OPTICAL MICRODISK RESONATOR WITH A SMALL BUT FINITE SIZE SCATTERER

C. P. Dettmann [1], *G. V. Morozov* [1*], *M. Sieber* [1], *H. Waalkens* [1,2]

[1]Department of Mathematics, University of Bristol, Bristol BS8 1TW, United Kingdom
[2]Department of Mathematics, University of Groningen, 9747 AG Groningen, The Netherlands
*Email: g.morozov@bristol.ac.uk

ABSTRACT

Circular microresonators (microdisks) are natural candidates for the realization of low-threshold miniature laser sources since some of their modes have extremely high Q-factors (low thresholds). In those modes, which are called whispering gallery modes, light circulates around the circumference of the disk trapped by total internal reflection. Although the microdisk cavities can provide ultra-low threshold lasing, their applicability faces the problem of isotropic light emission which is due to the rotational symmetry of the system. Contrary to usual procedure, where a geometric deformation of the microdisk boundary is used to break the symmetry and, as a result, to achieve the output directionality, we propose a scenario inducing rotational symmetry breaking by placing a small but finite size circular scatterer inside the microdisk itself. We calculate positions of the new resonant modes and show that some of them possess clear emission directionality while preserving high Q-factors.

1. INTRODUCTION

The technological progress of recent years has made possible the construction of microresonators in the μm-domain. These resonators have great potential for a wide range of applications and studies, like the realization of low-threshold miniature laser sources, the creation of dynamical filters for optical communications and even the development of sensitive optical biosensors [1, 2].

In contrast to ideal (closed) cavities which possess discrete eigenmodes at real frequencies, resonators are open systems coupled to the external world. As a result, their eigenmodes (resonances) are characterized by complex frequencies $\omega - i\Delta\omega/2$ where $\Delta\omega$ is the linewidth or inverse lifetime, related to the so-called Q-factor as $Q = \omega/\Delta\omega$.

Circular microresonators (microdisks) are natural candidates for lasing since some of their modes have extremely high Q-factor (low thresholds) [3, 4]. In those modes, which are called whispering gallery modes, light circulates around the circumference of the disk trapped by total internal reflection. Although the microdisk cavities can provide ultra-low threshold lasing, their applicability faces the problem of isotropic light emission which is due to rotational symmetry of the system.

In order to obtain a directional output one has to break the rotational symmetry, for example, by deforming the boundary of the cavity [5, 6]. This significantly improves the emission directionality but typically spoils the Q-factors. Another approach to breaking the symmetry is to insert an obstacle like a linear defect [7] or a hole [8] into the microdisk. This indeed allows one to obtain resonances with large Q-factors and relatively high directionality.

Following the second approach, we have recently suggested to place a point scatterer inside the microdisk, at some distance away from the centre [9]. We have demonstrated that the presence of the scatterer leads to significant enhancement in the directionality of the outgoing light in comparison with whispering gallery modes of a circular resonator without scatterer, while preserving their high Q-factors. However, an experimental realization of that model requires the detailed interpretation of the coupling strength of the point scatterer.

In this paper we suppose to use a small but finite size circular scatterer instead of a point scatterer to facilitate a practical realization of our model. Using a Green function method, we calculate positions of the new resonant modes and show that some of them possess clear emission directionality while still preserving high Q-factors.

2. THEORY OF 2D MICRODISK CAVITIES

The time-harmonic modes of frequency $\omega = ck$, where k is the wave number and c is the light velocity, of any passive microcavity filled with nonmagnetic dielectric material of refractive index $n(\mathbf{r})$ are described by 3D Maxwell's equations. For a microcavity, with the thickness of only a fraction of the mode wavelength, modes themselves can be studied in the 2D approximation with the aid of the effective refractive index $n_{\text{eff}}(\mathbf{r})$ which takes into account the material as well as the thickness of the cavity. In that approximation we omit the coordinate dependence of the EM field in the z direction and, as a result, separate the field into TM ($H_z = 0$) and TE modes ($E_z = 0$). For brevity we consider only TM modes in this paper. In polar coordinates we have

$$\frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r}\frac{\partial E_z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 E_z}{\partial \varphi^2} + k^2 n_{\text{eff}}^2 (r,\varphi) E_z = 0\,, \quad (1)$$

with

$$H_r = -\frac{i}{kr}\frac{\partial E_z}{\partial \varphi}\,, \quad H_\varphi = \frac{i}{k}\frac{\partial E_z}{\partial r}\,.$$

For a homogeneous microdisk of radius R in a medium of effective refractive index equal to 1 we have

$$n_{\text{eff}}(r,\varphi) = \begin{cases} n, & r < R\,, \\ 1, & r > R\,. \end{cases} \quad (2)$$

Separation of variables and physical conditions in the middle of the disk and at infinity lead to the field E_z in the form of whispering gallery modes

$$E_z^m = \begin{cases} N_m J_m(knr)\, e^{im\varphi}, & r < R, \\ H_m(kr)\, e^{im\varphi}, & r > R, \end{cases} \quad (3)$$

where J_m and H_m are Bessel and Hankel functions of the first kind respectively, m is the azimuthal modal index. Then, the boundary conditions (continuity of the EM fields) at the point $r = R$ lead to a set of independent transcendental equations for the microdisk resonances

$$J_m(knR)H_m'(kR) - nJ_m'(knR)H_m(kR) = 0, \quad (4)$$

and constants $N_m = H_m(kR)/J_m(knR)$. We use the radial modal index q to label different resonances with the same azimuthal modal index m.

The corresponding Green function for the microdisk is given by the following expressions where $r_<$ ($r_>$) is the smaller (larger) of r and r_0 [9]

$$G(\mathbf{r},\mathbf{r}_0,k) = -\frac{i}{4}H_0\left(kn\left|\mathbf{r}-\mathbf{r}_0\right|\right) + \frac{i}{4}\sum_{m=0}^{\infty}\frac{C_m}{A_m}\epsilon_m \cos\left[m\left(\varphi-\varphi_0\right)\right] J_m\left(knr_<\right) J_m\left(knr_>\right), \quad (5)$$

if both $r_<$ and $r_>$ lie inside the disk of radius R,

$$G(\mathbf{r},\mathbf{r}_0,k) = \frac{1}{2\pi kR}\sum_{m=0}^{\infty}\frac{1}{A_m}\epsilon_m \cos\left[m\left(\varphi-\varphi_0\right)\right] J_m(knr_<)H_m(kr_>), \quad (6)$$

if $r_< < R$ and $r_> > R$, and, finally,

$$G(\mathbf{r},\mathbf{r}_0,k) = -\frac{i}{4}H_0\left(k\left|\mathbf{r}-\mathbf{r}_0\right|\right) + \frac{i}{4}\sum_{m=0}^{\infty}\frac{B_m}{A_m}\epsilon_m \cos\left[m\left(\varphi-\varphi_0\right)\right] H_m(kr_<)H_m(kr_>), \quad (7)$$

if both $r_<$ and $r_>$ lie outside the disk of radius R. The coefficients are $\epsilon_m = 2$ if $m \neq 0$, $\epsilon_m = 1$ if $m = 0$, and

$$\begin{aligned} A_m &= \tilde{n} J_m(knR)\, H_m'(\tilde{n}kR) - nJ_m'(knR)\, H_m(\tilde{n}kR)\,, \\ B_m &= \tilde{n} J_m(knR)\, J_m'(\tilde{n}kR) - nJ_m'(knR)\, J_m(\tilde{n}kR)\,, \\ C_m &= \tilde{n} H_m(knR)\, H_m'(\tilde{n}kR) - nH_m'(knR)\, H_m(\tilde{n}kR)\,. \end{aligned}$$

with $\tilde{n} = 1$. The resonances k_{res} of the microdisk are determined by the poles of the Green function which are given by $A_m = 0$, which agrees with (4).

Now we place a small circular scatterer of radius a and refractive index n_a at a point $\mathbf{d}$ inside the microdisk. This geometry is similar to the one in Ref. [8]. In contrast to Ref. [8] we are interested in much smaller scatterers, of arbitrary refractive index, that are treatable analytically by a Green function approach. The Green function of the perturbed disk is found by treating the scatterer in the s-wave approximation. This results in

$$G^a(\mathbf{r},\mathbf{r}_0,k) \approx G(\mathbf{r},\mathbf{r}_0,k) + \frac{G(\mathbf{r},\mathbf{d},k)\mathcal{D}G(\mathbf{d},\mathbf{r}_0,k)}{1-\mathcal{D}G^{\text{sc}}(\mathbf{d},\mathbf{d},k)}\,, \quad (8)$$

where G^{sc} is the Green function in (5) without the H_0 Hankel function. The diffraction coefficient has the form $\mathcal{D} = -4i\,\mathcal{B}_0/\mathcal{A}_0$, where here and in the following $\mathcal{A}_m$ and $\mathcal{B}_m$ are given by A_m and B_m with n replaced by n_a, $\tilde{n}$ by n, and R by a. The resonances of the perturbed system are determined by the poles of the Green function and hence are defined by the transcendental equation

$$0 = -\frac{\mathcal{A}_0}{\mathcal{B}_0} + \sum_{m=0}^{\infty}\frac{C_m}{A_m}\epsilon_m J_m^2(knd)\,. \quad (9)$$

The corresponding field E_z follows from the residua of (8) as $E_z(\mathbf{r}) = NG(\mathbf{r},\mathbf{d},k)$ where N is a normalization factor and k is the wavenumber of the resonance. Outside of the microdisk, i.e. in the region $r > R$, the field is then of the form

$$E_z = \frac{N}{2\pi kR}\sum_{m=0}^{\infty}\frac{\epsilon_m \cos(m\varphi)\, J_m(knd)}{A_m} H_m(kr)\,, \quad (10)$$

if the scatterer is located on the positive x-axis. The s-wave approximation is valid if $|nk(R-d)| \gg 1$ and

$$\frac{\mathcal{A}_0}{\mathcal{B}_0} \ll \frac{\mathcal{A}_1}{\mathcal{B}_1}. \quad (11)$$

3. FAR-FIELD DIRECTIVITY

In order to quantify the far-field directionality of the electric field we consider its asymptotic behaviour for $r \to \infty$ which has the form

$$E_z(\mathbf{r},k) = E_z(r,\varphi,k) \propto \frac{\exp(ikr)}{\sqrt{r}} f(\varphi)\,.$$

To characterize the directionality we compute the directivity of the far-field intensity

$$D = \frac{2\pi \left|f_{\max}(\varphi_{\max})\right|^2}{\int\limits_0^{2\pi} \left|f(\varphi)\right|^2 d\varphi}.$$

From this definition it follows that $D = 1$ and $D = 2$ for unperturbed resonances (no small scatterer in the disk) which have $m = 0$ and $m \neq 0$, respectively.

To illustrate the gain in the emission directionality we consider a microdisk of effective refractive index $n = 3$ and radius $R = 1$ μm with a small scatterer of radius $a = 0.01$ μm and refractive index $n_a = 1.00$ (a small hole) placed at the distance $d = 0.5$ μm. The complex wave numbers of its resonant modes in the s-wave approximation can be found from Eq. (9). We are interested in modes that have both high directionality and low threshold characteristics (high Q-factors). In a spectral range of green light, the resonant mode $kR = 12.52676 - 0.00026\,i$ has both high directivity $D = 6.58$ and a very high factor $Q = 24382$. In Fig. 1 we compare the function $|f(\varphi)|^2$ for this mode with the resonant mode $kR = 12.52385 - 0.00015\,i$ ($m = 18$, $q = 5$), which is the closest one of the microdisk without scatterer. We should note that the condition (11) is perfectly fulfiled for such parameters of the microdisk and the small scatterer.

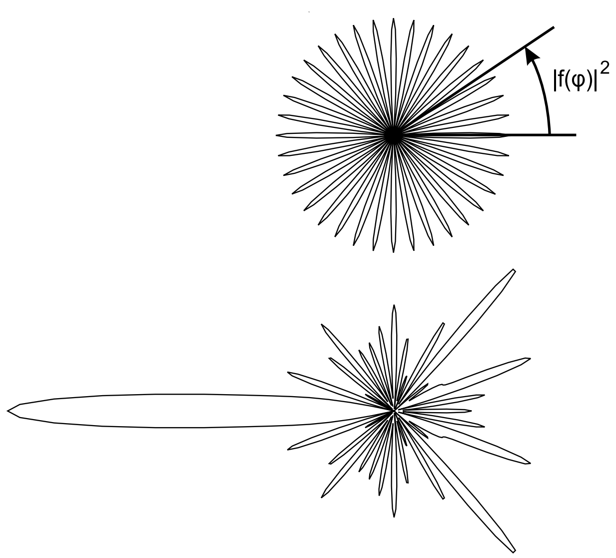

Fig. 1. Polar plot of the far-field intensity $|f(\varphi)|^2$ for the TM mode $kR = 12.52385 - 0.00015\,i$ ($m = 18$, $q = 5$), of the microdisk without scatterer (upper panel) and for the perturbed TM mode $kR = 12.52676 - 0.00026\,i$ of the disk with the small scatterer (lower panel).

4. CONCLUSIONS

In summary, we demonstrated the existence of directional TM-modes in the emission spectrum of a two-dimensional passive microdisk cavity with a small but finite size scatterer. The directional modes, in fact, can be observed in various frequency regions depending on the position and refractive index of the finite size scatterer. It would be interesting and potentially very useful to get a more detailed explanation of these results by relating the resonant modes to the ray dynamics in the semiclassical limit.

5. REFERENCES

[1] T. M. Benson, S. B. Boriskina, P. Sewell, A. Vukovic, S. C. Greedy, and A. I. Nosich. *Frontiers in Planar Lightwave Circuit Technology*, volume 216 of *NATO Science Series II: Mathematics, Physics and Chemistry*, section Micro-optical resonators for microlasers and intergrated optoelectronics, pages 39–70. Springer, 2006.

[2] V. S. Ilchenko and A. B. Matsko. Optical resonators with whispering-gallery modes - part ii: applications. *IEEE Journal of Selected Topics in Quantum Electronics*, 12:15–32, 2006.

[3] S. L. McCall, A. F. J. Levi, R. E. Slusher, S. J. Pearton, and R. A. Logan. Whispering-gallery mode microdisk lasers. *Appl. Phys. Lett.*, 60:289–291, 1992.

[4] A. F. J. Levi, R. E. Slusher, S. L. McCall, S. J. Pearton, and R. A. Logan. Threshold characteristics of semiconductor microdisk lasers. *Appl. Phys. Lett.*, 63:1310–1312, 1993.

[5] J. U. Nöckel and A. D. Stone. Ray and wave chaos in asymmetric resonant optical cavities. *Nature*, 385:45–47, 1997.

[6] C. Gmachl, F. Capasso, E. E. Narimanov, J. U. Nöckel, A. D. Stone, J. Faist, D. L. Sivco, and A. Y. Cho. High-power directional emission from microlasers with chaotic resonators. *Science*, 280:1556–1564, 1998.

[7] V. M. Apalkov and M. E. Raikh. Directional emission from a microdisk resonator with a linear defect. *Phys. Rev. B*, 70:195317, 2004.

[8] J. Wiersig and M. Hentschel. Unidirectional light emission from high-q modes in optical microcavities. *Phys. Rev. A*, 73:031802(R), 2006.

[9] C. P. Dettmann, G. V. Morozov, M. Sieber, and H. Waalkens. Directional emission from an optical microdisk resonator with a point scatterer. *Europhysics Letters*, 82:34002, 2008.