
Applied Dynamical Systems Problem Sheet 4

1. For the logistic map, give a numerical plot of all unstable periodic orbits with periods up to 10 for $3 < r < 5$.
2. Give examples of topological Markov chains satisfying each of the following properties:
 - (a) Are irreducible but not aperiodic
 - (b) Are aperiodic but not irreducible
 - (c) Has a greater topological entropy than when restricted to its essential symbols.

For each of your examples state the ergodic properties (discussed in lectures) of the corresponding dynamical system.

3. For the skew shift

$$x \rightarrow \begin{cases} 3x & 0 \leq x < 1/3 \\ \frac{3x-1}{2} & 1/3 \leq x < 1 \end{cases}$$

calculate

- (a) The topological entropy
 - (b) The invariant density
 - (c) The Lyapunov exponent
 - (d) The KS entropy
4. For the doubling map with hole at $[3/4, 1]$ find
 - (a) The topological entropy
 - (b) The conditionally invariant density
 - (c) The escape rate
 - (d) The dimension of the non-escaping set
 5. Show that the logistic map with $r = 4$ has invariant density

$$\rho(x) = \frac{1}{\pi\sqrt{x(1-x)}}$$

Find another (not necessarily positive) density which is an eigenvector of the transfer operator.

6. Write some code to implement the cycle expansions, and hence confirm the claim in the example for the escape rate of $\Phi(x) = 5x(1-x)$.