Advanced Fluids 2017

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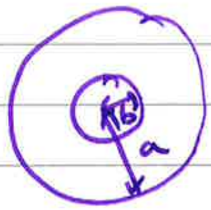

$\underline{u} = w(r)\,\underline{\hat{z}} \qquad a \leq r \leq b$

Unseen, although flow driven by pressure gradient in lectures

(a) $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) = 0$ subject to $w(a) = V_2 \quad w(b) = V_1$

$$w = A\log r + B \qquad \begin{aligned} A\log a + B &= V_2 \\ A\log b + B &= V_1 \end{aligned}$$

$$\Rightarrow \quad A\log\frac{a}{b} = V_2 - V_1 \quad \text{and} \quad B = V_2 - \frac{(V_2-V_1)\log a}{\log a/b}$$

Hence $$w = \frac{(V_2-V_1)}{\log a/b}\log r + V_2 - \frac{(V_2-V_1)\log a}{\log a/b}$$

5 $$= V_2 + (V_2-V_1)\frac{\log r/a}{\log a/b}$$

Unseen, similar to lecture example

(b) Viscous dissipation per unit axial length $= 2\mu\int_0^{2\pi}\int_b^a e_{ij}e_{ij}\, r\,dr\,d\theta$

$$e_{ij} = \frac{1}{2}\begin{pmatrix} 0 & 0 & \partial w/\partial r \\ 0 & 0 & 0 \\ \partial w/\partial r & 0 & 0 \end{pmatrix} \qquad e_{ij}e_{ij} = \frac{1}{2}\left(\frac{\partial w}{\partial r}\right)^2 = \frac{1}{2}\left[\frac{(V_2-V_1)}{\log a/b}\frac{1}{r}\right]^2$$

$$\text{Viscous dissipation} = 4\pi\mu\,\frac{1}{2}\int_b^a\left(\frac{V_2-V_1}{\log a/b}\right)^2\frac{1}{r}\,dr$$

6 $$= 2\pi\mu\,\frac{(V_2-V_1)^2}{\log a/b}$$

(c) Net Flux $$Q = 2\pi\int_b^a r w\,dr = 2\pi\left[\frac{V_2 r^2}{2} + \frac{(V_2-V_1)}{\log a/b}\left(\frac{r^2}{2}\log\frac{r}{a} - \frac{1}{4}r^2\right)\right]_b^a$$

$$= \pi\left(V_2(a^2-b^2) + \frac{V_2-V_1}{\log a/b}\left(-\frac{1}{2}a^2 - b^2\log b/a + \frac{1}{2}b^2\right)\right)$$

For $Q=0$ $$0 = V_2(a^2-b^2) + \frac{(V_2-V_1)}{\log a/b}\left(\frac{1}{2}b^2 - \frac{1}{2}a^2 - b^2\log b/a\right)$$

Unseen

Define $R = \frac{a}{b}$ $$0 = V_2(R^2-1) + \frac{(V_2-V_1)}{\log R}\left(\frac{1}{2} - \frac{1}{2}R^2 + \log R\right)$$

$$0 = V_2\left(R^2\log R - \log R + \tfrac{1}{2} - \tfrac{1}{2}R^2 + \log R\right) - V_1\left(\tfrac{1}{2} - \tfrac{1}{2}R^2 + \log R\right)$$

$$\Rightarrow \quad V_2 = V_1\left[\frac{\log R^2 + 1 - R^2}{R^2\log R^2 + 1 - R^2}\right]$$

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When $R = 1+\varepsilon$ $(\varepsilon \ll 1)$ $\quad R^2 = 1 + 2\varepsilon + \varepsilon^2$

$\log R^2 = 2\log R = 2\left(\varepsilon - \varepsilon^2/2 + \dots\right)$

$$V_2 = V_1\left[\frac{2\varepsilon - \varepsilon^2 + 1 - 1 - 2\varepsilon - \varepsilon^2 + \dots}{(1+2\varepsilon+\varepsilon^2)(2\varepsilon-\varepsilon^2) + 1 - 1 - 2\varepsilon - \varepsilon^2}\right]$$

$$= V_1\left[\frac{-2\varepsilon^2 + \dots}{2\varepsilon - \varepsilon^2 + 4\varepsilon^2 - 2\varepsilon - \varepsilon^2 + \dots}\right] = -V_1 + \dots$$

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Alternatively argue that when $R = 1+\varepsilon$, symmetry demands $V_2 = -V_1$

$\leftarrow$ $r=a$

$\rightarrow$ $r=b$

(d) Now solve $\mu\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) = -G$

$$\Rightarrow \quad w = A\log r + B - \frac{Gr^2}{4\mu} \qquad w(a) = 0 \quad w(b) = V_1$$

and $\quad \dfrac{\partial w}{\partial r} = \dfrac{A}{r} - \dfrac{Gr}{2}$

From boundary conditions $\quad A\log a + B - \dfrac{Ga^2}{4} = 0$

and $\quad A\log b + B - Gb^2/4 = V_1$ $\quad\Big\}\quad A\log\dfrac{a}{b} = \dfrac{G}{4\mu}(a^2-b^2) - V_1$

Viscous dissipation per unit length $\dot{\varepsilon} = 2\pi\mu\displaystyle\int_b^a\left(\frac{A}{r} - \frac{Gr}{2\mu}\right)^2 r\,dr = 2\pi\mu\int_b^a \frac{A^2}{r} - \frac{GAr}{\mu} + \frac{1}{4}\frac{G^2r^3}{\mu^2}\,dr$

$$= 2\pi\mu\left(A^2\log\frac{a}{b} - \frac{GA}{2\mu}(a^2-b^2) + \frac{1}{16}\frac{G^2}{\mu^2}(a^4-b^4)\right)$$

answer

Minimise wrt V_1 : $\quad \dfrac{\partial\dot{\varepsilon}}{\partial V_1} = \dfrac{\partial\dot{\varepsilon}}{\partial A}\dfrac{\partial A}{\partial V_1} = 0$

$$\Rightarrow \quad \left[2A\log\frac{a}{b} - \frac{G}{2\mu}(a^2-b^2)\right]\frac{\partial A}{\partial V_1} = 0$$

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$\Rightarrow \quad V_1 = 0$ to minimise viscous dissipation.

Alternatively : argue that must minimise work done by boundary $\Rightarrow V_1 = 0$

in lectures

2.(a) Define $u_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$ $\quad u_\theta = -\frac{\partial \psi}{\partial r}$

Vorticity $\quad \omega = \frac{1}{r}\frac{\partial}{\partial r}(r u_\theta) - \frac{1}{r}\frac{\partial u_r}{\partial \theta} = -\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi}{\partial r}\right) - \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2} = -\nabla^2 \psi$

Also vorticity is $\perp^r$ to plane as flow is 2-D $\Rightarrow \underline{\omega} = \omega \hat{\underline{z}}$.

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in lectures

(b) Stokes flow $\quad 0 = -\nabla p + \mu \nabla^2 \underline{u}$

Curl $\quad 0 = \mu \nabla^2 \nabla \wedge \underline{\omega} \quad \Rightarrow \quad \nabla^4 \psi = 0$

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straightforward but unseen

(c) In far-field $\psi = \psi_\infty = Exy$ in Cartesian coordinates

$= Er^2 \cos\theta \sin\theta$

$= \frac{1}{2} E r^2 \sin 2\theta$ in polars

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Flows with this type of separable solution in lectures but this is an unseen example

(d) Since $\psi \propto \sin 2\theta$ in far-field, seek a complete solution with $\psi = f(r)\sin 2\theta$.

$$\nabla^2 \psi = \left(\frac{1}{r}(rf')' - 4f\right)\sin 2\theta$$

And if $f(r) = r^\alpha$ $\quad \nabla^2 \psi = (\alpha^2 - 4) r^{\alpha-2} \sin 2\theta$.

Then $\quad \nabla^4 \psi = (\alpha^2 - 4)((\alpha-2)^2 - 4) r^{\alpha-4} \sin 2\theta$.

Possible solutions $\quad \alpha = \pm 2, 4, 0$.

Seek solution of form $\quad \psi = (Ar^2 + B + Cr^{-2} + Dr^4)\sin 2\theta$.

Boundary conditions : (i) $\psi \to \psi_\infty$ as $r \to \infty$ (far-field)

(ii) $u_r = 0$ } on $r = a$ (no slip)

(iii) $u_\theta = 0$ }

From (i) : $D = 0$ and $A = E/2$

From (ii) $\frac{\partial \psi}{\partial \theta} = 0 \Rightarrow Aa^2 + B + \frac{C}{a^2} = 0$

From (iii) $\frac{\partial \psi}{\partial r} = 0 \Rightarrow 2Aa - 2C/a^3 = 0$

Here $C = \frac{E}{2}a^4$. $B = -\frac{E}{2}a^2 - \frac{E}{2}a^2 = -Ea^2$

$$\psi = \left(\frac{E}{2}r^2 - Ea^2 + \frac{E}{2}\frac{a^4}{r^2}\right)\sin 2\theta$$

$$= \frac{E}{2}\left(r - \frac{a^2}{r}\right)^2 \sin 2\theta.$$

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(c) Flow field $u_r = \frac{1}{r}\frac{\partial \psi}{\delta \theta} = \frac{1}{r}E\left(r - a^2/r\right)^2 \cos 2\theta.$

$u_\theta = -\frac{\partial \psi}{\partial r} = E\left(r - a^2/r\right)\left(1 + \frac{a^2}{r^2}\right)\sin 2\theta.$

On surface of disk $\underline{n} = -\hat{\underline{r}}$ (normal vector)

Need to evaluate $\underline{t}\cdot\sigma\cdot\underline{n}$ where $\underline{t}$ is tangential vector ($\underline{t} = \hat{\underline{\theta}}$)

i.e. $\underline{t}\cdot\underline{\underline{\sigma}}\cdot\underline{n} = -\sigma_{r\theta}.$

$$\sigma_{r\theta} = 2\mu e_{r\theta} = \mu\left(r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right) + \frac{1}{r}\frac{\partial u_r}{\delta\theta}\right)\Big|_{r=a}$$

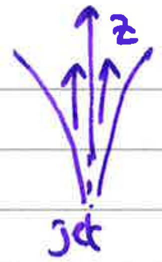
$$= \mu r \frac{\partial}{\partial r}\left(E\left(1 - \frac{a^4}{r^4}\right)\sin 2\theta\right) + \frac{1}{r}E\frac{1}{r}\left(r - a^2/r\right)^2 \cdot -2\sin 2\theta \Big|_{r=a}$$

$$= 4\mu E \sin 2\theta$$

So tangential stress exerted by fluid on disk $= 4\mu E \sin 2\theta$.

Unseen

3 (a) Navier Stokes



$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)+\frac{\partial^2 w}{\partial z^2}\right)$$

$$\frac{1}{r}\frac{\partial}{\partial r}(ru)+\left.\frac{\partial w}{\partial z}\right)=0$$

Thin jet $\Rightarrow$ $r/z \ll 1$ (r & z typical lengthscales)

$\Rightarrow$ $u \sim \frac{r}{z} w$ (radial velocity is small)

Steady $\Rightarrow$ $\frac{\partial w}{\partial t}=0$

Thin $\Rightarrow$ $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) \gg \frac{\partial^2 w}{\partial z^2}$

No imposed pressure gradient $\Rightarrow$ $\partial p/\partial z = 0$

Remaining inertial terms of axial momentum equation $u\frac{\partial w}{\partial r} \sim w\frac{\partial w}{\partial z}$

And these must balance viscous stresses at leading order

4 $\Rightarrow$ $$u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}=\frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)$$

2-D (Cartesian) example in lectures

(b) $$F=2\pi\rho\int_0^\infty r w^2\,dr$$

$$\frac{dF}{dz}=2\pi\rho\int_0^\infty 2r w\frac{\partial w}{\partial z}\,dr=2\pi\rho\int_0^\infty 2r\left(\frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)-u\frac{\partial w}{\partial r}\right)dr$$

$$=\left[4\pi\rho\nu r\frac{\partial w}{\partial r}\right]_0^\infty-4\pi\rho\int_0^\infty \frac{\partial}{\partial r}(ruw)-w\frac{\partial}{\partial r}(ru)\,dr$$

$$=\left[4\pi\rho\left(\nu r\frac{\partial w}{\partial r}-ruw\right)\right]_0^\infty+4\pi\rho\int_0^\infty rw\frac{\partial w}{\partial z}\,dr$$

$$=\left[4\pi\rho\left(\nu r\frac{\partial w}{\partial r}-ruw\right)\right]_0^\infty-\frac{dF}{dz}$$

Similar to 2-D manipulation

In the far-field $w \to 0$ as $r \to \infty$

$$\frac{\partial w}{\partial r} \to 0$$

Hence $\frac{dF}{dz} = 0$

4 $\Rightarrow F = 2\pi\rho\int_0^\infty r w^2\, dr = \text{constant}$

(c) Scaling analysis $F \sim \rho\delta^2 w^2$ and $\frac{w^2}{z} \sim \nu\frac{w}{\delta^2}$ $\Bigg\}$ $w \sim \nu z/\delta^2$, $F \sim \rho\delta^2 \cdot \frac{\nu^2 z^2}{\delta^4}$

Standard method, but unique.

$$\Rightarrow \delta^2 \sim \frac{\nu^2\rho z^2}{F} \Rightarrow \delta \sim \nu z\left(\frac{\rho}{F}\right)^{1/2}$$

4 and $w \sim \nu z\frac{F}{\nu^2\rho z^2} \Rightarrow w \sim \frac{F}{\rho}\frac{1}{\nu z}$

(d) Define $\delta(z) = \nu z\left(\rho/F\right)^{1/2}$ $\eta = r/\delta(z)$

Unseen example, but follows many examples in lectures

Stokes stream function $\Psi = \nu^2 z^2\frac{\rho}{F}\cdot\frac{F}{\rho}\frac{1}{\nu z}f(\eta) = \nu z f(\eta)$

$$u = -\frac{1}{r}\frac{\partial\Psi}{\partial z} = -\frac{1}{r}\left(\nu f + \nu z\frac{\partial\eta}{\partial z}f'\right) \qquad \frac{\partial\eta}{\partial z} = -\frac{r}{\delta^2}\delta' = -\frac{\eta}{z}$$

$$= -\frac{1}{r}\nu\left(f - \eta f'\right) \qquad \frac{\partial\eta}{\partial r} = \frac{1}{\delta}$$

$$w = \frac{1}{r}\frac{\partial\Psi}{\partial r} = \frac{\nu z}{r}\cdot\frac{1}{\delta}f' = \left(\frac{F}{\rho}\right)^{1/2}\frac{1}{r}f'$$

$$\partial w/\partial r = \left(\frac{F}{\rho}\right)^{1/2}\left(-\frac{1}{r^2}f' + \frac{1}{r}\frac{1}{\delta}f''\right)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{F}{\rho}\right)^{1/2}\left(-\frac{1}{r}f' + \frac{1}{\delta}f''\right) = \frac{1}{r}\left(\frac{F}{\rho}\right)^{1/2}\left(\frac{1}{r^2}f' - \frac{1}{r\delta}f'' + \frac{1}{\delta^2}f'''\right)$$

$$\frac{\partial w}{\partial z} = \left(\frac{F}{\rho}\right)^{1/2}\frac{1}{r}\cdot\frac{-\eta}{z}f''$$

Governing equation: $-\frac{\nu}{r}(f - \eta f')\left(\frac{F}{\rho}\right)^{1/2}\left(\frac{-1}{r^2}f' + \frac{1}{\delta r}f''\right)$

$$+ \left(\frac{F}{\rho}\right)^{1/2}\frac{1}{r}f'\cdot\left(\frac{F}{\rho}\right)^{1/2}\frac{1}{r}\cdot\frac{-\eta}{z}f'' = \frac{\nu}{r}\left(\frac{F}{\rho}\right)^{1/2}\left(\frac{1}{r^2}f' - \frac{1}{r\delta}f'' + \frac{1}{\delta^2}f'''\right)$$

$$-\frac{\nu}{r^3}(f-\eta f')(-f'+\eta f'') + \left(\frac{F}{\rho}\right)^{1/2}\cdot\frac{-\eta}{r^2 z} f'f''$$

$$= \frac{\nu}{r^3}\left(f' - \eta f'' + \eta^2 f'''\right)$$

$$\Rightarrow (f-\eta f')(f'-\eta f'') + \left(\frac{F}{\rho}\right)^{1/2}\cdot\frac{-r\eta}{\nu z} f'f''$$

$$= f' - \eta f'' + \eta^2 f'''$$

but $\left(\frac{F}{\rho}\right)^{1/2}\frac{r}{\nu z} = \frac{r}{\delta} = \eta$

$$\Rightarrow (f-\eta f')(f'-\eta f'') - \eta^2 f'f'' = f' - \eta f'' + \eta^2 f'''$$

10 $$\Rightarrow ff' - \eta(f'^2 + ff'') = f' - \eta f'' + \eta^2 f'''$$

(e) Boundary conditions:

Similar to 2-D example

(i) $\cancel{w(0)=0}$ $w \to 0$ as $r \to \infty$ $\Rightarrow$ $f' \to 0$ as $\eta \to \infty$

(ii) $u(0) = 0$ $\Rightarrow$ $f(0) = 0$

(iii) $2\pi\int_0^\infty \rho r w^2\, dr = F$ $\Rightarrow$ $2\pi\int_0^\infty \rho\cdot\delta\cdot\eta\cdot\frac{F}{\rho}\frac{1}{\delta^2\eta^2}f'^2\cdot\delta\, d\eta = F$

$$\Rightarrow \quad 2\pi\int_0^\infty \frac{f'^2}{\eta}\, d\eta = 1$$

4(a) Vortex sheet $\quad \underline{u}_0 = \begin{cases} u_0\hat{\underline{x}} & z>0 \\ -u_0\hat{\underline{x}} & z<0 \end{cases} \qquad p_0 = \text{constant}$

Perturbation equations $\qquad \nabla\cdot\underline{u}_1 = 0$

$$\rho\left(\frac{\partial \underline{u}_1}{\partial t} + \underline{u}_0\cdot\nabla\underline{u}_1 + \underline{u}_1\cdot\nabla\underline{u}_0\right) = -\nabla p_1$$

$\Rightarrow$ as $\underline{u}_0$ piecewise constant $\quad \rho\left(\frac{\partial}{\partial t} + \underline{u}_0\cdot\nabla\right)\underline{u}_1 = -\nabla p_1$

Take divergence $\Rightarrow \quad \nabla^2 p_1 = 0$

Take ∇^2 of Euler equation $\Rightarrow \quad \nabla^2\underline{u}_1 = 0$

If $\underline{u}_1$ and p_1 proportional to $e^{i(kx+ly)+st}$

write $\quad \underline{u}_1 = \begin{pmatrix} u_1(z) \\ v_1(z) \\ w_1(z) \end{pmatrix} e^{i(kx+ly)+st}$ and $p_1 = \hat{p}_1(z)\, e^{i(kx+ly)+st}$

Since $\nabla^2\underline{u}_1 = 0 \Rightarrow \quad \frac{d^2}{dz^2}\begin{pmatrix} u_1 \\ v_1 \\ w_1 \end{pmatrix} - (k^2+l^2)\begin{pmatrix} u_1 \\ v_1 \\ w_1 \end{pmatrix} = 0$ and similarly for $\hat{p}_1(z)$

Pick solution that decays away from interface

$$u_1 = \begin{cases} A\, e^{-\Delta z} & z>0 \\ B\, e^{\Delta z} & z<0 \end{cases} \qquad \Delta^2 = k^2 + l^2$$

and $\quad v_1 = \begin{cases} C\, e^{-\Delta z} & z>0 \\ D\, e^{\Delta z} & z<0 \end{cases} \qquad w_1 = \begin{cases} E\, e^{-\Delta z} & z>0 \\ F\, e^{\Delta z} & z<0 \end{cases}$

<u>Boundary conditions</u>:

(i) Kinematic condition linearises to $\left(\frac{\partial}{\partial t} + \underline{u}_0\cdot\nabla\right)\eta = w$

$$\Rightarrow \quad \begin{aligned} (s + iu_0k)\,\eta_0 &= E \\ (s - iu_0k)\,\eta_0 &= F \end{aligned}$$

2-D perturbations in lectures + Squire's Thm. This also for 3-D perturbation

(ii) Dynamic condition : p is continuous $\Rightarrow$ $p_1(0^+) = p_1(0^-)$
$\Rightarrow$ $\partial_x p_1(z=0^+) = \partial_x p_1(z=0^-)$

$$\Rightarrow \quad (s+ikU_0)A = (s-ikU_0)B \quad \text{from Euler equation}$$

Likewise $(s+ikU_0)C = (s-ikU_0)D$

Incompressibility $\Rightarrow$ $ikA + ilC - \Delta E = 0$
and $ikB + ilD + \Delta F = 0$

Hence $(s+ikU_0)\Delta E = -(s-ikU_0)\Delta F$

$$\Rightarrow \quad (s+ikU_0)^2 = -(s-ikU_0)^2$$

$$\Rightarrow \quad s^2 + 2ikU_0 s - k^2U_0^2 = -s^2 + 2ikU_0 s + k^2U_0^2$$

$$\Rightarrow \quad s^2 = k^2U_0^2$$

10 $\Rightarrow$ $s = kU_0$ Growth rate

(b) $\underline{u} = (u(z), +Ey, -Ez)$

$\underline{\omega} = \nabla\wedge\underline{u} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ u & +Ey & -Ez \end{vmatrix} = \frac{\partial u}{\partial z}\hat{\underline{y}}$ Define $\omega = \frac{\partial u}{\partial z}$

Vorticity equation $\frac{\partial \underline{\omega}}{\partial t} + \underline{u}.\nabla\underline{\omega} = \underline{\omega}.\nabla\underline{u} + \nu\nabla^2\underline{\omega}$

Hence $-Ez\frac{\partial\omega}{\partial z} = +\omega E + \nu\frac{\partial^2\omega}{\partial z^2}$

5 $\Rightarrow$ $\frac{\partial}{\partial z}\left(\nu\frac{\partial\omega}{\partial z} + Ez\omega\right) = 0$

Unseen example

(c) $\nu\frac{\partial\omega}{\partial z} = -Ez\omega$ (as $z\to\infty$ $\omega\to 0$)
$\Rightarrow$ $\omega = A\exp(-Ez^2/2\nu)$ A = constant

Unseen example

$$\frac{\partial u}{\partial z} = A \exp\left(-\frac{E z^2}{2\nu}\right)$$

$$u = A \int_0^z \exp\left(\frac{-E s^2}{2\nu}\right) ds$$

$s = \left(\frac{2\nu}{E}\right)^{1/2} y \qquad u = A\left(\frac{2\nu}{E}\right)^{1/2} \int_0^{\left(\frac{E}{2\nu}\right)^{1/2} z} e^{-y^2}\, dy$

$$= A\left(\frac{\nu\pi}{2E}\right)^{1/2} \cancel{\left(\ \right)^{1/2}}\, \mathrm{erf}\left(\left(\frac{E}{2\nu}\right)^{1/2} z\right)$$

As $z \to \infty \quad \mathrm{erf}\left(\left(\frac{E}{2\nu}\right)^{1/2} z\right) \to 1 \quad \Rightarrow A.\left(\frac{\nu\pi}{2E}\right)^{1/2} = U_0$

6 $\quad \Rightarrow \quad u = u_0\, \mathrm{erf}\left(\left(\frac{E}{2\nu}\right)^{1/2} z\right)$

2 Thickness of vortex sheet $= \left(\frac{\nu}{E}\right)^{1/2}$

Unseen

(d) For long wavelengths, ~~the fastest~~ $\left(\frac{1}{k} \gg \left(\frac{\nu}{E}\right)^{1/2}\right)$ the disturbances does not "see" the viscous layer. It appears like an inviscid vortex sheet $\Rightarrow$ unstable.

[Short wavelengths could be stabilised – but this is not shown here]

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