidize the particles, a long, thin current forms. The motion of such a current appears to involve its bulk, and so conservation equations can be written down to describe the motion of the particles. The voidage within the current is not only important for the mechanics of the current, but also very difficult to measure. The currents appear to be homogeneous and dense when inter-particle collisions are not significant, and so it is assumed that voidage is a constant. The particle conservation equations (Eames & Gilbertson 2000) are for mass,

$$-\frac{\partial \epsilon}{\partial t} + \nabla \cdot ((1-\epsilon)\boldsymbol{v}) = 0, \tag{3.1}$$

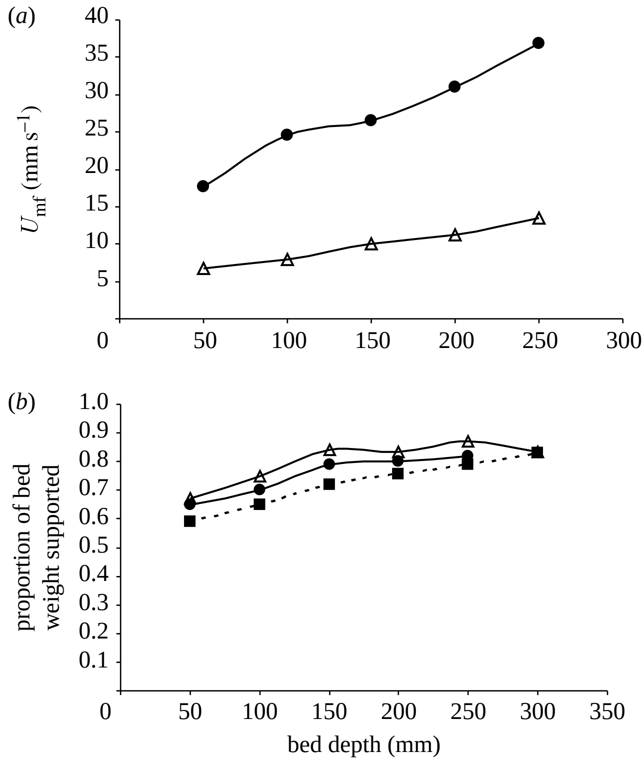

Figure 2. Variation of the behaviour of a rectangular bed of dimensions 8.0×9.4 cm with bed depth from pressure measurements at the bottom of the bed. The beds consist of glass particles of diameter 45–90 μm (group A, triangles), 106–212 μm (group B, circles) and 250–425 μm (group B, squares).

where $\boldsymbol{v}$ is the particle velocity; and for momentum,

$$(1-\epsilon)\rho_\mathrm{p}\frac{\mathrm{D}\boldsymbol{v}}{\mathrm{D}t} = (\rho_\mathrm{p}-\rho_\mathrm{g})(1-\epsilon)\boldsymbol{g} - \nabla p_\mathrm{s} + \boldsymbol{f}_\mathrm{gs} + \nabla\cdot\boldsymbol{\sigma}, \tag{3.2}$$

where the second term on the r.h.s. expresses the effect of the pressure transmitted through the particle phase, $\boldsymbol{f}_\mathrm{gs}$ is the interaction force per unit volume between the gas and the particles, which can be expressed using equation (2.3), and $\boldsymbol{\sigma}$ is the stress within the current less the static pressure p_s. For the small-scale currents being considered here, the inertial terms are negligible compared with the drag force (Eames & Gilbertson 2000).

In the absence of particle collisions and frictional forces, the static pressure within the solid phase driving the current will be generated within the flow owing to the drag exerted by the fluid on the particles and will equal the gradient owing to the weight of particles that is equivalent to the hydrostatic pressure in a liquid (Eames & Gilbertson 2000).

With a suitable choice of expression for stress within the current, it is possible to solve the momentum equation subject to the volume condition

$$\int_0^{x_\mathrm{f}} h\,\mathrm{d}x = qt, \tag{3.3}$$

for a continuous release of particles, as that which happens when they are passed through a funnel (Neddarman 1992). The currents are long and thin, and so the shallow water approximation can be made so that depth-averaged equations can be developed. Furthermore, on a horizontal surface, the equations are suitable for scaling and then solving through the use of similarity to arrive at analytical solutions.

(*a*) *Modelling of stress in a continuously fluidized current*

There are a number of different possible descriptions of stress within the current. Because the currents can be depth averaged, what happens at the boundary becomes very important. There are a large number of papers concerned with the interaction between a granular flow and the boundary, either through collisions or friction. In addition, the shear behaviour of a granular medium is of great interest, though most have treated it as a Newtonian material. Measurements of fluidized beds using rheometers have come up with values of between 0.1 and 1 Pa s, though the specific value changes with method (Davidson *et al.* 1977).

There are therefore several descriptions available for the shear stresses within the current. One approach is to make a direct analogy with a Newtonian viscous fluid with a no-slip or a partial slip boundary condition, and the presence of a boundary layer up to a no-stress boundary condition on the surface of the current; therefore, by fixing $\tau = \mu(\partial u_z/\partial z)$, we obtain for a constant particle flux in one dimension

$$x \sim q^{3/5} t^{4/5}, \tag{3.4}$$

$$h \sim q^{2/5} t^{1/5}, \tag{3.5}$$

where the quantities have been non-dimensionalized by

$$\left(\frac{9\mu^2}{(\rho_\mathrm{p}-\rho_\mathrm{f})^2 g}\right)^{1/3}, \quad \left(\frac{3\mu}{(\rho_\mathrm{p}-\rho_\mathrm{f}) g^2}\right)^{1/3}$$

for length and time, respectively. Alternatively, the variation in the horizontal gas speed could be confined to a small layer of thickness δ, so that

$$x \sim q^{1/2} t^{3/4} \delta^{1/4}, \tag{3.6}$$

$$h \sim q^{1/2} t^{1/4} \delta^{1/4}. \tag{3.7}$$

A third point of view is that the interaction is provided by drag with the air, so that $\tau_{xz} = (1/2)\rho_\mathrm{f} C_\mathrm{D} u_x^2$, where C_D is a constant, which results in

$$x \sim q^{2/5} t^{4/5}, \tag{3.8}$$

$$h \sim q^{3/5} t^{1/5}. \tag{3.9}$$

Experiments were performed using glass ballotini of diameter 250–425 μm along a surface of length 1 m through which gas was passed at a sufficient rate to fluidize the particles strongly. The particles were introduced at a continuous flux through a funnel. The results of the experiments are ambiguous. The scaling of experimental measurements with flow rate of particles is shown in figure 3. The

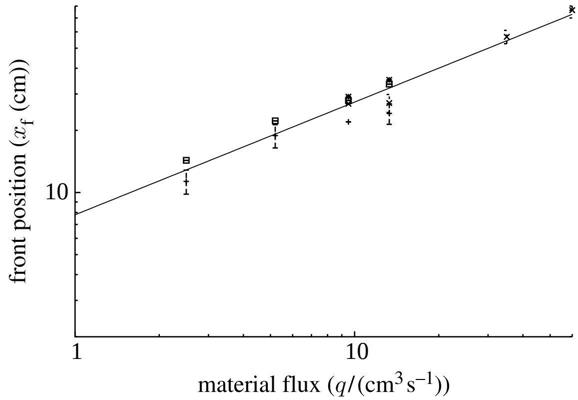

Figure 3. Scaling of a continuously fluidized particle current with the mass flux of particles into the system. The currents were measured at a fixed time of 2 s after the introduction of particles, which was sufficiently short to accommodate all the currents (best fit: solid line, $x_f \sim q^{0.546}$; pluses, $U_g/U_{mf}=1.3$; crosses, $U_g/U_{mf}=1.486$; asterisks, $U_g/U_{mf}=1.86$; squares, $U_g/U_{mf}=2$).

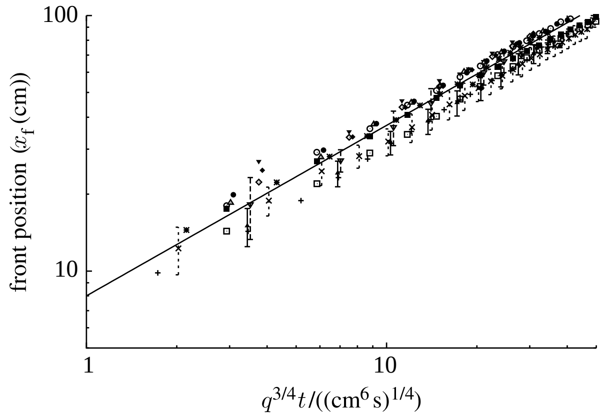

Figure 4. Scaling of a continuously fluidized particle current with time on logarithmic axes. The line corresponds to a power of 2/3. The error bars show the standard deviation values from repeated experiments.

best fit of the data results in a value for the exponent of 0.56, which is closest to that for the treatment of the particle current like a Newtonian fluid (0.6); however, in the case of time it can be seen from figure 4 that none of the expected scalings apply, but under a wide range of conditions the data collapse well to a power of 2/3. This corresponds to what would be expected if there was an additional stress on the particle phase proportional to velocity, but the physical origin of such a stress is not clear.

4. Granular currents formed from initially fluidized particles

Modelling the particle current in a similar manner to a fluid focuses attention on the stresses that exist within it. This has been a successful approach, but the only concession to the particulate nature of the current is through the calculation of

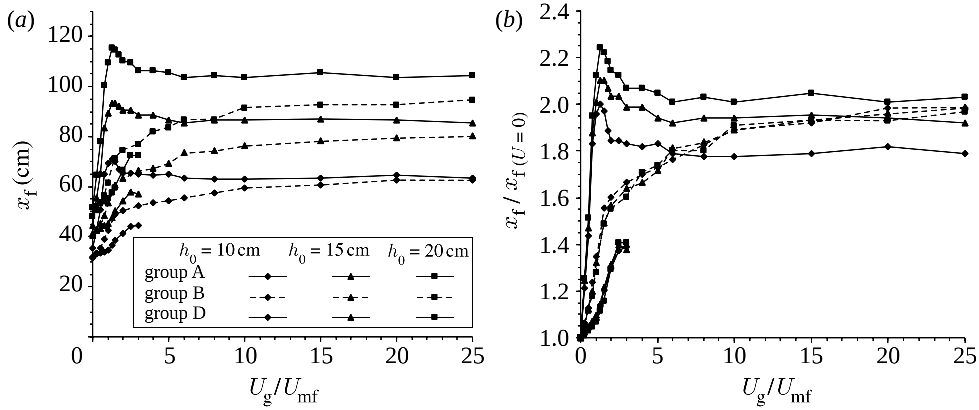

Figure 5. The mobility of constrained particle currents that are initially fluidized. (*a*) The length of run-out is plotted against the gas velocity normalized with respect to the minimum fluidization velocity for differently sized particles and different values for initial height, h_0. (*b*) The run-out has been normalized with respect to the run-out when there is no initial fluidization. The particles are glass ballotini, with sizes for group A of 40–90 μm, for group B of 106–212 μm and for group D of 600–800 μm. The group A particles are sufficiently fine for there to be an interval above U_{mf} where the bed will freely expand without the presence of bubbles. Adapted with permission from Roche *et al.* (2004).

Figure 6. Picture of a particle current formed from an initially fluidized bed of particles described in Roche *et al.* (2004). The particles are of group A and are released from the reservoir at the left of the picture. The spacing between the background lines is 50 mm.

the interaction force between the fluid and the particles. This view is not consistent with the currents that form when the particles are initially fluidized, but are not subject to a gas flow once they have formed. From the classical viewpoint, it might be expected that at first a mobile current would be formed that would become retarded fairly quickly once the gas initially within the bed had left the bed and friction becomes dominant. In fact, this does not appear to happen, but instead a mobile current forms that remains so beyond the time taken for the gas to escape, but appears to behave differently from a continuously fluidized current.

Roche *et al.* (2004) conducted experiments where a wall was removed from a rectangular fluidized bed. The resulting flow was over an impermeable surface and contained within two parallel walls so the motion was effectively one dimensional. Figure 5 shows some of the results from these experiments. It can be seen that the effect of initially fluidizing the powder is to significantly increase its mobility, by up to 70% when the powder is well fluidized. Long, thin currents are formed where the particles move as a bulk, as shown in figure 6, for continuously fluidized currents. When initially non-fluidized, all the particles had a similar, triangular height profile with a little run-out at the front; when fluidized, the finest particles took on the long, humped profile shown, as did larger particles when they were more strongly fluidized.

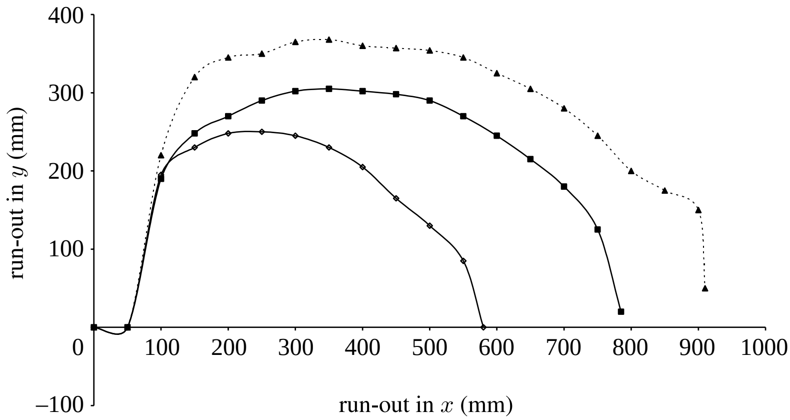

Figure 7. Plan view of the deposits made by a current of initially fluidized group A particles allowed to form on a flat, impermeable surface. The particles are released from a reservoir at the lower, left-hand corner of the figure in which they could be fluidized. The line corresponding to $y=0$ is at the edge of the reservoir, which therefore extends from 0 to -100 mm. The different lines correspond to different initial gas velocities within the reservoir. The initial height of the column of particles was 300 mm (diamonds, $U=0$; squares, $U=U_{\mathrm{mf}}$; triangles, $U=2U_{\mathrm{mf}}$).

When a mixture of differently sized particles is put in the bed, then it is notable that very little segregation of differently sized components takes place, unlike non-fluidized and continually fluidized currents. The flows typically move at a constant speed before quickly coming to a halt (Roche *et al.* 2004). The same increase in mobility has been seen in tall columns of particles (Lube *et al.* 2004): when the columns are short, then friction dominates and the expected triangular pile is formed; when the column is tall, then the particles run-out further and form a shallow pile over a large area.

It can be seen from figure 5 that particle size is very important in determining the behaviour of the fluidized particles and that the behaviour of these different types of particles as defined by the Geldart classification is distinct. The group A particles are very much more mobile at lower gas speeds than the larger particles, and the mobility of group A particles is dependent on the initial height of the bed. Particle size was also important when the particles were continuously fluidized, but in that case the dependency was the other way round: instead of larger particles being less mobile, the currents of the finer particles were less mobile, possibly owing to the action of weak inter-particle forces (Gilbertson & Eames 2003).

When the particles are released using the same apparatus, but are allowed to expand sideways, the behaviour of initially fluidized beds of particles does not change markedly when they are allowed to expand laterally. For example, figure 7 shows the deposits for different degrees of initial fluidization. It can be seen that a roughly elliptical shape is obtained and the increase in mobility results from increased initial fluidization. Similar shaped deposits are obtained at all heights and degrees of fluidization apart from the flat back for the deepest beds. In addition, the aspect ratio of the deposit appears to be linear with time after an initial lag.

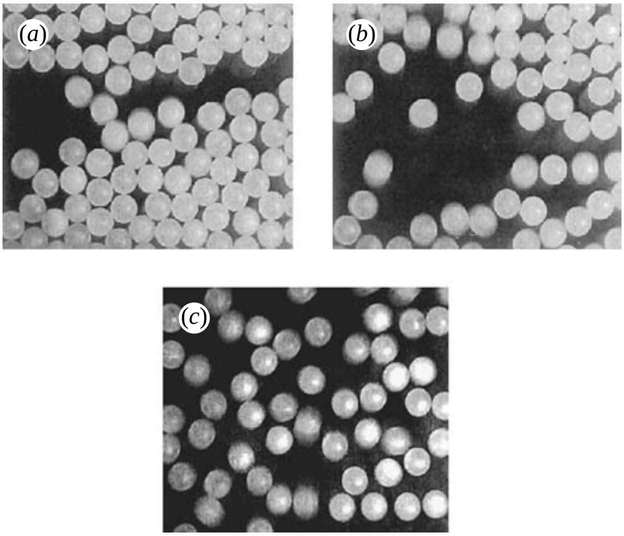

Figure 8. Pictures of the interior of a bubbling, planar fluidized bed. The particles are made from polypropylene and are large and light ($d_p = 7.9$ mm and $\rho_p = 1290$ kg m^{-3}) and constrained between two clear walls. (*a*) Point of minimum fluidization $U/U_{mf} = 1$; (*b*) weakly fluidized $U/U_{mf} = 1.08$; and (*c*) strongly fluidized $U/U_{mf} = 1.40$.

5. Discussion

The reasons for the greater mobility of currents of fluidized particles than non-fluidized particles are clear—the support of the particles' weight by the gas flow. The physical effects of doing this on a particle current are not yet well understood. It is obvious that the gas flow changes the force balance on the particles in a current so that friction between them is greatly reduced and they can move in bulk. However, the particle currents appear to reach equilibrium quickly, and it is not clear what is balancing the gradients of hydrostatic pressure driving the current. The currents described here are too small and slow for inter-particle collisional stress to be important, and the conventional models that emphasize rheological behaviour and interfacial friction do not generate the required dependencies on time and flow rate.

Understanding the dynamics of continuously fluidized currents is not sufficient to explain the behaviour of initially fluidized particles: once the particles have moved out of the bed then it would be expected that the gas would escape and they will become defluidized and their behaviour dominated by friction. This is not the case for a significant period of time, at the end of which the role of friction in arresting the current appears to become dominant very quickly, and the current stops.

A probable explanation for this is that the internal structure of the current is changed when the particle is fluidized, and that this allows the mobile behaviour to take place. Figure 8 shows the particles in a planar fluidized bed when static, when close to the minimum point of fluidization, and when well fluidized. In all cases, the bed is fluidized, in that the pressure drop over the bed does not vary with flow rate; however, the structure changes markedly: close to the minimum point of fluidization, the particles are mostly touching each other, while when they are well fluidized, the particles are not touching each other. It has been shown that the dynamic behaviour of fluidized beds is different when a bed is close to the minimum point of fluidization from when it is fluidized and is different at low bed depths than large bed depths (Croxford *et al.* 2005).

For models where the current is treated as a continuous medium, the structure of the bed can only be included through the void fraction or voidage ϵ. For the currents in this study, this appears to be constant on average, and aside from the presence of bubbles, they are homogeneous: no dilute layer on the surface of the current is apparent. The voidage of a bed can change with gas flow rate when the particles are sufficiently fine (Geldart's group A particles). An explanation for the increased mobility of the initially fluidized particles would then be that the beds become expanded and then during the collapse of the particle current, once it is moving, the particles continue to be fluidized over an appreciable period of time and it is this behaviour that maintains its high mobility; however, there was no evidence that the time taken for the bed to collapse was at all related to the motion of the current (Roche *et al.* 2004). In addition, many other features of the currents from the initially fluidized bed differ from those that are continually fluidized. For example, no segregation is seen in mixtures of particles and finer particles are more mobile than currents of larger ones, not less, indicating that initial fluidization gives rise to currents of different characteristics from those that are continually fluidized.

All these approaches treat currents, and the stresses generated within them, as continuous. The classical modelling of static assemblies of particles was to treat them in a similar manner as a continuous, yielding solid. Experimental measurements (e.g. Howell *et al.* 1999) and numerical models (e.g. Makse *et al.* 2000) for static assemblies of particles have shown that, in fact, they are not uniform and continuous, but that the stresses within them are concentrated in a relatively small number of particles that make up chains through which forces are transmitted. In the same direction as the force along them, the chains are very stiff; however, in shear they are very weak. Campbell (2006) has proposed that the presence of force chains may be important in moving assemblies of particles as well as stationery ones. The chains may form in a direction oblique to that of the current and are anchored at the bottom of the surface they are moving over. This anchor point can act as a pivot point about which the chain can rotate until it is perpendicular to the direction of the current and hence very weak and it dissolves. Eventually, the force is not sufficient to overcome these weak chains and a strong network of force chains forms, stopping the current quickly. Such an approach would provide a force balancing the static pressure gradient, and it would provide a connection between the motion of the current and the surface over which it is passing; however, there is as yet no experimental evidence connecting structure with motion. Other structures are conceivable and it is likely that the strength of them will depend on the local fluid flow pattern as well.

6. Conclusion

Currents of fluidized particles are physically interesting because the dominating effects of friction are removed within the overall current. Modelling particle currents using similar techniques to fluid gravity currents has been very successful, but as yet does not predict the results discussed in this paper. One of the characteristics of multiphase flows that make them intriguing subjects of study is that often it is not possible to get the scale separation that can be achieved in single phase fluid between the components of a flow and its overall, apparent nature. The challenge of making further progress in understanding and modelling particle currents may be to understand the particular behaviour of particles: how they interact with one another, the fluid through which they travel and the surfaces with which they come into contact. The inclusion of instantaneous collisions in calculations of particulate systems has had a great effect, but as yet the influence of longer term contacts and the structure and arrangement of particles in flows and the effects these have on their motion is poorly understood and this presents a substantial and interesting challenge for the future.

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References

Anderson, K. & Jackson, R. 1992 A comparison of the solutions of some proposed equations of motion of granular materials for fully developed flow down inclined planes. *J. Fluid Mech.* **241**, 145–168. (doi:10.1017/S0022112092001988)

Campbell, C. 1990 Rapid granular flows. *Annu. Rev. Fluid Mech.* **22**, 57–92. (doi:10.1146/annurev.fl.22.010190.000421)

Campbell, C. 2006 Granular material flows—an overview. *Powder Tech.* **162**, 208–229. (doi:10.1016/j.powtec.2005.12.008)

Croxford, A., Harrison, A. & Gilbertson, M. A. 2005 The optimisation of pressure measurements for the control of bubbling fluidised beds. *Int. J. Chem. Reactor Eng.* **3**, A39. (doi:10.2202/1542-6580.1272)

Davidson, J., Harrison, D. & Guedes de Carvalho, J. 1977 On the liquidlike behaviour of fluidized beds. *Annu. Rev. Fluid Mech.* **9**, 55–86. (doi:10.1146/annurev.fl.09.010177.000415)

Eames, I. & Gilbertson, M. 2000 Aerated granular flow over a horizontal rigid surface. *J. Fluid Mech.* **424**, 169–195. (doi:10.1017/S0022112000001920)

Ergun, S. 1952 Fluid flow through packed beds. *Chem. Eng. Prog.* **48**, 89–94.

Geldart, D. 1972 The effect of particle size and size distribution on the behaviour of gas-fluidized beds. *Powder Tech.* **6**, 201–215. (doi:10.1016/0032-5910(72)83014-6)

Gilbertson, M. A. & Eames, I. 2003 The influence of particle size on the flow of fluidised powders. *Powder Tech.* **131**, 197–205. (doi:10.1016/S0032-5910(02)00343-1)

Howell, D., Behringer, R. & Veje, C. 1999 Fluctuations in granular media. *Chaos* **9**, 559–572. (doi:10.1063/1.166430)

Johnson, P. & Jackson, R. 1987 Frictional–collisional constitutive relations for granular materials, with application to plane shearing. *J. Fluid Mech.* **176**, 67–93. (doi:10.1017/S0022112087000570)

Johnson, P. C., Nott, P. & Jackson, R. 1990 Frictional–collisional equations of motion for particulate flows and their application to chutes. *J. Fluid Mech.* **201**, 501–535. (doi:10.1017/S0022112090001380)

Lube, G., Huppert, H., Sparks, R. & Hallworth, M. 2004 Axisymmetric collapses of granular columns. *J. Fluid Mech.* **508**, 175–199. (doi:10.1017/S0022112004009036)

Lun, C. K. K., Savage, S. B., Jeffrey, D. J. & Chepurniy, N. 1984 Kinetic theories for granular flow: inelastic particles in Couette flow and slightly inelastic particles in a general flowfield. *J. Fluid Mech.* **140**, 223–256. (doi:10.1017/S0022112084000586)

Makse, H., Johnson, D. & Schwartz, L. 2000 Packing of compressible granular materials. *Phys. Rev. Lett.* **84**, 4160–4163. (doi:10.1103/PhysRevLett.84.4160)

Menon, N. & Durian, D. 1997 Particle motions in a gas-fluidized bed of sand. *Phys. Rev. Lett.* **79**, 3407–3410. (doi:10.1103/PhysRevLett.79.3407)

Neddarman, R. M. 1992 *The statics and kinematics of granular materials*. Cambridge, UK: Cambridge University Press.

Nott, P. & Jackson, R. 1992 Frictional–collisional equations of motion for granular materials and their application to flow in aerated chutes. *J. Fluid Mech.* **241**, 125–144. (doi:10.1017/S0022112092001976)

Pouliquen, O. 1999*a* Scaling laws in granular flows down rough inclined planes. *Phys. Fluids* **11**, 542–548. (doi:10.1063/1.869928)

Pouliquen, O. 1999*b* On the shape of granular fronts down rough inclined planes. *Phys. Fluids* **11**, 1956–1958. (doi:10.1063/1.870057)

Rahman, K. & Campbell, C. 2002 Particle pressures generated around bubbles in gas-fluidized beds. *J. Fluid Mech.* **455**, 103–127. (doi:10.1017/S002211200100725X)

Roche, O., Gilbertson, M. A., Phillips, J. C. & Sparks, R. S. J. 2004 Experimental study of gas-fluidized granular flows with implications for pyroclastic flow emplacement. *J. Geophys. Res.* **109**, B10201. (doi:10.1029/2003JB002916)

Savage, S. B. 1984 The mechanics of rapid granular flows. *Adv. Appl. Mech.* **199**, 177–215.

Savage, S. B. & Hutter, K. 1989 The motion of a finite mass of granular material down a rough incline. *J. Fluid Mech.* **199**, 177–215. (doi:10.1017/S0022112089000340)

Tsinontides, S. C. & Jackson, R. 1993 The Mechanics of gas-fluidized beds with an interval of stable fluidization. *J. Fluid Mech.* **255**, 237–244. (doi:10.1017/S0022112093002472)