Applied Dynamical Systems Problem Sheet 3

1. Identify the bifurcation in the system

$$\frac{dy}{dt} = a \ln y + y - 1$$

find a change of variables to reduce it to normal form given in lectures, and give an approximate form for orbits starting close to the fixed point at the value of a corresponding to the bifurcation.

2. Consider the map $rx(1-x^2)$. Find exact or approximate values of r for which there is

   - A pitchfork bifurcation
   - A period doubling bifurcation
   - A fold bifurcation
   - An attractor merging crisis
   - A boundary crisis

   For the local bifurcations give an approximate form for orbits that start near the fixed point at the bifurcation value. What is the maximum number of stable fixed points that can coexist at any parameter value? Is there a period doubling cascade with the same Feigenbaum constants as the logistic map?

3. For the two dimensional flow $(\dot{x}, \dot{y}) = (x - y, x^2 - y^2)$ give an analytical form for the invariant manifolds of the origin, and an approximate location of the other fixed point on one of these manifolds.

4. A damped pendulum has equations $(\dot{x}, \dot{v}) = (v, -\omega^2 \sin x - \gamma v)$ where x is an angle (2π-periodic) variable and $\gamma \geq 0$ the damping parameter. Sketch the phase space for $\gamma = 0$ (noting that in this case, the energy $v^2/2 - \omega^2 \cos x$ is a conserved quantity). and for $\gamma > 0$. Describe what happens to the fixed points, their stable and unstable manifolds, and the ω-limit sets of general points with the addition of damping.

