General relativity problem sheet 1

Unstarred problems follow quickly from definitions and equations in the lectures; single starred problems require more effort; double starred questions are designed as a challenge. The exam will contain questions similar to the first two categories. To obtain credit points, either get 50 on the exam, or submit reasonable attempts at 1/3 of the problems AND 30 on the exam. Parts (a), (b) count as separate problems for this purpose, so this sheet has SEVEN problems.

1. From http://www-history.mcs.st-andrews.ac.uk/ go to “History Topics” then “Mathematical Physics” then “General Relativity”. To what extent should Hilbert (rather than Einstein) be given credit for general relativity?

2. * Despite the quote on the unit home page, E. Mach’s ideas had an important impact on the development of relativity. Look into this and write a brief (eg half page) account of what you find.

3. Approximately how much older is your head than your feet?

4. The Earth rotates once every 24 hours, leading to an equatorial radius of 6378 km which is greater than its polar radius of 6357 km.
   *(a)Give a physical argument that clocks at the equator and the pole should run at exactly the same rate.
   *(b)Noting that special relativity predicts time dilation (ie slowing) of clocks at the moving equator by a factor $\gamma = 1/\sqrt{1 - v^2/c^2}$ where v is the velocity of the equator, and noting the general relativistic time dilation of the gravitational field discussed in lectures, calculate the difference in gravitational potential between the equator and the pole, and compare it with gd.

5. Does the surface of a cube have intrinsic curvature, and if so, where? What about a cylinder?

6. Show that the constants G and c can be combined to make a quantity with dimensions of power (energy per unit time), and find its value in SI units. What theoretical significance do you expect this power to have?