General relativity solution sheet 9

Here is a sample essay, with relevant points marked by stars* - please don't put these in your essays! At least seven points should be given for each of the three topics. Note that an ideal essay would involve not only facts but also some understanding (but not details) of the derivation. Pictures are helpful too. Style is hard to quantify; essays in point form rather than in complete sentences are likely to attract few marks for style, while essays with a flow and synthesis of argument are likely to attract more marks for style; there are exceptions to both rules. I have tried to incorporate these ideas in introductory and concluding paragraphs, but this may have made it longer than can reasonably be expected in an exam, and I am sure some of you can do better.

Tests of general relativity

General relativity is a physical theory, so in contrast to pure mathematics, its validity depends on the outcome of experimental and observational work in addition to logical consistency. Tests of the theory can be broadly divided into weak and strong field effects; because gravity is such a weak interaction, evidence for strong field effects is restricted to astronomical sources for which repeatable experiments cannot be performed. On the other hand, weak field effects can be tested within the solar system. In order to make useful comparisons with competing theories, the weak field parameters have been fully classified; the result is called the parametrised post-Newtonian (PPN) formalism. The remainder of this essay will focus on two weak field effects - the bending of light by a massive object and the perihelion precession of nearly Newtonian orbits, and a strong field effect, the existence of black holes. This will allow a comparison of the different information that can be obtained in tests of the theory.

The perihelion precession is significant as it is effectively the only data Einstein could use to test the theory when it was first developed*. Apart from this there was only the fact that the theory should reduce to Newtonian gravity and special relativity in appropriate limits, which is not nearly sufficient to specify the theory. The observation that the perihelion (point of closest approach to the sun) of Mercury's orbit precessed, ie rotated slowly in the direction of the orbit*, had been known for some time, and measurements of this effect were extremely precise - to seconds of arc per century. The fact that Newtonian two body orbits are closed is quite remarkable, due to an additional constant of motion in addition to the obvious energy and angular momentum. This means that virtually any perturbation will cause precession*. In fact 5600 seconds per century is caused by Newtonian perturbations from the other planets*, however an additional 43 seconds per century remained unaccounted for*. Although the fact of precession is unsurprising from the theory, the amount of precession is a testable aspect of the theory, PPN parameter β which characterises nonlinearity in the field equations*. The reason that the effect was discovered in Mercury is that it is the planet closest to the sun (strongest gravitational effect), and also has a very eccentric orbit (easy to measure precession)*. The effect has since been observed in other planets, and in binary pulsar systems (the latter are relatively strong field)*.

The bending of star light by the sun is also significant as the first prediction of the theory to be confirmed, at a solar eclipse in 1919*. The bending of light is a straightforward consequence of Einstein's Equivalence Principle*

A gravitational field is locally equivalent to an accelerated frame of reference.

since straight light rays are clearly bent when viewed in an accelerated reference frame*. The relevant PPN parameter γ measures the amount of spacetime curvature associated with a given mass*. A detailed calculation shows that the amount of bending is given by $4GM/bc^2$ where M is the mass and b is the impact parameter*. The strongest effect in the solar system is in light that passes very close (small b) to the sun (large M), which, due to the brightness of the sun, is only visible during an eclipse*. More recently, Very Long Baseline Interferometry (VLBI) radio techniques, which have a much smaller angular resolution than visible light, have observed the bending of radio waves at angles far from the sun, to an accuracy of about 10^{-3}*. Another recent observation of light bending is that of gravitational lensing, where brighter, displaced or multiple images give valuable information about the massive object causing the deflection*.

Finally, we turn to the strong field example of black holes, which are predictions of the vacuum Einstein equations, imposing only sperical symmetry*. Because we cannot observe a black hole in the solar system, evidence for black holes is necessarily indirect*. If we accept the strong theoretical arguments that any object of greater than about two solar masses and no energy source must collapse, the measurement of binary systems where one partner is very massive but not a star is indirect evidence for a black hole*. Properties that may be inferred include

1. The last stable orbit is at $R = 6M$ (redshift data from infalling matter)*
2. Infalling matter does not radiate as it hits a solid surface (in constrast to neutron stars)*
3. There is no definite rotation period (again in constrast to neutron stars)*
4. Fluctuating intense radiation from infalling matter accelerating to relativistic speeds*

so that "black" is probably not a good adjective to describe these objects, at least in the presence of infalling matter from a companion star. More massive black holes have been observed at the centre of galaxies, inferred by strong radiation as above and the very rapid orbiting motion of nearby stars*. The theoretical challenge is now to understand the complex phenomena generated by black holes absorbing extremely hot magnetised plasma, and hence make closer connection with observational evidence*.

We have investigated only three of many possible tests of general relativity, with the result that it is still completely consistent with observation. Solar system tests, while precise and flexible, allow only the weak field properties of the theory to be tested. More distant objects provide the possibility of more stringent tests, at a cost of more unknown parameters and in some cases, complicated physics. This must remain the case until, in the distant future, we perform the ultimate test of general relativity, creating a black hole in the laboratory. In addition to clean strong field tests, this would also permit verification of quantum effects such as Hawking radiation. Such a feat would dwarf the present atomic age in scientific knowledge and unlimited energy - but also moral hazard.