RANDOM GRAPHS AND WIRELESS COMMUNICATION NETWORKS

Part 8: Mobility

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Summary

Most literature focuses on human mobility, eg cellular or mobile ad-hoc network (MANET) applications. Can also consider animals, robot swarms, underwater sensors etc.

1. Random walk

2. Random waypoint

3. Truncated Lévy walk

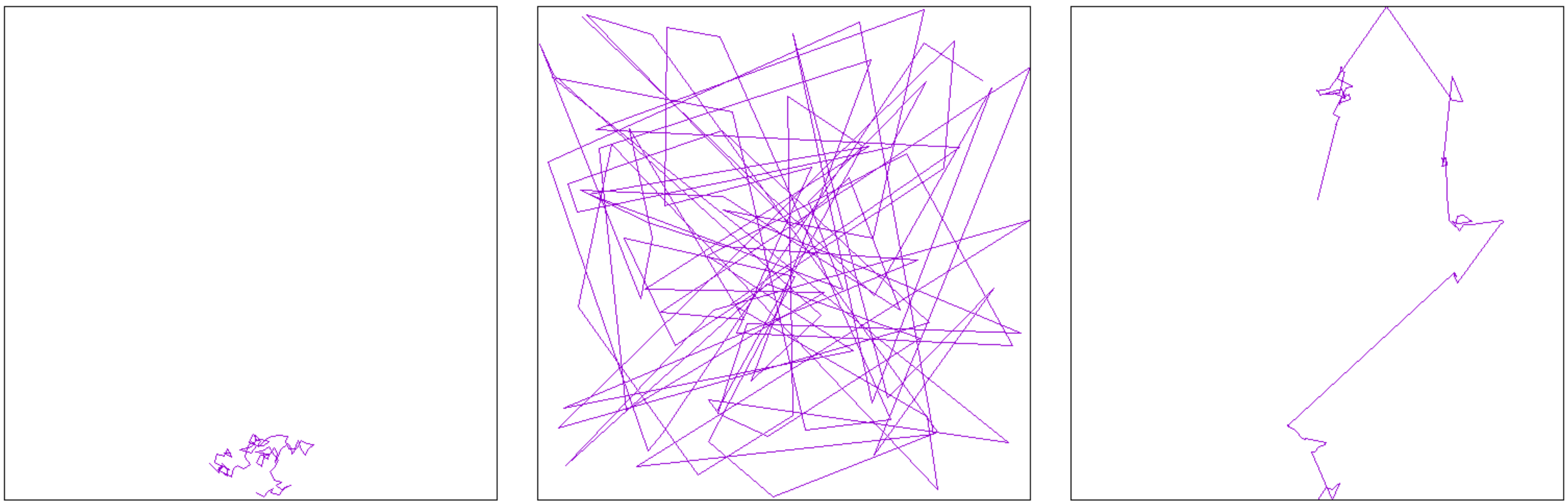
4. Self-similar least action walk (SLAW)

5. Periodic and social mobility model (PSMM)

For each of these we will be thinking about simulation algorithms, short and long time properties, effects of boundaries.

Random models

100 steps, differing characteristics...



Random walk | Random Waypoint | Truncated Lévy

The random walk, neglecting boundaries

The simplest, but also usually least realistic, mobility model is the random walk (RW). In each time step T, move wth a random velocity (for example speed $\sim U(v_{min}, v_{max})$, and angle $\sim U(0, 2\pi)$) independent of all previous steps. Thus the displacement is

$$\Delta(t) = \mathbf{V}_{[t/T]} \left\{ \frac{t}{T} \right\} T + \sum_{i=0}^{[t/T]-1} \mathbf{V}_i T$$

We see that at short times $t < T$ each node moves ballistically,

$$\Delta(t) = \mathbf{V} t$$

whilst for longer times motion is diffusive, controlled by the central limit theorem (assuming as usual that $\mathbb{E}(\mathbf{V}) = 0, \ \mathbb{E}(\mathbf{V}^2) < \infty$)

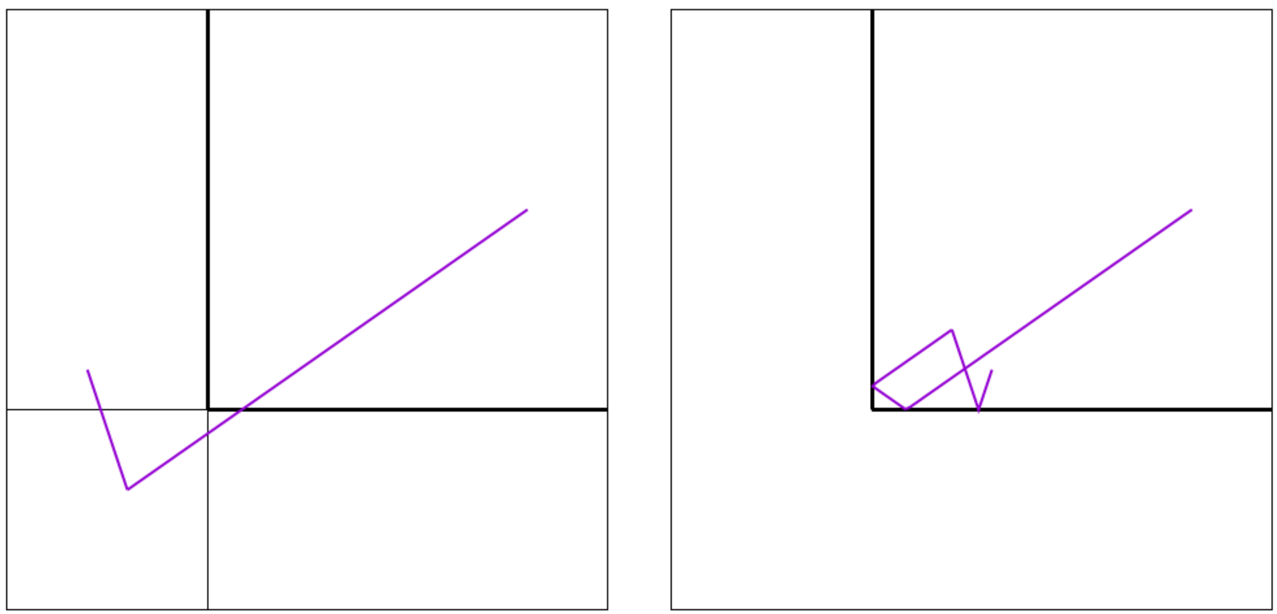
$$\mathbb{P}\left(\Delta(t) \in t^{1/2} \mathcal{D} \right) \to \int_{\mathcal{D}} \frac{\exp\left(-\frac{\mathbf{x}^2}{2\mathbb{E}(\mathbf{V}^2) T t / d} \right)}{(2\pi \mathbb{E}(\mathbf{V}^2) T t / d)^{d/2}} d\mathbf{x}$$

So, with high probability $\Delta(t)$ is of order $\sqrt{\mathbb{E}(\mathbf{V}^2) T t}$.

Recurrence: The node returns arbitrarily close to its starting point with probability 1. Well known to hold for $d \leq 2$ and to fail for $d = 3$ (generalisation of Pólya's recurrence theorem for random walks on lattices).

Boundary effects

If a boundary is reached, reflect from it like a mirror and continue for the remainder of the time interval (also for other mobility models). If all boundaries are straight and and corners of angle π/n (ie rectangles, a few triangles), we can consider the whole trajectory in free space, then do reflections as needed to obtain the bounded trajectory.



In the RW at long times, the nodes are distributed uniformly, hence recurrent.

Random waypoint

The random waypoint mobility (RWPM) model is a popular and somewhat more realistic approach. Each node undergoes a sequence of alternating flights and pauses.

Flight Choose a random destination point (eg uniformly) in the (bounded, convex) domain, and a speed, for example $v_k \sim U(v_{min}, v_{max})$. Move uniformly to the destination point, taking a time $\tau_k = |\mathbf{x}_{k+1} - \mathbf{x}_k|/v_k$.

Pause Stop for a random time T_k, for example $T_k \sim U(T_{min}, T_{max})$.

Thus

$$t_k = \sum_{j=0}^{k-1}(\tau_j + T_j) \qquad \mathbf{x}(t) = \begin{cases} \mathbf{x}_k + (\mathbf{x}_{k+1} - \mathbf{x}_k)\frac{t-t_k}{\tau_k} & t_k < t < t_k + \tau_k \\ \mathbf{x}_{k+1} & t_k + \tau_k < t < t_{k+1} \end{cases}$$

Simulation: Typically all nodes are started unformly in the domain at the beginning of a flight. This differs from the steady state which is reached after a few times the longest possible flight plus pause time, namely

$$\frac{D}{v_{min}} + T_{max}$$

where D is the diameter of the domain.

Average node density I

The waypoints $\mathbf{x}_k$ are chosen with respect to a given probability density $f_s(\mathbf{x})$, perhaps uniform. However the average node density $f(\mathbf{x})$ differs because much of the time is spent at intermediate points:

$$f(\mathbf{x}) = p_s f_s(\mathbf{x}) + (1 - p_s) f_m(\mathbf{x})$$

where $f_m(\mathbf{x})$ is the density of mobile nodes and the probability that a given node is static is

$$p_s = \frac{\mathbb{E}(T)}{\mathbb{E}(T) + \mathbb{E}(\tau)}$$

If $T_k \sim U(T_{min}, T_{max})$ we have

$$\mathbb{E}(T) = \frac{T_{min} + T_{max}}{2}$$

while the average flight time is

$$\mathbb{E}(\tau) = \mathbb{E}(v^{-1})\bar{l}$$

where $\bar{l}$ is the average flight length. If $v_k \sim U(v_{min}, v_{max})$ we have

$$\mathbb{E}(v^{-1}) = \frac{\ln v_{max} - \ln v_{min}}{v_{max} - v_{min}}$$

Average node density II

We still need $\bar{l}$ and $f_m(\mathbf{x})$. These are found by integrating over source and destination points, weighted by the flight length since each node spends proportionately longer times on longer flights. The proportion of paths that pass through a region A gives the integral of $f_m(\mathbf{x})$ over A [HLV06]. Remarkably, $f_m(\mathbf{x})$ also gives the expected betweenness centrality of a node at $\mathbf{x}$ [GGD15].

We use polar coordinates centred at $\mathbf{x}$ in which the domain has equation $r = R(\phi)$. The source node is (r_1, ϕ) for $0 < r_1 < a_1 \equiv R(\phi)$ and the destination node $(r_2, \phi + \pi)$ for $0 < r_2 < a_2 \equiv R(\phi + \pi)$.

1D Here, $x \in [0, L]$, $\phi = 0$, $a_1 = L - x$, $a_2 = x$.

$$f_m(x) = \frac{2}{\bar{l}} \int_0^{a_1} g(x + r_1)\, dr_1 \int_0^{a_2} g(x - r_2)\, dr_2$$

$$\bar{l} = \int_0^L \int_0^L g(x) g(y) |y - x|\, dy\, dx$$

2D

$$f_m(\mathbf{x}) = \frac{2}{\bar{l}} \int_0^{2\pi} d\phi \left[\int_0^{a_1} g(\mathbf{x} + r_1 \mathbf{e}_\phi)\, r_1\, dr_1 \int_0^{a_2} g(\mathbf{x} + r_2 \mathbf{e}_{\phi+\pi})\, dr_2 \right]$$

$$\bar{l} = \int \int g(\mathbf{x}) g(\mathbf{y}) |\mathbf{y} - \mathbf{x}|\, d\mathbf{y}\, d\mathbf{x}$$

In both cases $\bar{l}$ can also be calculated using the normalisation of f.

Uniform waypoints

1D uniform We find on the unit interval $[0,1]$

$$f_1(x) \equiv f_m(x) = 6x(1-x), \qquad \bar{l} = \frac{1}{3}$$

On $[0,L]$ we have simply

$$f_m(x) = \frac{f_1(x/L)}{L}, \qquad \bar{l} = \frac{L}{3}$$

2D uniform The above expression reduces to

$$f_m(\mathbf{x}) = \frac{1}{\bar{l}V^2}\int_0^\pi a_1 a_2(a_1+a_2)d\phi$$

where V is the area of the domain. For a disk of radius R this gives

$$f_m(r,\theta) = \frac{2(R^2-r^2)}{\bar{l}(\pi R^2)^2}\int_0^\pi \sqrt{R^2 - r^2\cos^2\phi}\,d\phi, \qquad \bar{l} = \frac{128R}{45\pi}$$

On a rectangle with side lengths (a,b), the approximation

$$f_m(x,y) \approx \frac{f_1(x/a)f_1(y/b)}{ab}$$

is often used; see [BRS03] for more sophisticated approximations. The full analytical result was finally obtained in Ref. [PDG16]...

In the rectangle $[-a,a]\times[-b,b]$ with $1>\frac{y}{b}>\frac{x}{a}>0$ (and similarly for other regions)

$$
\begin{aligned}
f(x,y) = \frac{1}{16a^2b^2\bar{l}}\Bigg\{ & \frac{d_1(b-y)(a+x)(a-2x)}{(a-x)} + \frac{d_4(a-x)(a+2x)(b-y)}{(a+x)} \\
& + [d_3(a-x)+d_2(a+x)]\left[\frac{(b-y)(b+2y)}{(b+y)}\right] + 2x(b-y)^2 \ln\left|\frac{a-x}{a+x}\right| + 4ax(b-y)\ln\left|\frac{b-y}{b+y}\right| \\
& + c_1\left[\ln\left|\frac{b-y+d_1}{a-x}\right| + \ln\left|\frac{y-d_4-b}{a+x}\right|\right] + c_2\ln\left|\frac{d_3+a-x}{d_2-a-x}\right| + c_3\ln\left|\frac{d_2+b+y}{d_1+b-y}\right| \\
& + c_4\ln\left|\frac{d_2-a-x}{d_1+x-a}\right| - c_5\ln\left|\frac{y-d_4-b}{-d_3-b-y}\right| + c_6\ln\left|\frac{d_4+a+x}{d_3+a-x}\right| \Bigg\}
\end{aligned}
$$

where

$$
\begin{aligned}
c_1 &= a(a-x)(a+x) \\
c_2 &= b(b-y)(b+y) \\
c_3 &= (a+x)(b-y)^2 \\
c_4 &= (a+x)^2(b-y) \\
c_5 &= (x-a)(b-y)^2 \\
c_6 &= (x-a)^2(b-y) \\
d_1 &= \sqrt{(a-x)^2+(b-y)^2} \\
d_2 &= \sqrt{(a+x)^2+(b+y)^2} \\
d_3 &= \sqrt{(a-x)^2+(b+y)^2} \\
d_4 &= \sqrt{(a+x)^2+(b-y)^2}.
\end{aligned}
$$

Short time properties

We are now in a position to analyse short time properties, once the system has reached a steady state. Given a node, its location is distributed according to $f(\mathbf{x})$ and has a probability p_s of being static.

If it is static, the probability that it will remain static for at least a time $\Delta < T_{min}$ is found by weighting the distribution of times by T, and integrating the probability that the time is not within the last Δ of the pause:

$$\begin{aligned}\mathbb{P}(t_{k+1} > t + \Delta | t_{k+1} < t + T_k) &= \int_{T_{min}}^{T_{max}} \frac{2T}{T_{max}^2 - T_{min}^2} \frac{T - \Delta}{T} dT \\ &= 1 - \frac{2\Delta}{T_{max} + T_{min}}\end{aligned}$$

and similarly for $T_{min} < \Delta < T_{max}$ and in a more involved manner if it is mobile. Thus, a probability distribution for $\mathbf{x}(t + \Delta)$ given $\mathbf{x}(t)$ can be constructed, corrected by contributions from situations where the node stops and/or starts to move in a new direction during the time interval.

This analysis has been carried out in detail in a simplified model (lattice in 1D) to understand time correlation of interference in Ref. [KD16].

Nonuniform waypoints and generalisations

There have also been a number of studies on nonuniform waypoints and generalisations of RWPM, for example

- A nonuniform waypoint density and/or a position-dependent pause time distribution can lead to a uniform average mobile density [MRS14].

- Uniform on the boundary, the random waypoint on border (RWPB) model [HLV06]. The density can be obtained as a limit of the above integrals, and typically increases towards the boundary; if there are straight segments on the boundary, the node spends a finite fraction of time there.

- In nonconvex domains, the uniform motion between waypoints can be replaced by a path that moves to the nearest vertex to the destination that is visible, the mission critical mobility model (MCM) [PBDK12]. In this case, a node can spend a finite fraction of its time on the boundary of the obstacles.

Lévy flights and walks

The standard central limit theorem applies only when the variance of the random variable is finite. For heavy tailed symmetric distributions, with densities

$$f_X(x) \sim x^{-\alpha-1}, \qquad x \to \infty, \qquad 0 < \alpha < 2$$

the sum of iid random variables approaches a Lévy stable law with density

$$f_{stable}^{\alpha,c}(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp[-itx - |ct|^{\alpha}]\, dt$$

The constant $c > 0$ is a scale factor. For most α there is no simple analytic form; for $\alpha \to 2$ we return to the Gaussian distribution, and $\alpha = 1$ gives a Cauchy distribution:

$$f_{stable}^{1,c}(x) = \frac{c/\pi}{x^2 + c^2}$$

Nonsymmetric stable distributions with mean μ and "skewness parameter" β have been similarly defined, but we will not need them here.

A **Lévy flight** is a random walk with a heavy tailed symmetric distribution. Realistically it takes a finite time to travel long distances; including a finite velocity leads to **Lévy walk** models. These models exhibit a variety of anomalous diffusion properties, for example the super-ballistic Richardson's law of turbulent diffusion $\mathbb{E}(\Delta(t)^2) \sim t^3$ [SKW86].

Truncated Lévy walk mobility model

A popular and experimentally validated model of human mobility is the following truncated Lévy walk model [RSHLKC11].

Each node undergoes a sequence of alternating flights and pauses, similar to RWPM. First choose a random position and values of the constants $\alpha, \beta \in (0, 2)$, $c_f, c_p, k, \tau_l, \tau_p > 0$, $\rho \approx 0.5$. Then,

Flight Choose a random path length $l \sim f_{stable}^{\alpha, c_f}(l)$; reject and resample if $l < 0$ or $l > \tau_l$. The flight time is $\Delta t_f = k l^{1-\rho}$. The direction $\theta \sim U(0, 2\pi)$. Move uniformly to the destination point, reflecting from the boundaries if needed.

Pause Stop for a random time $\Delta t_p \sim f_{stable}^{\beta, c_p}(l)$; reject and resample if $\Delta t_p < 0$ or $\Delta t_p > \tau_p$.

Simulating the symmetric stable distribution

This is not obvious; an algorithm was given in Ref. [CMS76] for the more general case of nonzero mean and skewness parameter; in our case:

1. Generate $U \sim U\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $W \sim \exp(1)$.

2. Then the desired random variable with density $f_{stable}^{\alpha,c}$ is

$$X = \begin{cases} c\frac{\sin(\alpha U)}{\cos^{1/\alpha} U}\left[\frac{\cos(U(1-\alpha))}{W}\right]^{\frac{1-\alpha}{\alpha}} & \alpha \neq 1 \\ c \tan U & \alpha = 1 \end{cases}$$

Note that due to the generalised central limit theorem, the results of a truncated Lévy walk are unlikely to depend sensitively on the choice of distribution with given tail exponents α (for the path length) and β (for the pause time).

Group, social and periodic mobility models

Recently, a number of very detailed mobility models have been introduced to describe mobility in specific applications, taking account of effects such as

Groups A group of nodes has a leader and other members, whose mobility is related.

Social interaction Nodes have preferential attachment, moving to areas with more nodes. Or, there is an underlying network of friends, which affects node behaviour and may be affected by node locations.

Time-periodic behaviour Nodes are likely to spend certain times of the day/week at fixed home or work locations, together with commuting and social travel at other times.

SLAW

A self-similar least action walk (SLAW) was proposed in Ref. [LHKRC09]. The aim was to include

1. Truncated power law flights and pause times

2. Heterogeneously bounded mobility areas

3. Truncated power law inter-contact times

4. Fractal waypoints

SLAW construction

Fractal waypoints Construct a fractal set of points by recursively subdividing the region and allocating points nonuniformly at each stage.

Clusters Partition these points into clusters of size roughly 100 metres (typical connection range).

Walkers Each walker chooses 3 to 5 clusters with a probability weighted by the number of points in each cluster. Then a set of roughly 10% of points in each of the chosen clusters, thenone of these points that will be its home.

Daily walk Each walker chooses a new cluster randomly as before and walk from home to each of the usual and new chosen points according to a "least action trip planning" (LATP) algorithm, returning home. Pause times are chosen from a truncated Lévy distribution as with the TLW and the total time normalised to 12 hours.

LATP: Choose the next waypoint from all unvisited points with a probability weighted by distance to the power $-\alpha$, $\alpha \approx 2$.

Simulation code for truncated Lévy walk and SLAW:

`http://research.csc.ncsu.edu/netsrv/?q=content/human-mobility-models-download-tlw-slaw`

PSMM

The Periodic and Social mobility Model (PSMM) of Ref. [CML11] is:

- Each user has a probability that varies throughout the day of being in Home or Work states.
- If Home or Work, the location is distributed Normally around these locations
- There is also a probability of being in a Social state, in which case the time and location are distributed close (with power law decay) to those of the social check-ins of the user's friends.

Fitting this to real data, these authors concluded that social relationships explain 10-30% of human mobility, while periodic behaviour explains 50-70%.

Dynamic network properties

Mobility models affect the rate of breaking links (both cellular and MANET):

- Low in RW
- Moderate in TLW and related models
- High in RWP.

This then impacts the choice of protocols, eg routing protocol in MANET applications should be more “reactive” than “proactive” if links break frequently.

Summary

Mobility models are important to describe both short and long timescale behaviour in networks.

Waypoints can be distributed randomly, from a fixed set, or a combination of these.

There is generally a trade-off between mathematical tractability/insight and realism.

Many of the effects of nonuniform spatial distributions, correlated and periodic behaviour, dynamic and collective processes on networks have yet to be explored.

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