STOCHASTIC STABILIZATION OF THE COSMIC MICROWAVE BACKGROUND RADIATION

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Photon trajectories in models of the Universe that have constant negative spatial curvature are exponentially unstable. We demonstrate that they can be stabilized by additional random fluctuations in the curvature. The mechanism is analogous to the one responsible for stabilizing the stochastic Kapitsa pendulum. We discuss the consequences for the cosmic microwave background fluctuations.

The cosmological assumptions of homogeneity and isotropy imply a spatial geometry of constant curvature, as in the standard Friedmann-Roberston-Walker Universe. Recent observations[1] have shown that this curvature is small, but have not yet determined its sign. The characteristic feature of negatively curved spaces is the exponential separation of initially close geodesics, leading to chaotic instability of cosmic microwave background (CMB) photons, for example, hence Gaussian fluctuations in the CMB[2,3]. Similar chaotic mixing mechanisms have also been proposed to explain pre-inflationary homogeneity[4] and arrows of time[5].

It is obvious, however, that the Universe is not exactly homogeneous and isotropic. Matter is quite unevenly distributed, in that galaxies and voids lead to fluctuations of the curvature. In particular, when the spatial curvature becomes positive at some points, it is no longer clear that the chaotic instability will remain. Intuitively, the fact that the fluctuations are random, and that the average curvature is still negative point towards a chaotic instability still more unpredictable than in the homogeneous case. In this paper, we show that the exact opposite is true: fluctuations of sufficiently high (spatial) frequency and amplitude tend to stabilize the photon trajectories. A back of the envelope calculation[6] suggests that this is possible in the present cosmological epoch.

Since our aim is to establish the principle of stochastic stabilization, rather than give a detailed calculation, we will focus on the essential ingredients for the effect, and ignore many phenomena with weaker effects on the stabilization. Thus we ignore vector and tensor perturbations (such as gravitational waves) and pressure fluctuations (for example due to relativistic neutrinos), writing in the conformal Newtonian gauge the perturbed line element in the form[7]

$$ds^2 = R^2(\tau)\{(1+2\Phi)d\tau^2 - \frac{1-2\Phi}{[1+\frac{K}{4}(x^2+y^2+z^2)]^2}(dx^2+dy^2+dz^2)\}. \quad (1)$$

Here $R(\tau)$ is the scale factor or spatial curvature radius of the Universe, $\tau = \int R^{-1}dt$ is conformal time, $\Phi \ll 1$ is the Newtonian gravitational potential, $K = -1$ is the spatial curvature corresponding to a hyperbolic geometry, (x, y, z) are comoving coordinates, and units are chosen in which $G = c = 1$. Since the equations for null geodesics are conformally invariant (c.f. 8), the overall factor R appears only in the definition of the conformal time.

The separation of geodesics is given by the geodesic deviation equation[9]; computing the spatial projection (that is, on hypersurfaces of constant τ) with respect to a basis aligned to the instantaneous direction of motion (that is, undergoing Fermi-Walker transport[9]), leads to an equation for the separation of the two spatial directions orthogonal to the direction of motion:

$$\frac{d^2}{d\tau^2}\begin{pmatrix}\xi\\ \eta\end{pmatrix} = -\begin{pmatrix}K+2\Phi_{\xi\xi}+\Phi_{\zeta\zeta} & 2\Phi_{\xi\eta}\\ 2\Phi_{\xi\eta} & K+2\Phi_{\eta\eta}+\Phi_{\zeta\zeta}\end{pmatrix}\begin{pmatrix}\xi\\ \eta\end{pmatrix} \quad (2)$$

where the constant exponential separation arising from the spatial curvature K is modified by the second derivatives of the Newtonian potential, namely tidal forces. Our Newtonian assumptions imply that Φ is constant in time; we assume that it is random in space, leading to a rapidly fluctuating force on the CMB photons, which we represent schematically by

$$\frac{d^2u}{d\tau^2} = -(Af(\tau)-1)u. \quad (3)$$

where A is the amplitude and $f(\tau)$ is some stochastic forcing function with zero mean and characteristic frequency ω. This equation also describes an inverted pendulum which can be stabilized by appropriate oscillations of the pivot[10]; the smooth hyperbolic geometry corresponds to the gravitational force on the pendulum, and the matter fluctuations correspond to the stochastic forcing.

Without forcing the solutions clearly grow exponentially. In the forced case, the $\omega \to \infty$ limit has been shown to stabilize using Lyapunov exponent techniques[11]; we now find the dependence of the stability on ω by separating u asymptotically into fast and slow components: $u(\tau) = \langle u(\tau)\rangle + u_f(\tau)$ where the $\langle\rangle$ denote an average over a time scale T_{av} with $\omega^{-1} \ll T_{av} \ll 1$; we will have in the high frequency limit $Af(\tau) \ll \langle u(\tau)\rangle \approx 1 \ll u_f$. Substituting into (3) and averaging, we have

$$\frac{d^2\langle u\rangle}{d\tau^2} = \langle u\rangle - A\langle f(\tau)u_f(\tau)\rangle. \quad (4)$$

Subtracting (4) from (3) gives, as $\omega \to \infty$,

$$\frac{d^2u_f}{d\tau^2} \approx -Af(\tau)\langle u(\tau)\rangle. \quad (5)$$

This equation can be integrated directly, treating $\langle u(\tau)\rangle$ as a constant, leading to $u_f(\tau) \approx -A\langle u(\tau)\rangle x(\tau)$ where $x = \int v d\tau$, $v = \int f d\tau$ and the constants of integration are chosen so that the averages of x and v are zero.

Substituting into (4) and computing the integral implicit in the average by parts, we obtain

$$\frac{d^2\langle u\rangle}{d\tau^2} \approx (1 - A^2\langle v^2(\tau)\rangle)\langle u\rangle. \tag{6}$$

It follows from (6) that if $A^2\langle v^2\rangle < 1$ then u grows exponentially as $\tau \to \infty$, but if $A^2\langle v^2\rangle > 1$ then u is linearly stabilized. This effect, and the accuracy of (6), are illustrated by numerical simulations presented elsewhere[6].

In the cosmological context, an analogous calculation in two dimensions[6] based on Eq. (2) shows that stabilization follows if $A^2/\omega^2|K| > 1$ and $\omega \gg 1$. The latter simply says that the photon has had a chance to be influenced by the gravitational field of many clusters of galaxies; as the Universe expands ω will decrease until the photon ceases to interact. On the other hand, $A^2 \gg 1$ since the mass density in clusters is much greater than the average cosmological density, so for some range of times (including our own) both inequalities are satisfied. Note that the exact value of $|K| \ll 1$ is not required: the parallel geodesics of a spatially flat Universe are equally well stabilized, and (6) continues to hold.

Stabilisation leads to alternate focussing and defocussing of photons from the largest cosmological distances; we expect this to limit the resolution of the CMB fluctuations, and possibly also affect observations of distant galaxies. More detailed calculations will follow in a future paper.

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