Mathematical Billiards and their Applications
21st - 24th of June 2010
List of Abstracts

Florian Lenz (University of Hamburg, Centre for Optical Quantum Technologies)

Title: Evolutionary phase space in driven elliptical billiards.

Abstract: We perform the first long-time exploration of the classical dynamics of a driven billiard with a four dimensional phase space. With increasing velocity of the ensemble we observe an evolution from a large chaotic sea with stickiness due to regular islands to thin chaotic channels with diffusive motion leading to Fermi acceleration. As a surprising consequence, we encounter a crossover, which is not parameter induced but rather occurs dynamically, from amplitude dependent tunable subdiffusion to universal normal diffusion in momentum space.

Christoph Petri (University of Hamburg, Institute for Laserphysics)

Title: Particle focusing in oscillating dissipative billiards.

Abstract: We develop and analyze a scheme to achieve both spatial and energetic focusing of an ensemble of neutral particles which is based on an oscillating billiard with frictional forces. The interplay of two competing mechanisms, acceleration due to collisions with the oscillating billiard walls and deceleration caused by friction, leads to the emergence of attractors in phase space. Their specific properties, i.e. spatial localization and energy spread, can be controlled and tuned by varying e.g. the frequency of the time-dependent billiard.

Andrew Ferguson (University of Warwick)

Title: Escape rates for Gibbs measures.

Abstract: In this talk I will present a result relating to the convergence of the spectral radius of a singularly perturbed transfer operator. I will then discuss applications to the convergence of escape rates and topological pressure for subshifts of finite type. This is joint work with Mark Pollicott.

Orestis Georgiou (University of Bristol)

Title: Transmission and Reflection in the Stadium Billiard: Time reversal symmetry breaking.

Abstract: We investigate the transmission and reflection survival probabilities for the chaotic stadium billiard with two holes (one on a straight segment and one on a curved segment). Classically, these distributions are shown to be algebraic or exponential depending on the choice of the injecting hole. This offers an example of an unexpected type of time-reversal symmetry breaking not accounted for in existing semiclassical and RMT treatments of ballistic transport through quantum dots. Exact expressions are given and confirmed numerically suggesting new means of calibrating and controlling experimental devices.

Chris Joyner (University of Bristol)
Title: Quantum Chaotic Systems with Discrete Symmetries.
Abstract: When a quantum system has completely chaotic classical dynamics the energy levels cannot be determined explicitly. Instead the focus shifts to a statistical analysis of the distribution of level spacings in the complete spectrum. It has been conjectured this distribution should, in the semiclassical limit, agree with predictions concerning the eigenvalue distribution of large random matrix ensembles (with the choice of ensemble depending on the spin and time-reversal invariance properties of the Hamiltonian). Interestingly, the choice of ensemble for quantum chaotic systems with extra discrete spatial symmetries is somewhat counterintuitive. However by analysing correlations between classical periodic orbits in systems such as chaotic billiards we are able to determine the correct ensembles and predict the overall energy level distribution corresponding to the symmetry.

Nick Simm (University of Bristol)
Title: Orthogonal polynomial approach to the random matrix theory of chaotic cavities.
Abstract: The scattering of a particle through a cavity attached to two ideal leads is described using quantum mechanics by a finite dimensional unitary matrix called the scattering matrix, S. If the classical dynamics inside the cavity are chaotic then it is thought that the universal behaviour of the system is well described by random matrix theory. We use the connection with orthogonal (and skew orthogonal) polynomials to explicitly compute higher order moments, including quantities like delay time, conductance, and shot noise of the system. In the limit of infinite scattering matrices (the semiclassical limit), our results take on a highly combinatorial form, with connections to Narayana numbers, Catalan numbers and Schroeder numbers.

Wang Feiran (Department of Physics and Astronomy, University of Nottingham)
Title: The dramatic effect of semi-classical stochastic webs on the quantised states of superlattices.
Abstract: Electrons in a semiconductor superlattice can be used to realize, and exploit, the unique dynamics of the driven harmonic oscillator that were discovered by George Zaslavsky and colleagues. Under the action of an electric and tilted magnetic field, each quantum well acts as a two-dimensional electron billiard with a parabolic profile enclosed by oblique barriers. Quantum tunnelling through the barriers couples the billiards, thus creating energy bands that give rise to the semi-classical electron trajectories. These semi-classical trajectories exhibit non-KAM chaos.[1] At certain critical field parameters, the electron trajectories change abruptly from fully localized to completely unbounded, and map out intricate stochastic webs in phase space, through which the electrons rapidly diffuse. [2] When an electron starts from rest at the web centre, it becomes trapped on the inner ring of the web; due to the exponentially decreasing width of radial filament with distance from the web centre. In real space, this trapping confines the electron to a particular region of the superlattice, whose width is determined by ring diameter and, hence, the roots of Bessel Functions.[3,4] In a quantum picture, the quantised energy eigenfunctions of the superlattice resemble a superposition of Landau states whose index increases by 1 on moving from one well to the next. Consequently, the tunnel coupling of adjacent wells is described by the overlap integrals of adjacent Laudau states. We show that these overlap integrals can be expressed in terms of Bessel functions and, consequently, equal zero at the roots of those Bessel functions. This means that the eigenstates vanish at precisely the same position where, semi-classically, the electron is trapped on the first ring of the stochastic web.
Reference:
[1] D.Fowler *et al*, Phys. Rev. B 76, 245303, (2007).
[2] T.M. Fromhold *et al*, Nature 428, 726, (2004).

[3] D.Hardwick, Ph.D. thesis, University of Nottingham, (2007).
[4] George M. Zaslavsky, Hamiltonian Chaos and Fractional Dynamics, Oxford University Press, Oxford (2005).

Hanya A M Ben Hamdin (Nottingham University, Mathematics school)
Title: Wave energy transport in multicomponent systems.
Abstract: We investigate the transport of acoustic wave energy through coupled polygonal domains with the aim of finding efficient methods for large scale, multicomponent systems. We present numerical computations using the Boundary Element Method (BEM) for two dimensional, coupled polygonal domains that have different material properties. BEM is an advantageous technique in the sense that it reduces the dimensionality of the problem by one. This means, the solution of a two dimensional system can be obtained by performing a one dimensional integral over the boundary of the domain. In the realistic applications, the domain under consideration might be governed by individual differential equations and/or constructed of different materials. Different parameters in the wave equation then apply for each sub-domain such as different wavenumbers and wave velocities. In these circumstances, the straightforward BEM breaks down; then, one needs to decompose the system into subdomains and incorporate the interfacial conditions, i.e the continuity of the wave function and the energy flux across the interface. Implementing the BEM in this way gives rise to hypersingular integrals due to the logarithmic singularity of the free space Green function. To overcome this obstacle, we derive a singularityfree formula that regularises the integral kernel.

Michael White (Nottingham University, Mathematics school)
Title: Directional Emission from Optical Resonators.
Abstract: Light emission from optical resonators can show strong directionality even for small perturbations of a circular resonator. In order to model the emission patterns produced, it is necessary to understand the underlying ray dynamics which must be extended to include complex coordinates. These ray dynamics are produced by considering a canonical perturbation of a circular billiard and a model for the emission pattern of the wave problem is introduced.