1. Introduction

An optical microresonator is a device of micron size that partially traps light. A microcavity is a device that pumps active medium that emits coherent light and microcavities have a multitude of uses in reading DVDs to welding. The design of very efficient, very small and very directional laser emission is of major industrial interest.

Aiming for lasers with these characteristics, we look for resonances (decaying, unstimulated, linear modes) with high Q-factor and Directivity (definitions below).

The most common resonator geometry is Fabry-Perot, consisting of two parallel mirrors. More recently, resonators have been designed using total internal reflection [1], which allows higher reflectivity in a smaller device, and does not fix the wavefront [2].

We can keep both high Q-factor and directivity by putting an obstacle in the interior of the disk. This could be a point, line or hole [2,3], but we prefer a point (physically very small defect or hole) for which a lot more can be done analytically, and which allows a classical orbit is a trajectory in a 2D or 3D billiard, the wavenumber, which for resonances is analytically continued into the lower complex half-plane. Perturbing the shape of the boundary improves the directivity, but at the expense of the Q-value.

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2. Unperturbed microdisk

For a thin passive microdisk, Maxwell’s equations reduce approximately to the 2D scalar Helmholtz equation in the plane of the microdisk

$$\nabla^2 + k^2 n_{eff}(r^2) \psi = 0 \quad (1)$$

Here $n_{eff}$ is the effective refractive index (a function of the true refractive index and thickness) which we take to be $n$ inside the disk and 1 outside. $k = \omega/c$ is the wavenumber, which for resonances is analytically continued into the lower complex half-plane. $\Psi$ is the field variable, equal to $E_z$ for TM modes ($n_1 = 0$ or $H_z/n_{eff}$ for TE modes ($E_z = 0$). Other components of the field may be obtained by differentiating $\Psi$.

The unperturbed circle is separable in polar coordinates $(r, \phi)$ leading to modes with $e^{in\phi}$ dependence multiplied by $J_m(k r)$ inside the microdisk and $H_m(k r)$ outside. $J$ and $H$ are Bessel and Hankel functions of the first kind, respectively, to satisfy boundary conditions at the origin and at infinity.

The Green function for the microdisk $G(r, r_0, k)$ is given by (see Ref. [4])

$$G(r, r_0, k) = \frac{1}{2\pi k R} \sum_{m=0}^{\infty} \frac{1}{A_m} e^{i \alpha_m}$$
$$\times \cos \{ m(\varphi - \varphi_0) \} J_m(k r) H_m(k r_0)$$
$$+ \sum_{m=0}^{\infty} \frac{B_m}{A_m} e^{i \alpha_m}$$
$$\times \cos \{ m(\varphi - \varphi_0) \} H_m(k r) J_m(k r_0) \quad (2)$$

for $r$ and $r_0$ both less than $R$.

for one of $r$ and $r_0$ greater than $R$,

$$- \frac{i}{4} H_0^0(k r - r_0) + \frac{i}{4} \sum_{m=0}^{\infty} \frac{D_m}{A_m} e^{i \alpha_m}$$
$$\times \cos \{ m(\varphi - \varphi_0) \} J_m(k r) H_m(k r_0)$$
$$+ \sum_{m=0}^{\infty} \frac{E_m}{A_m} e^{i \alpha_m}$$
$$\times \cos \{ m(\varphi - \varphi_0) \} H_m(k r) J_m(k r_0) \quad (3)$$

for $r$ and $r_0$ both greater than $R$, where $r_0 > R$ is the smaller (larger) of $r$ and $r_0$. The coefficients are $\epsilon_m = 2 - \epsilon_m, 0$ and

$$A_m = a_0 J_m(k R) H_m^0(k R) - a_2 J_2(k R) H_m(k R)$$
$$B_m = a_1 J_m(k R) H_m^0(k R) - a_2 J_2(k R) J_m(k R)$$
$$D_m = a_1 H_m(k R) H_m^0(k R) - a_2 H_2(k R) H_m(k R)$$

with $(a_1, a_2) = (1, 0)$ for TM and $(n, 1)$ for TE. The resonances are given by the poles of the Green function, ie $A_m = 0$ which is the matching condition between the two regions.

In the semiclassical limit $k R \gg m$ we find resonances at

$$k R \approx \frac{\pi m}{n} - \frac{3}{4} \frac{2m}{n} + \frac{i}{2} \ln \frac{n - 1}{n + 1} \quad (5)$$

If instead we take $n \rightarrow \infty$ we find that $k R$ approaches the real axis at zeros of $J_{m+1}$, for the TM case and $J_m$ for the TE case.

3. Perturbed microdisk

We have previously considered the case of a point scatterer using self-adjoint extension theory [4]. The equation for the resonance is $G_x (d, d, k_{res}) = \lambda^{-1}$ where $G_x$ is a regularised Green function evaluated at the location of the scatterer $d$ and $\lambda$ is a parameter related to the strength of the scatterer. The results were similar to those below.

Here we use an explicit small scatter of radius $a$ and effective refractive index $n_a$ at a point $d$ inside the microdisk. For brevity we restrict to the TM case. The Green function of the perturbed disk is found by treating the scatter in the s-wave approximation. This results in

$$G^*(r, r_0, k) = G(r, r_0, k)$$
$$+ \frac{G(r, d, k) D G(d, r_0, k)}{1 - D^2 G^2 (d, d, k)} \quad (6)$$

where $G^*$ is the Green function in (2) without the Hankel function. The diffraction coefficient has the form $D = -4i k R / A_m$, where $A_m$ and $B_m$ are given by $A_m$ and $B_m$ with $(a_1, a_2) = (n, n_a)$ and $R$ replaced by $a$. The resonances of the perturbed system are determined by the poles of the Green function and hence are defined by the equation

$$0 = A_m B_m + \sum_{m=0}^{\infty} C_m$$
$$\times \sum_{m=0}^{\infty} \epsilon_m e^{i \alpha_m}$$
$$\cos \{ m(\varphi - \varphi_0) \} J_m(k R)$$

if the scatterer is located on the positive x-axis. The s-wave approximation is valid if $|n k (R - d)| \gg 1$ and $A_0 / B_0 \ll A_1 / B_1$.

4. Results

As noted above, we are interested in high $Q = |R(k)/23(k)|$ and high directivity $D = 2\pi f |\gamma_{max}|^2 \int_0^\infty |\Psi|^2 dp$.

where the field at infinity takes the form $\Psi \sim f(\varphi) e^{i m / \gamma / r^2}$. The directivity is remarkable given the small size of the scatterer. Two observations can help to understand this:

1. The location of the scatterer at $d = 0.5$ is the point at which a parallel beam of light at infinity would focus according to ray optics in the paraxial approximation. Compare the different sections of Fig. 2.

2. The size of the scatterer appears only logarithmically in the expressions.

This gives a definite prediction of where to put the scatterer in the case $n > 2$, which would be good to test experimentally. There is much left for future exploration, including further analytics, other geometries, and extending the s-wave and effective refractive index approximations.

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References


