Avalanche Defence Schemes

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Abstract Snow avalanches are hazardous. They flow rapidly down mountain gulleys, destroying homes, built infrastructure and leading to fatalities. One way of mitigating against the hazards they pose is to build large structures in their flow tracks to deflect, retard and arrest the motion. This article describes some of the research that underpins modern guidelines of how to design these structures. It reports mathematical models that capture the transformation in the state of the flowing avalanche as it interacts with large-scale obstacles and the predictions that can be used to optimise engineering designs.

Introduction

Snow avalanches are potent hazards. They may flow at speeds in excess of 60 ms^{-1} and transport large volumes of snow downhill, potentially destroying and burying houses and posing a significant threat to human life. Snow avalanches are typically released on steep mountain slopes when the snowfall has been heavy and the snow pack on the ground becomes unstable, possibly due to weak layers, rapid loading, or perhaps some other external forces such as explosions used to trigger avalanches or skiers or other travellers in the mountains. They occur widely and owing to the increasing development of mountaineous regions for settlement and leisure, there is a pressing need to assess the hazards they pose to lives and livelihoods, and to develop strategies to mitigate against these hazards. Part of the solution relies on the accurate assessment of the area inundated by potential avalanches and this 'hazard' zoning is routinely employed to identify the risks. However, there are locations where human

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developments and even settlements are unavoidable in snow avalanche terrain and here alternative measures must be devised to defend against the potential effects of the avalanches. This article describes the mathematical modelling that underpins some of the guidelines of how to design structures to defend against avalanches [1].

Iceland is a country that is particularly susceptible to snow avalanches. High lying snow on the country's mountains is frequently mobilised and flows rapidly down steep mountainous gulleys towards some of the inhabited regions along the coastline. Historical analysis shows records of snow avalanche damage around the entire country, but particularly notable avalanches occured at Neskaupstaður in eastern Iceland in 1974 and at Súðavík and Flateyri in north-western Iceland in 1995. They led to fatalities, the destruction of buildings and substantial economic losses. It was of considerable concern that the some of the damage and fatalities due to the avalanche at Súðavík and Flateyri were outside of the zone that had been assessed as hazardous. This prompted the decision to build large engineered structures to defend against the effects of snow avalanches. However at the time there were only rudimentary guidelines of how to design such protective measures, limited understanding of the nature of the interaction between avalanches and solid obstacles and virtually no mathematical models for its prediction. These incidents led to a comprehensive programme of research that culminated in new guidelines for the design of avalanche protection measures in the run-out zone of snow avalanches [1] and some of the research that underpins these guidelines is reported here [2–4].

There are three types of obstacles that are built in the run-out zones of avalanches to protect locations farther downhill. These are deflecting dams, which turn the avalanche away from the protected infrastructure [2], catching dams, designed to be sufficiently high so that no flow may overtop it [3], and braking mounds, which retard the oncoming flow and reduce its subsequent runout [4] (see Fig. 1). We present in detail the mathematical results that enable design guidelines to be drawn up for deflecting dams.

Fig. 1 Photograph of the avalanche protection measures at Seljalandsmúli, Ísafjörður, north-western Iceland. The figure shows two rows of braking mounds, each mound is of height 7 m, and a 700 m long deflecting dam of height 16 m

Mathematical Models of Snow Avalanches

Snow avalanches comprise particles or clumps of snow surrounded by air. A useful idealised description is to treat a vertical section through an avalanche as being composed of three layers [1], although the interfaces between these layers are not sharp and the flow is inherently fluctuating. At its base the avalanche has a dense core in which particles directly interact with each other through dissipative collisions and enduring frictional contacts; air plays a negligible role in its mechanics. Typically the density of the dense core is $300\,\mathrm{kgm}^{-3}$, while the thickness is 1–3 m. Above the core is the fluidised layer in which particles undergo relatively long durations between the contacts with each other. This layer is less dense (10–$100\,\mathrm{kgm}^{-3}$) and of typical thickness 2–5 m. Above the fluidised layer there is sometimes a 'powder snow' cloud. Here the volumetric concentration is low and the particles are supported by the action of turbulence in the air. Powder snow clouds are highly mobile, since they experience smaller resistance than the denser layers. Their density is relatively low ($3\,\mathrm{kgm}^{-3}$), but their thickness may be in excess of 100 m and so may neverthess be associated with the movement of substantial masses of snow.

In this article we focus on the interaction of the dense core of the avalanche with obstacles, because the core is found to exert the highest pressures and cause the most damage. We treat the flowing snow as a continuum and so do not calculate the motion of individual particles, but rather deduce the bulk properties, such as the density, which observations suggest does not vary very much, and the velocity field. Furthermore, because the dense core is relatively shallow, the velocity is predominantly parallel with the underlying boundary with only a negligible component of velocity perpendicular to the boundary. This means that there is force balance between the bed-normal component of the weight of the snow per unit area plus inertial forces induced by the curvature of the bed and the corresponding components of the internal stresses within the avalanche body. It is then possible to deduce governing equations that express mass conservation and the balance of momentum downslope and across the slope; these equations are known as the shallow water equations, often used for modelling hydraulic flows, but here modified to account for the resistance due to granular interactions [2].

A key dimensionless parameter in this model is the Froude number, which is given by

$$F = \frac{|\mathbf{u}|}{\sqrt{g\cos\theta h}}, \tag{1}$$

where $\mathbf{u}$ denotes the velocity field, h the flow thickness, g gravitational acceleration and θ the inclination from the horizontal of the slope down which the avalanche flows. Large avalanches are often associated with relatively high Froude numbers with typical values in the range $F = 5$–10 [1].

The shallow water equations are unable to represent flows in which the depth and velocities of the flowing layer vary over relatively short distances, because the governing equations are based upon the neglect of appreciable bed-normal accelerations. Instead abrupt transitions are captured as 'jumps' in the flow variables. These discontinuities are termed 'shocks' and across a stationary shock we enforce the following conditions that encompass mass and momentum conservation [2]

$$[h(\mathbf{u}.\,\mathbf{n})]_-^+ = 0 \qquad \text{and} \qquad \left[h\mathbf{u}(\mathbf{u}.\,\mathbf{n}) + \tfrac{1}{2} g \cos\theta h^2 \mathbf{n}\right]_-^+ = 0, \tag{2}$$

where $\mathbf{n}$ is a unit normal vector perpendicular to the discontinuity and the square brackets, $[\ldots]_-^+$, denote the difference between the variables either side of the shock.

Deflecting Dams

Armed with the shock conditions (2), we may now calculate the interaction between an oncoming avalanche and a deflecting dam. In particular, we calculate the flow depth adjacent to the dam because this determines how high the barrier must be built. Additionally, we evaluate the magnitude of the pressure within the flow and the effects that the angle the barrier makes to the avalanche flow direction has upon the deflection, since these could both influence the design.

In the analysis that follows, the avalanche flows downslope with depth h_1 and velocity $\mathbf{u}_1 = U_1\hat{\mathbf{x}}$, where $\hat{\mathbf{x}}$ is a unit vector along the x-axis orientated downslope (Fig. 2), so that the oncoming Froude number $F = U_1/(g\cos\theta h_1)^{1/2}$. The avalanche encounters a rigid, stationary obstacle orientated at an angle γ to the x-axis and forms a steady shock, downslope of which the depth of the flow is h_2 and the velocity field is $\mathbf{u} = U_2(\cos\gamma\hat{\mathbf{x}} + \sin\gamma\hat{\mathbf{y}})$. The shock is assumed to be attached to the apex of the deflector and orientated at an angle β to the x-axis (such that $\beta > \gamma$); a unit normal vector to the shock is given by $\mathbf{n} = (\sin\beta, -\cos\beta)$. It is then possible to simultaneously solve (2) to determine the relative flow depth $H = h_2/h_1$, the relative speed, $V = U_2/U_1$ and the shock angle β in terms of the upslope conditions and the deflector angle, γ.

We find the following implicit expression for the deflection angle, β, as a function of the deflector angle, γ and Froude number, F, given by

$$\tan\gamma = \frac{4\sin\beta\cos\beta(F^2\sin^2\beta - 1)}{3 + 4\cos^2\beta(F^2\sin^2\beta - 1) + \sqrt{1 + 8F^2\sin^2\beta}}. \tag{3}$$

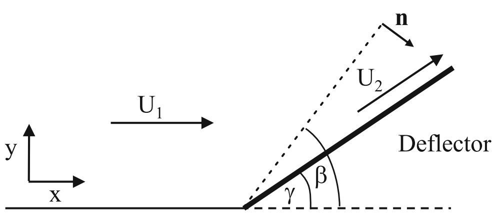

Fig. 2 Plan view of the flow configuration for a deflection dam. The oncoming motion with velocity U_1 is deflected to flow parallel with the dam at velocity U_2, via a shock attached to the apex of the dam

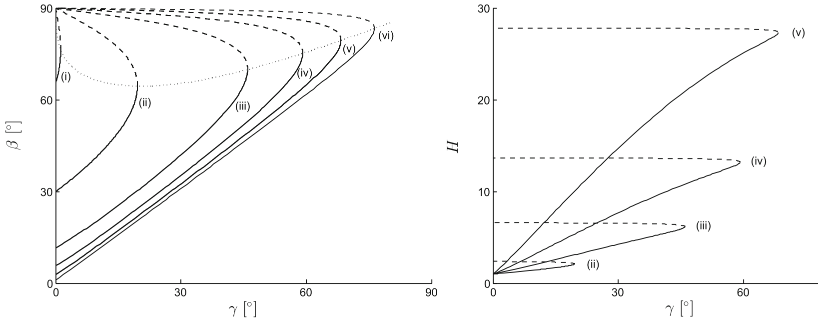

Fig. 3 The deflection angle, β, and the depth of the flow downslope of the shock relative to the upslope depth, H, as functions of the deflector angle for $F = 1.1, 2, 5, 10, 20$ & 50 (curves (i)–(vi)). Weak shock solutions are plotted in a *solid line*; strong shocks with a *dashed line*. The locus of the maximum deflector angle for which an attached steady shock exists is plotted with a *dotted line*

This relationship is plotted in Fig. 3 for a range of values of the upstream Froude number. We note several feature of these results. For a given Froude number greater than unity, there is a maximum deflector angle, γ_m, for which solutions exist; the locus of maximum deflector angles, γ_m, is also plotted in Fig. 3. Furthermore, we note that when there are solutions ($\gamma < \gamma_m$), then there are two solutions for the deflection angle β. We term these the 'weak' and 'strong' shock solutions, corresponding to the smaller and larger values of β, respectively. When the upstream Froude is less than unity $F < 1$, or when the deflector angle is greater than γ_m, there are no steady solutions with a shock attached to the apex of the dam. We also plot the relative height as function of the deflector angle in Fig. 3 for a range of Froude numbers. We observe the general trends that the relative depth of the flow for the weak shock solutions increases with increasing Froude number and with increasing deflector angle. When the Froude number of the oncoming avalanche is large ($F \gg 1$), we find

$$\beta = \gamma + \frac{1}{\sqrt{2}\, F \cos\gamma} + \cdots \quad \text{and} \quad H = \sqrt{2} F \sin\gamma + \cdots, \tag{4}$$

for the weak shock and these two asymptotic results are useful in the physical regime of interest.

Since the mechanics of granular materials are incompletely represented by all current mathematical models due to the different ways in which the grains may interact and due to their highly dissipative nature, it is vital to test predictions against results from experiments. Laboratory-scale experiments demonstrated that in the steady state, the predictions of the flow depth and deflector angles are in very good accord with this simple theory and that the flow adjusted to the weak shock solution [2]. The experiments also revealed additional features that may be useful in the design of avalanche defence dams, such as the height of the initial splash of the grains on impact with the dam before a steady state is fully established.

Application

The research reported in this article has underpinned and is embodied in modern guidelines for the design of avalanche defence barriers [1, 5]. These reference books and practical guides are used extensively by specialists across Europe who design deflecting and catching dams and have played a crucial part in securing very significant investment in infrastructure through large-scale civil engineering projects aimed at reducing the risk of avalanche damage to settlements.

For example, in Iceland since 2008, over €54M has been spent on large-scale installations, which were constructed on the basis of these new guidelines. There are a further projects in planning and design stages with the expected infrastructure investment running at over €5–10M per year until at least 2020. These current schemes and the further planned developments reduce the risk of avalanche damage to many endangered settlements. However, the use of the guidelines extends to many other countries. Norwegian, Swiss and Austrian engineers have designed several projects to defend lives and livelihoods partly based on the new guidelines, while notably the guidelines are also underpinning the design and current construction of the mounds, deflectors and catching dams at the base of the Taconnaz glacier, Chamonix, France, an investment in infrastructure of approximately €10M.

The modern guidelines and the research also form the basis of highly regarded training courses for avalanche professionals. Delivered by expert practitioners, there have been a series of courses for avalanche engineers from the public and private sectors in France, Italy and Spain, under the framework of the European Summer School on Avalanches.

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