On Modeling Cellular Networks by Using Inhomogeneous Poisson Point Processes

Marco Di Renzo

Paris-Saclay University
Laboratory of Signals and Systems (L2S) – UMR8506
CNRS – CentraleSupelec – University Paris-Sud
Paris, France
marco.direnzo@l2s.centralesupelec.fr

H2020-MCSA

H2020-MCSA

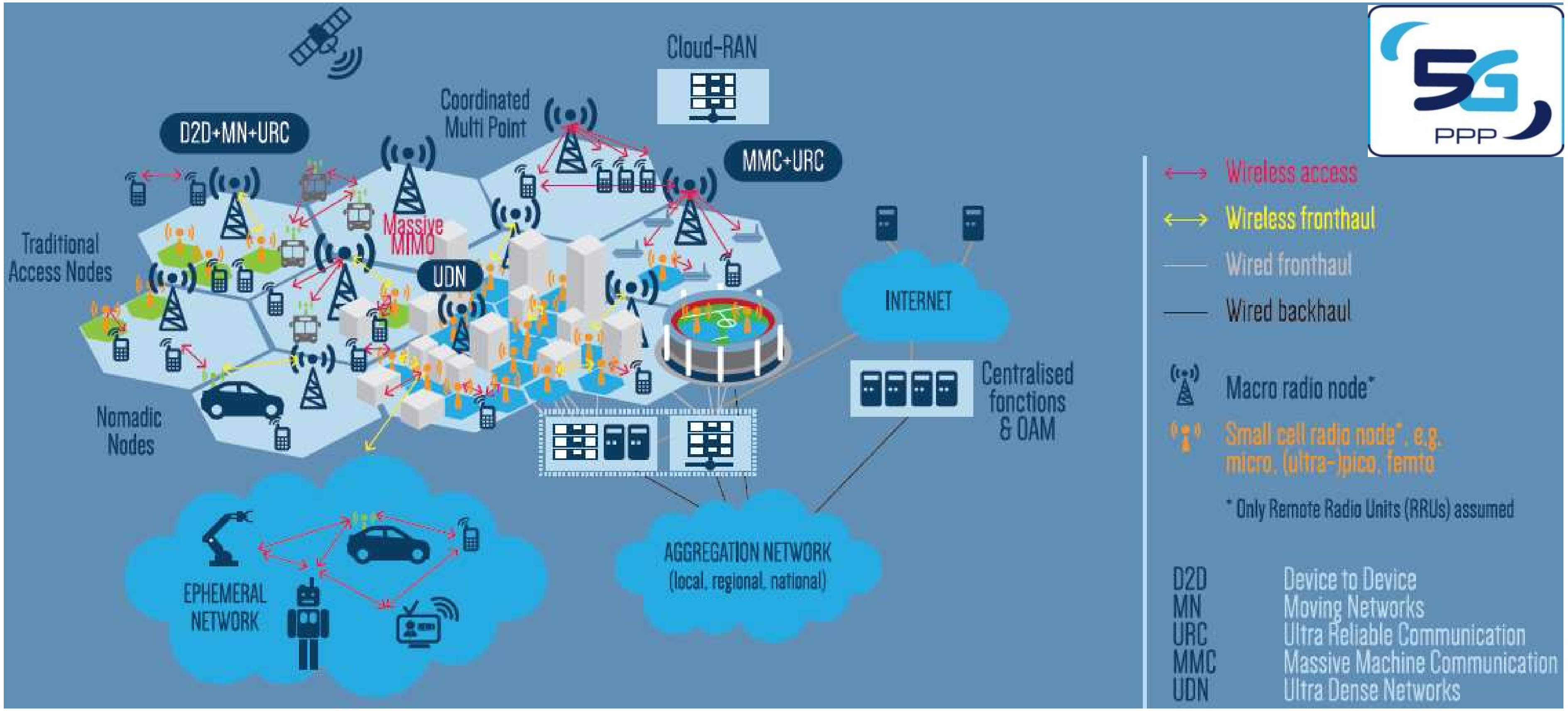
Complex Networks & Point Processes
Oxford University, September 2017

5G-PPP – 5G Network Vision

More information at
www.5g-ppp.eu

5G-PPP 5G Vision Document, "The next-generation of communication networks and services", March 2015. Available: http://5g-ppp.eu/wp-content/uploads/2015/02/5G-Vision-Brochure-v1.pdf.

5G-PPP – 5G Network Vision



5G-PPP 5G Vision Document, "The next-generation of communication networks and services", March 2015. Available: http://5g-ppp.eu/wp-content/uploads/2015/02/5G-Vision-Brochure-v1.pdf.

The 5G (Cellular) Network Of The Future

- **Buzzword 1: Densification**
  1. **Access Points (*Network Topology*, HetNets)**
  2. **Radiating Elements (Large-Scale/Massive MIMO)**

- **Buzzword 2: Spectral vs. Energy Efficiency Trade-Off**
  1. **Shorter Transmission Distance (Relaying, Femto, D2D)**
  2. **Total Power Dissipation (Single-RF MIMO, Antenna Muting)**
  3. **RF Energy Harvesting, Wireless Power Transfer, Full-Duplex**

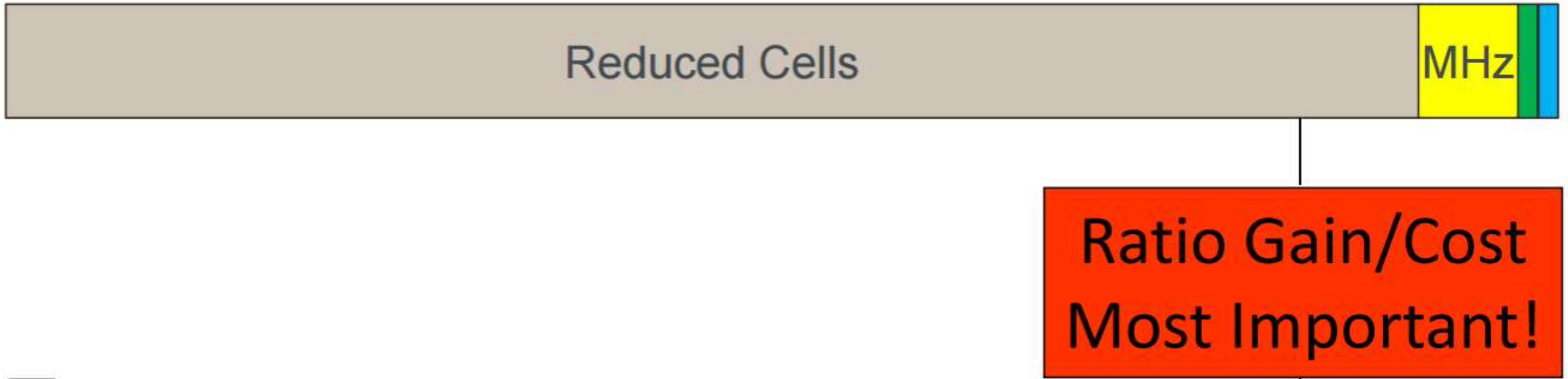
- **Buzzword 3: Spectrum Scarcity**
  1. **Cognitive Radio and Opportunistic Communications**
  2. **mmWave Cellular Communications**

- **Buzzword 4: Software-Defined, Centrally-Controlled, Shared, Virtualized**
  1. **SDN, NFV, Network Slicing**

Why Network Densification Is So Important ?

… Increase in Capacity Over the Last Decade …

- Breakdown of these gains:
  - 5 x PHY; 25 x spectrum; 1600 x reduced cells, 5 x rest



- Breakdown of (estimated) cost:

$$\text{bit/sec/m}^2 \leftrightarrow \lambda_{\text{BS}} \text{B}_{\text{w}} \log_2\left(1 + \text{SINR}\right)$$

M. Dohler, R. W. Heath Jr., A. Lozano, C. Papadias, R. A. Valenzuela, "Is the PHY Layer Dead?", IEEE Communications Magazine, Vol 49, No 4, pp. 159-165, April 2011.

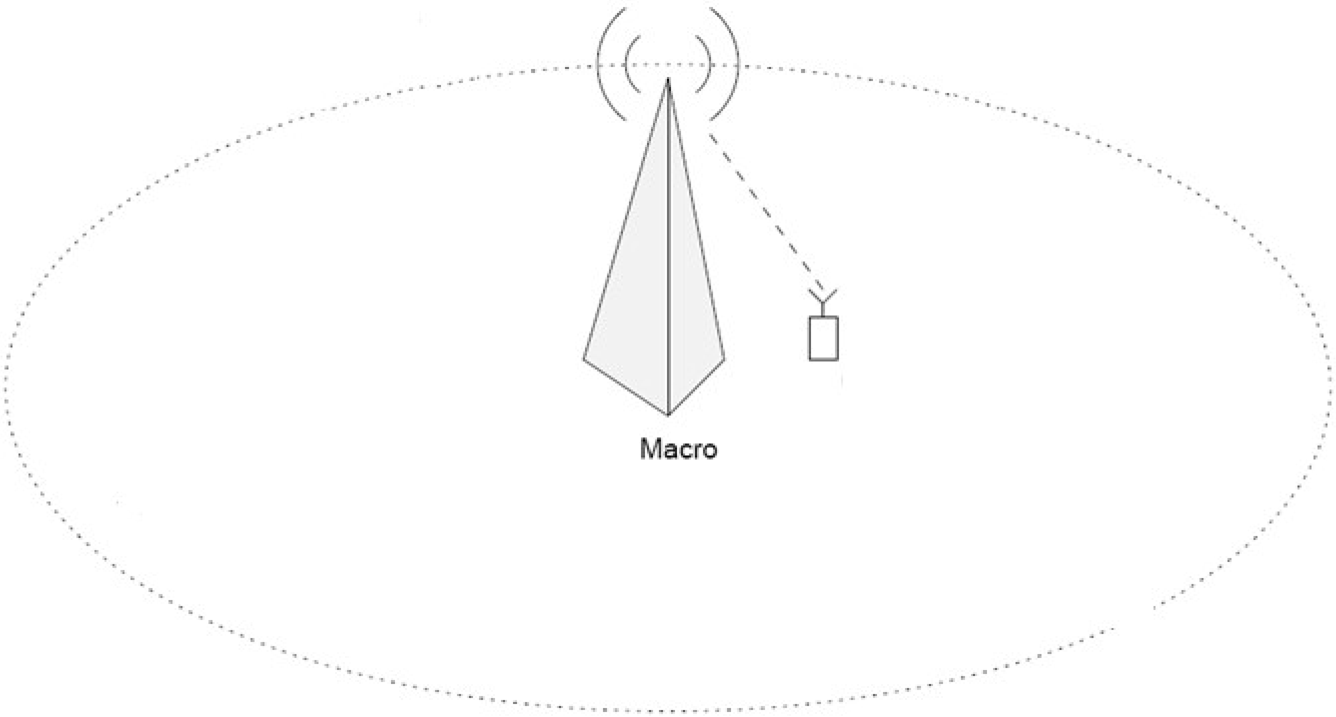
This Talk: Poisson Point Processes and Beyond

- **Part I: Poisson Point Processes for Cellular Networks**
  - **Why do we need Stochastic Geometry ?**
  - **How to use Stochastic Geometry for performance evaluation ?**
  - **Six years later… where are we now ?**

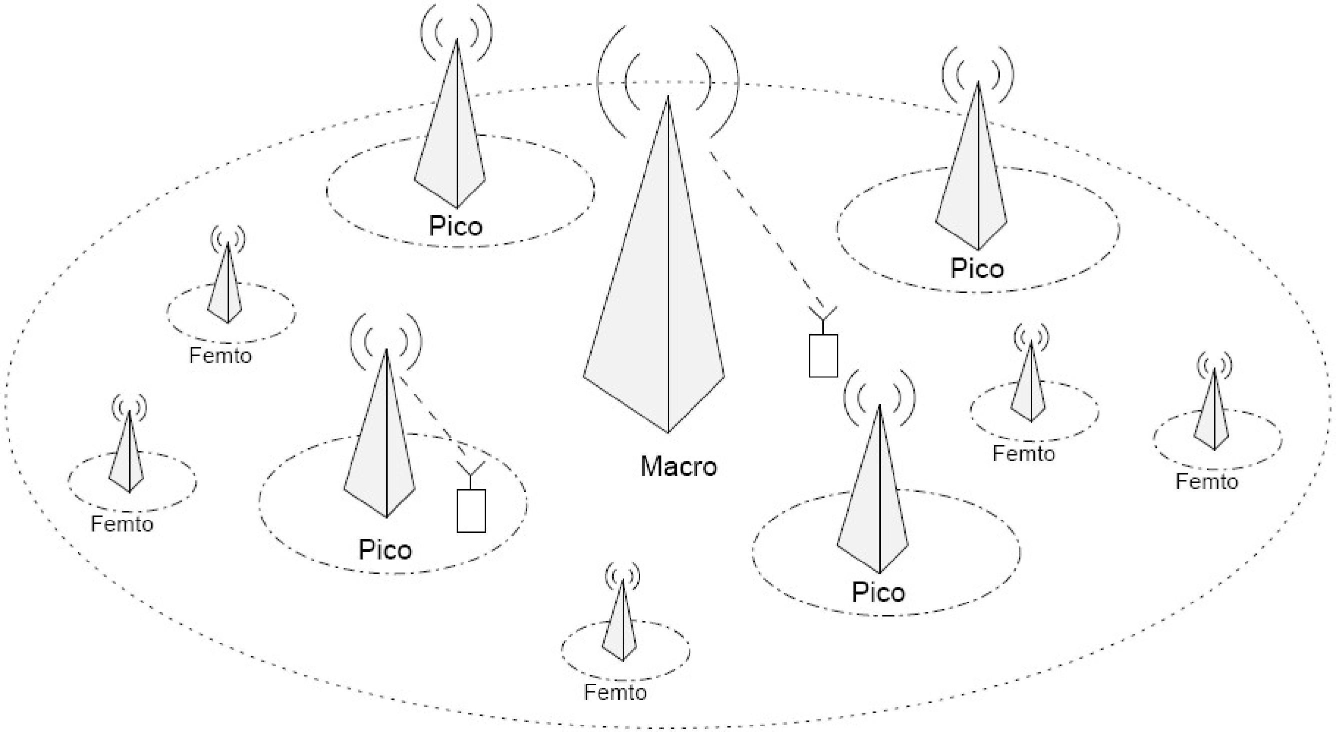
- **Part II: Beyond the Poisson Point Process**
  - **Why to go beyond the Poisson Point Process ?**
  - **What are the mathematical challenges of this generalization ?**
  - **Inhomogeneous Poisson Point Processes – An approximation**

… Part I …
(The PPP Saga)

Why? - Densification of Base Stations (your parents net)

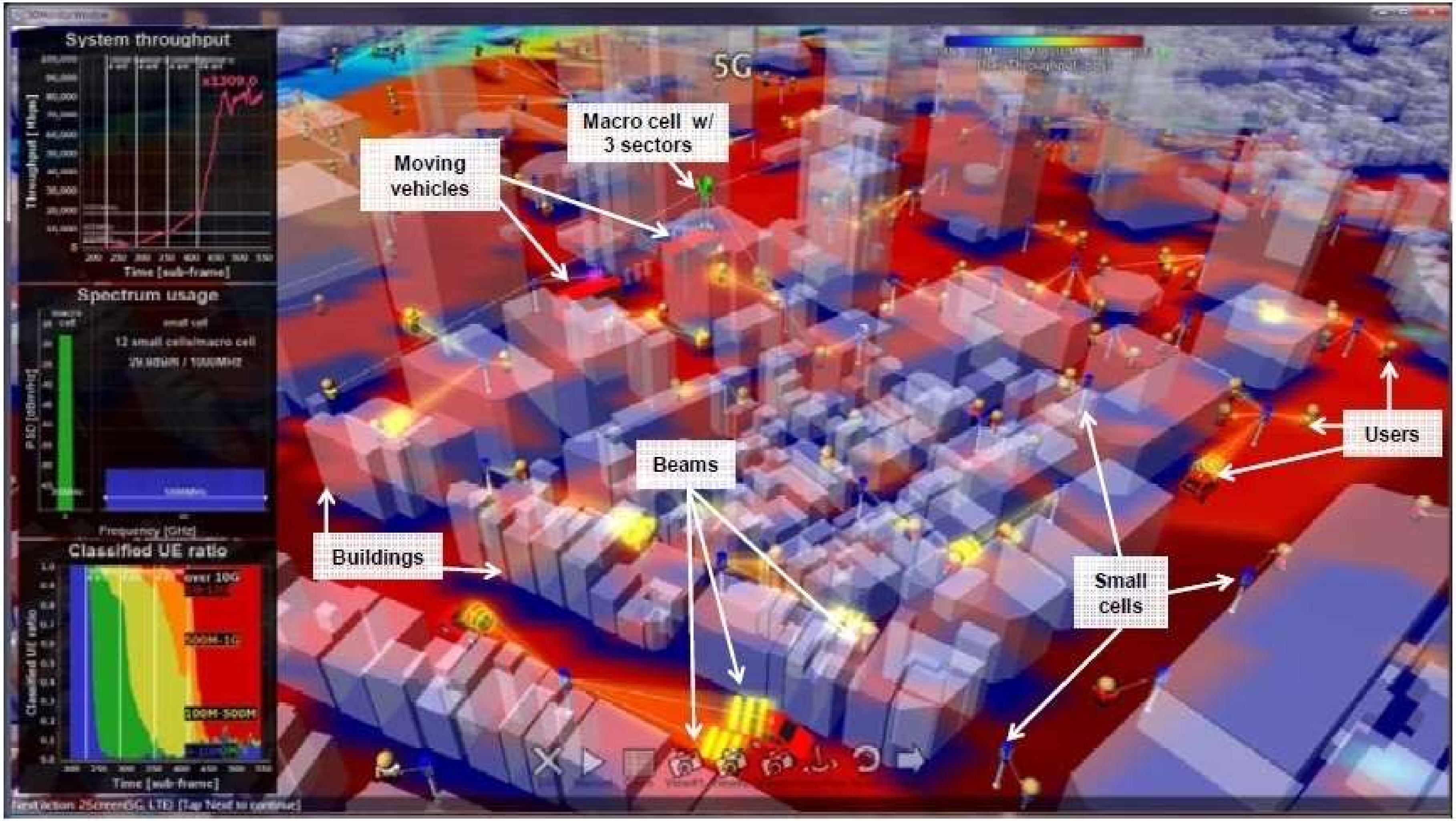


Why? - Densification of Base Stations (your kids net)



Modeling Cellular Networks – In Industry

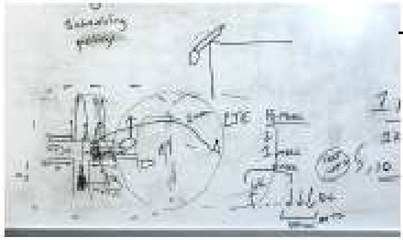
The NTT DOCOMO 5G Real-Time Simulator



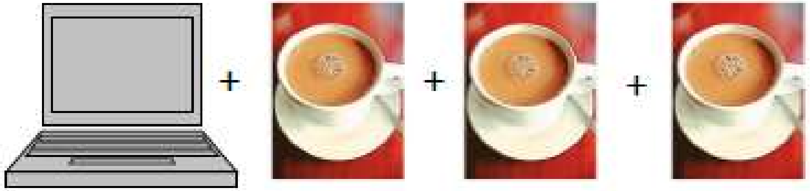
DOCOMO 5G White Paper, "5G Radio Access: Requirements, Concept and Technologies", July 2014.

Life of a 3GPP Simulation Expert (according to Samsung)

Inspiration strikes!



Weeks writing/debugging code:



There's a reason the server is called "grumpy":

```
[t.novlan@grumpy ~]$ bhosts
HOST_NAME          STATUS       JL/U    MAX  NJOBS
grumpy             unavail       -        1      0
mpisrv01dal        ok            -       24      5
mpisrv02dal        unavail       -        1      0
mpisrv03dal        ok            -       20     14
mpisrv04dal        ok            -       24     18
mpisrv05dal        ok            -       20      0
```

Come back after waiting this long:

```
runTime =

   1.4498e+05
```

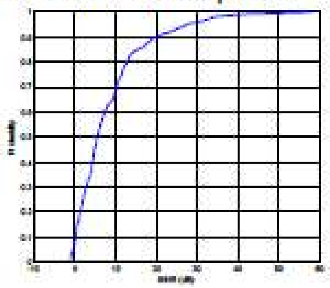
seconds!

Hope you don't find this:

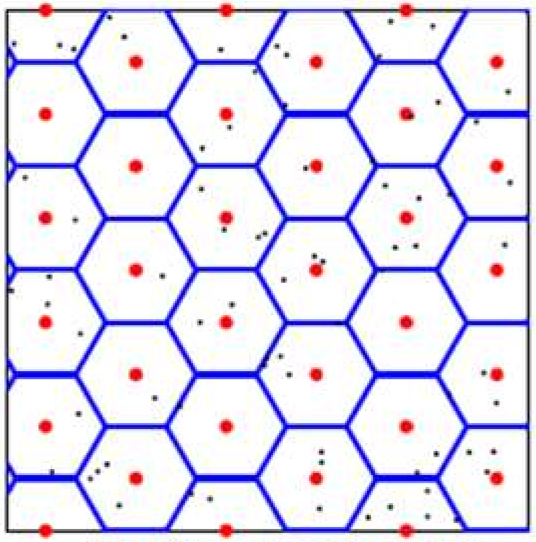
```
Segmentation violation detected-----
R8 = 000000002c81d6f5    R9 = 00002ac
_interpreter.so+02245206[  7] 0x0000
xa64/libmwm_interpreter.so+01923475[
13a/bin/glnxa64/libmwm_dispatcher.so-
[ 36] 0x00002ac2133eeb88 /opt/Hw/app:
```

All for a few numbers and a MATLAB plot:

```
Buffer Occupancy
    Operator 1 (laa): 0.560
    Operator 2 (wifi): 0.607
UE Throughput (Mbps)
    Operator 1 (laa): 10.804
    Operator 2 (wifi): 8.708
Packet Latency (s)
    Operator 1 (laa): 0.606
    Operator 2 (wifi): 0.702
```



Modeling Cellular Networks – In Academia

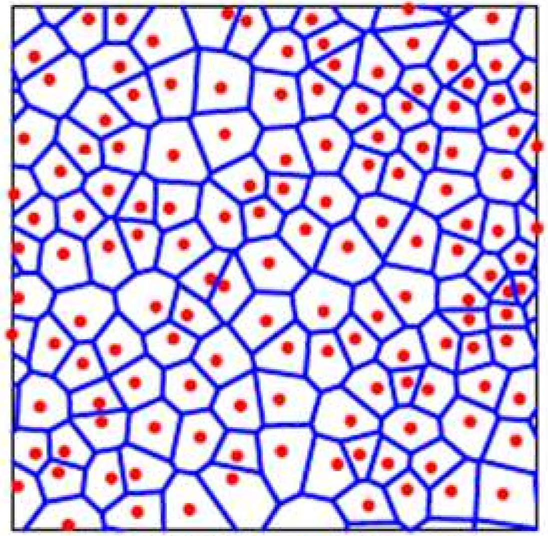


Traditional grid model

- Conventional approaches to the analysis and design of cellular networks (abstraction models) are:
  - The Wyner model
  - The single-cell interfering model or dominant interferers model
  - ***The regular hexagonal or square grid model***

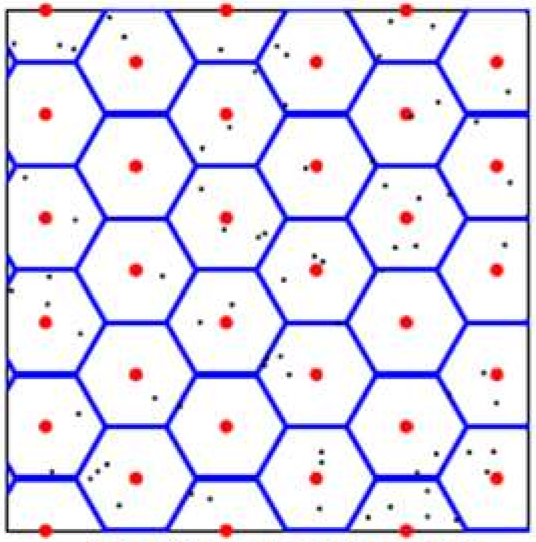
    D. H. Ring and W. R. Young, "The hexagonal cells concept", Bell Labs Technical Journal, Dec. 1947. http://www.privateline.com/archive/Ringcellreport1947.pdf.

Modeling Cellular Networks – In Academia



Actual 4G network today

← Reality
vs.
Abstraction
Modeling →

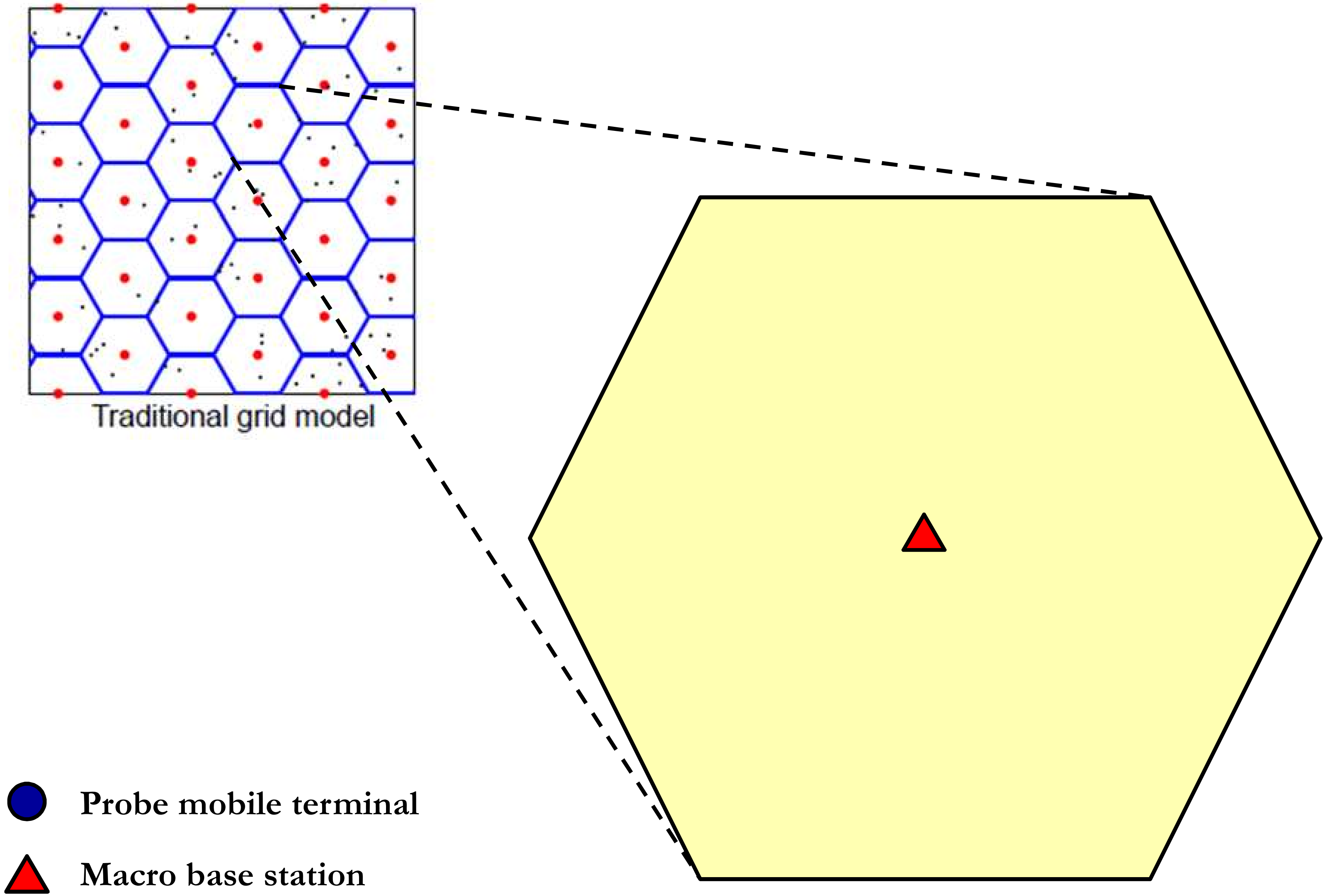


Traditional grid model

- **Conventional approaches to the analysis and design of cellular networks (abstraction models) are:**
  - **The Wyner model**
  - **The single-cell interfering model or dominant interferers model**
  - ***The regular hexagonal or square grid model***

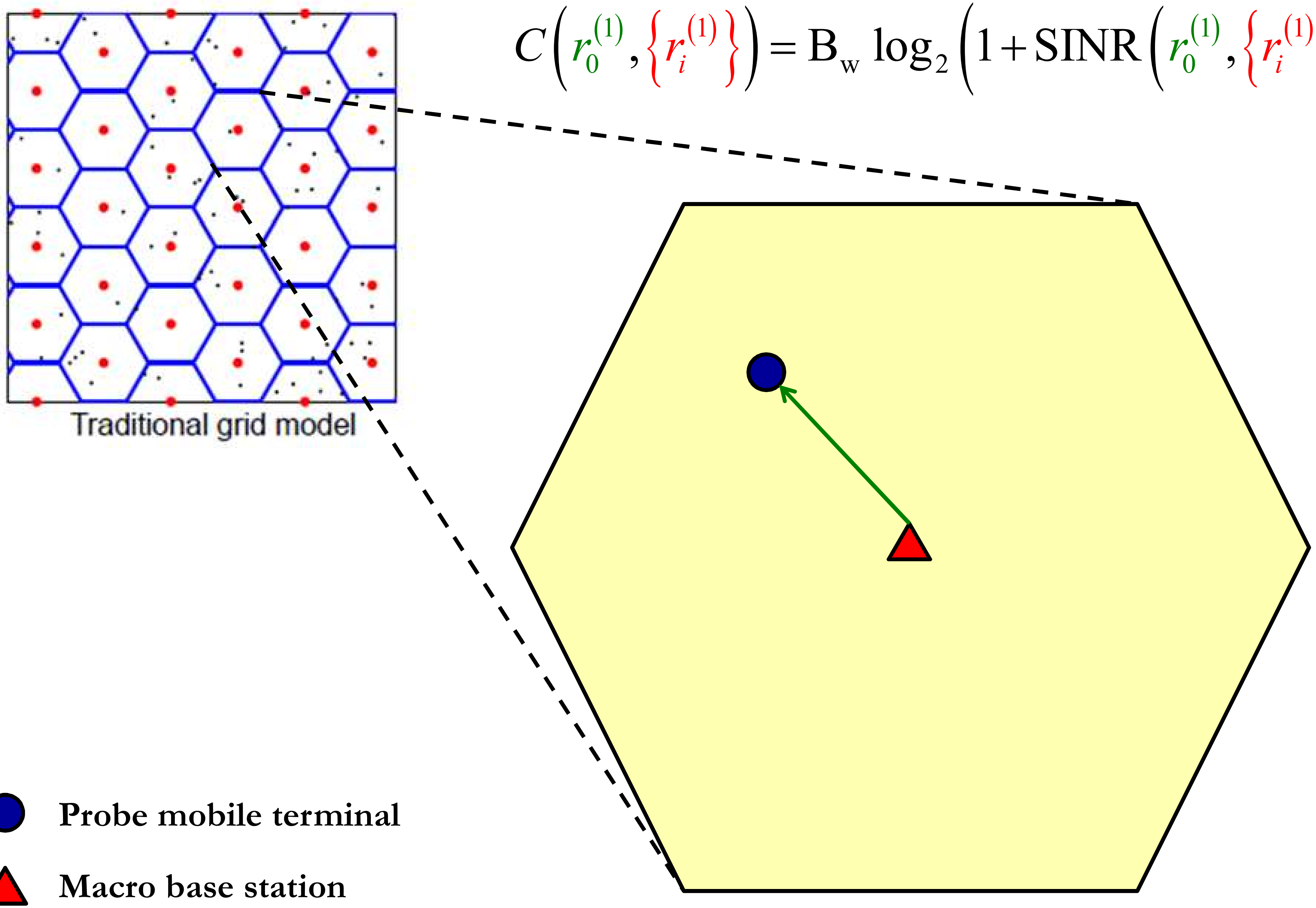
    D. H. Ring and W. R. Young, "The hexagonal cells concept", Bell Labs Technical Journal, Dec. 1947. http://www.privateline.com/archive/Ringcellreport1947.pdf.

The Conventional Grid-Based Approach



The Conventional Grid-Based Approach

$$C\left(r_0^{(1)},\left\{r_i^{(1)}\right\}\right)=\mathrm{B_w}\log_2\left(1+\mathrm{SINR}\left(r_0^{(1)},\left\{r_i^{(1)}\right\}\right)\right)$$



Probe mobile terminal

Macro base station

The Conventional Grid-Based Approach

… Signal-to-Interference-Plus-Noise Ratio (SINR) …

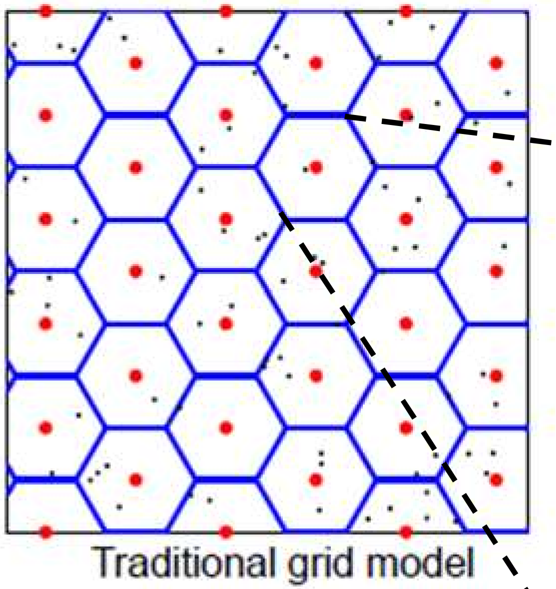
$$\mathrm{SINR} = \frac{P\left|h_o\right|^2 r_o^{-\alpha}}{\sigma^2 + I_{agg}\left(r_0\right)} \qquad I_{agg}\left(r_0\right) = \sum_{i \in \Phi \backslash BS_0} P\left|h_i\right|^2 r_i^{-\alpha}$$

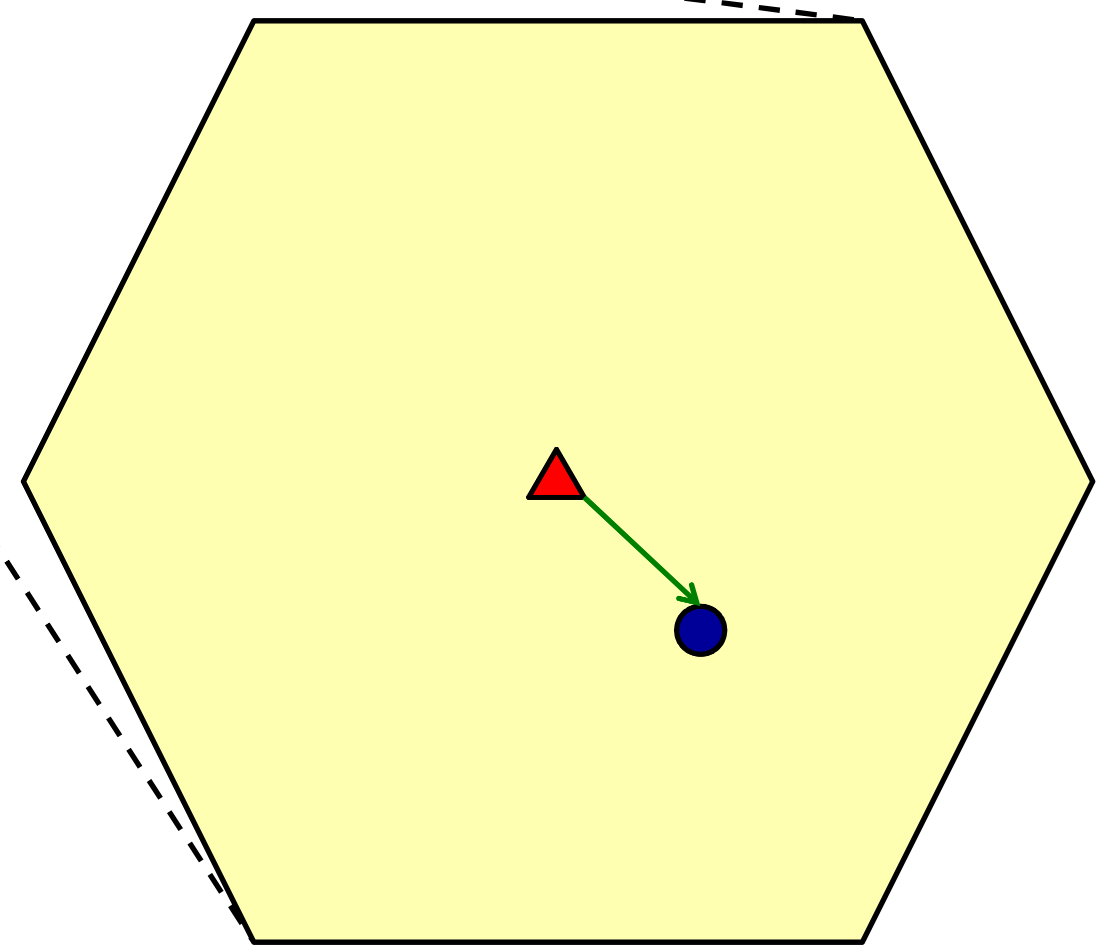
$$\mathrm{CCDF}\left(T\right) = \mathrm{P}_{\mathrm{cov}}\left(T\right) = \Pr\left\{\mathrm{SINR} > T\right\}$$

$$= \Pr\left\{\frac{P\left|h_o\right|^2 r_o^{-\alpha}}{\sigma^2 + I_{agg}\left(r_0\right)} > T\right\} = \ldots$$

The Conventional Grid-Based Approach

$$C\left(r_0^{(2)},\left\{r_i^{(2)}\right\}\right)=\mathrm{B_w}\log_2\left(1+\mathrm{SINR}\left(r_0^{(2)},\left\{r_i^{(2)}\right\}\right)\right)$$



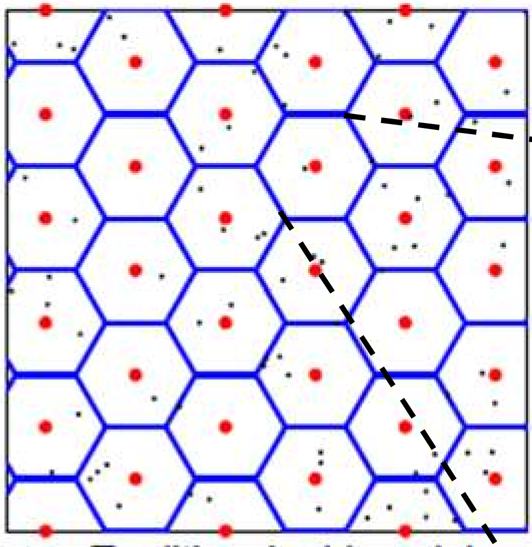


Probe mobile terminal

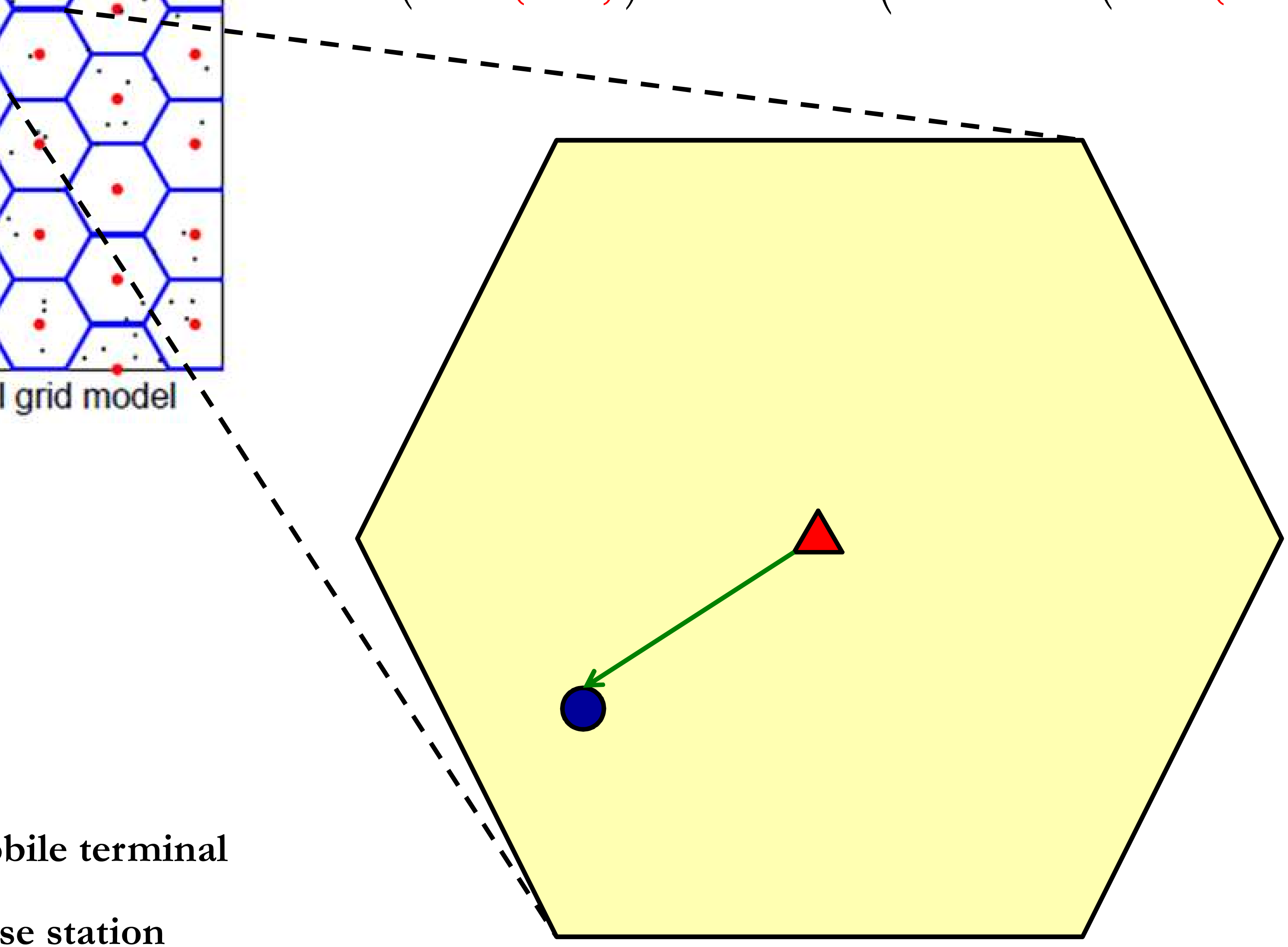
Macro base station

The Conventional Grid-Based Approach

$$C\left(r_0^{(3)},\left\{r_i^{(3)}\right\}\right)=\mathrm{B_w}\log_2\left(1+\mathrm{SINR}\left(r_0^{(3)},\left\{r_i^{(3)}\right\}\right)\right)$$



Traditional grid model



Probe mobile terminal

Macro base station

Simple enough… So, where is the issue?

The answer:

…this spatial expectation

***cannot* be computed mathematically…**

$$\bar{C} = \mathrm{E}_{r_0, \{r_i\}} \left\{ C\left(r_0, \left\{ r_i \right\} \right) \right\}$$

The Conventional Grid-Based Approach

… spatially-average metrics are difficult to be formulated in mathematical terms …

↓

Monte Carlo Approximations ($N \to \infty$)

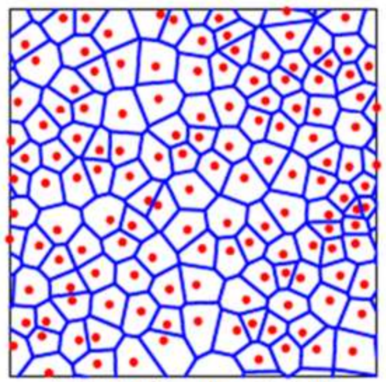
$$\bar{C} = \cancel{\mathrm{E}_{r_0,\{r_i\}}\left\{C\left(r_0,\{r_i\}\right)\right\}} \approx \frac{1}{N}\sum_{n=1}^{N} C\left(r_0^{(n)},\left\{r_i^{(n)}\right\}\right)$$

$$= \frac{1}{N}\sum_{n=1}^{N} \mathrm{B_w} \log_2\left(1+\mathrm{SINR}\left(r_0^{(n)},\left\{r_i^{(n)}\right\}\right)\right)$$

The Conventional Grid-Based Approach: (Some) Issues

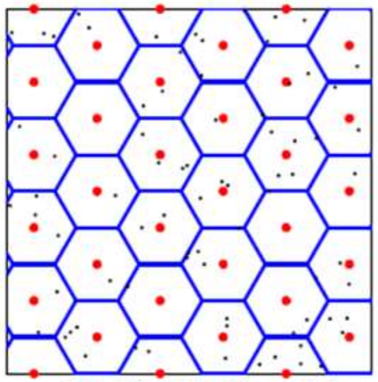
- **Advantages:**
  - Dozens of system parameters can be modeled and tuned in such simulations, and the results have been sufficiently accurate as to enable the evaluation of new proposed techniques and guide field deployments

- **Limitations:**
  - Actual coverage regions deviate from a regular grid
  - Mathematical modeling and optimization are not possible. Any elegant and insightful Shannon formulas for cellular networks?
  - The abstraction model is not scalable for application to ultra-dense HetNets (different densities, transmit powers, access technologies, etc…)

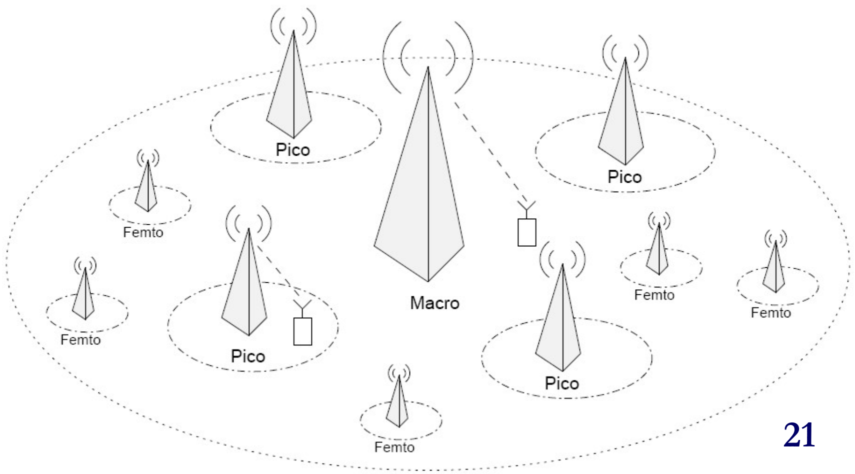


Actual 4G network today

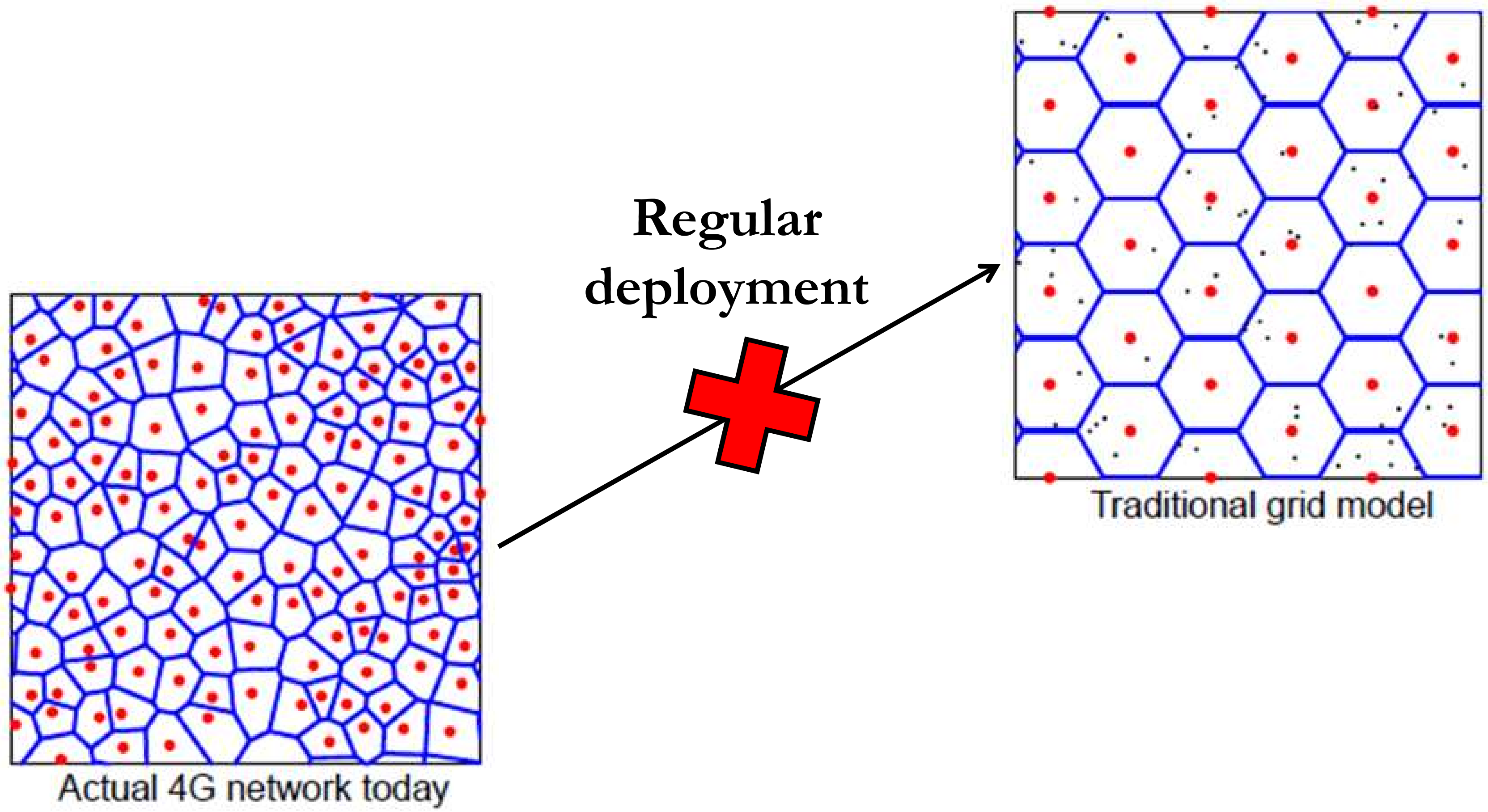
VS.



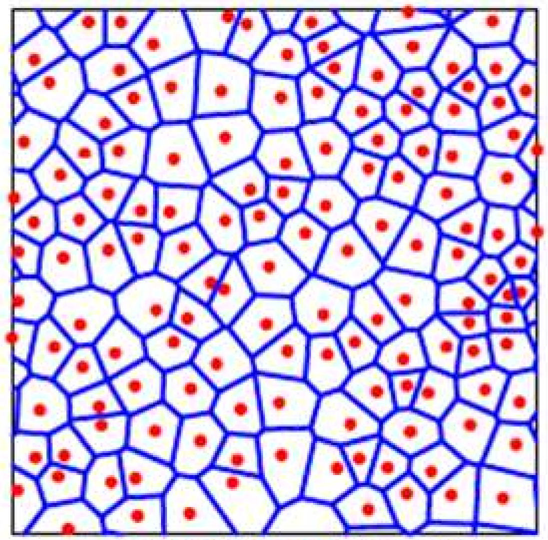
Traditional grid model



Let's Change The Abstraction Model, Then…

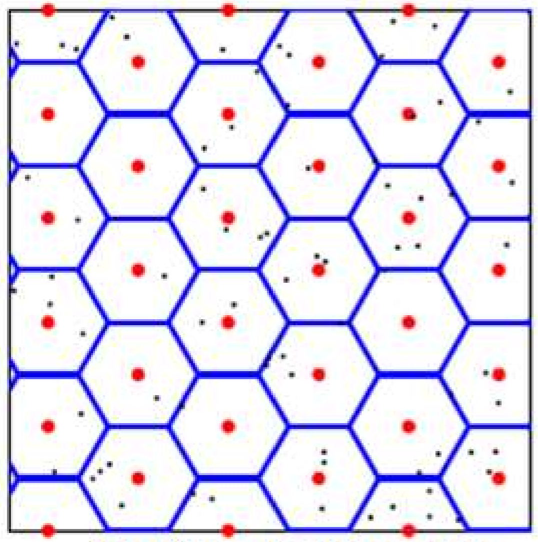


Let's Change The Abstraction Model, Then…



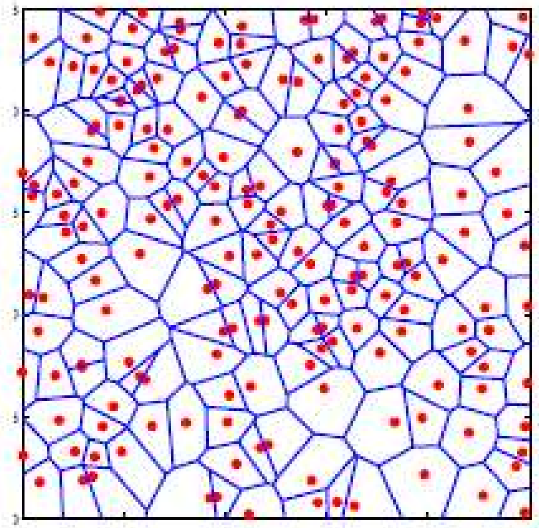
Actual 4G network today

Regular deployment



Traditional grid model

Random deployment (PPP)



Completely random BSs

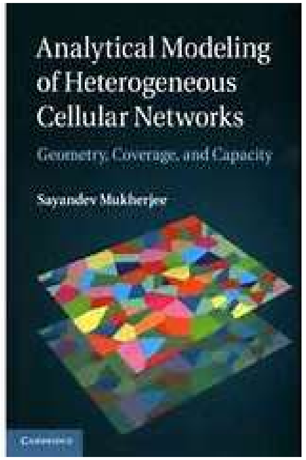
Stochastic Geometry Based Abstraction Model

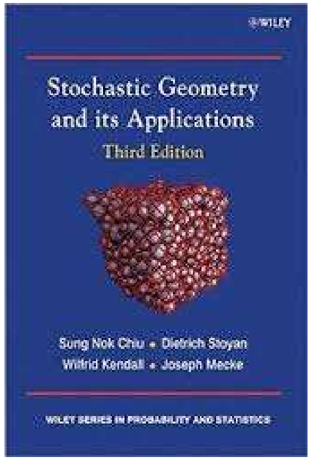
A Tractable Approach

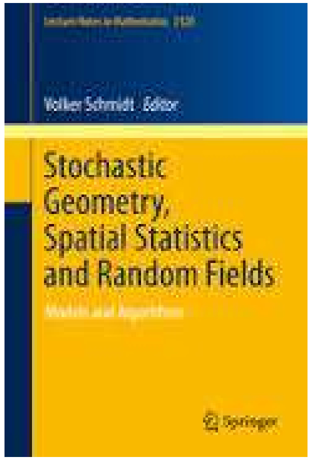
- **A RANDOM SPATIAL MODEL for Heterogeneous Cellular Networks (HetNets) modeling, analysis, and optimization**

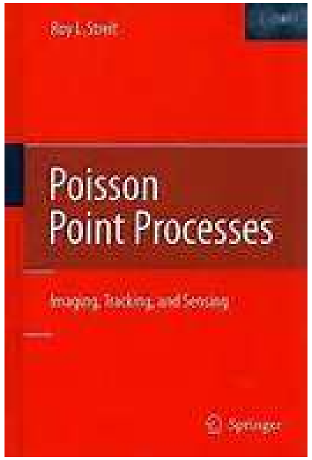
**Stochastic Geometry
emerges as a powerful tool for the
analysis, design and optimization
of ultra-dense HetNets**

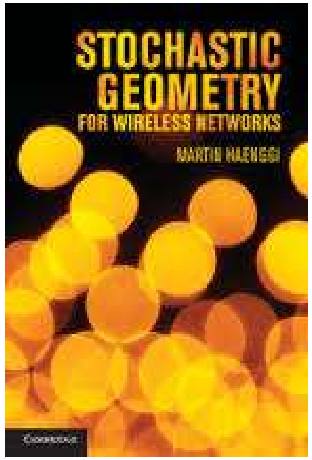
Stochastic Geometry: Well-Known Mathematical Tool

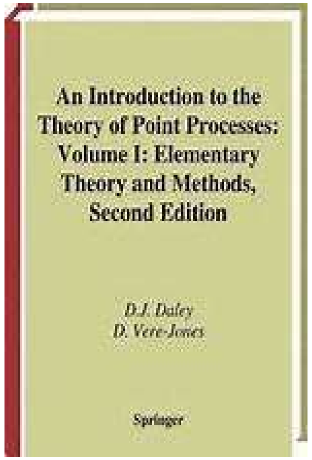


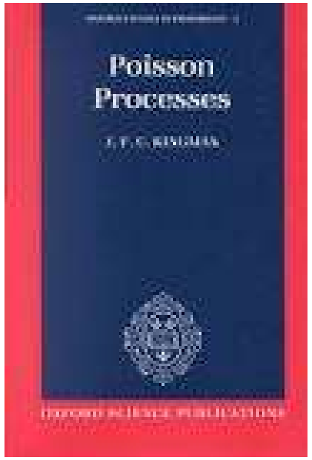


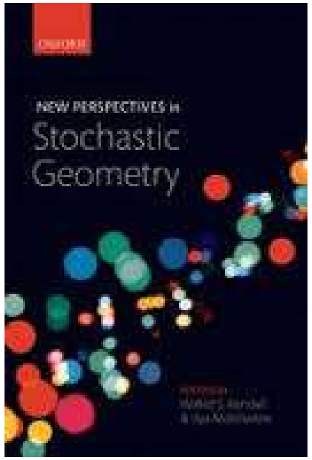


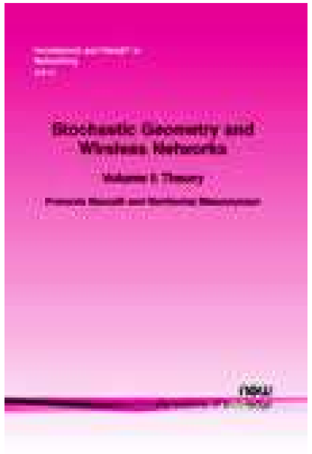


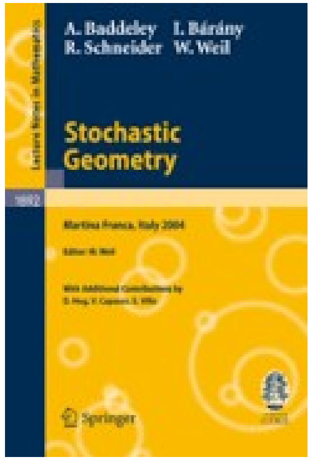








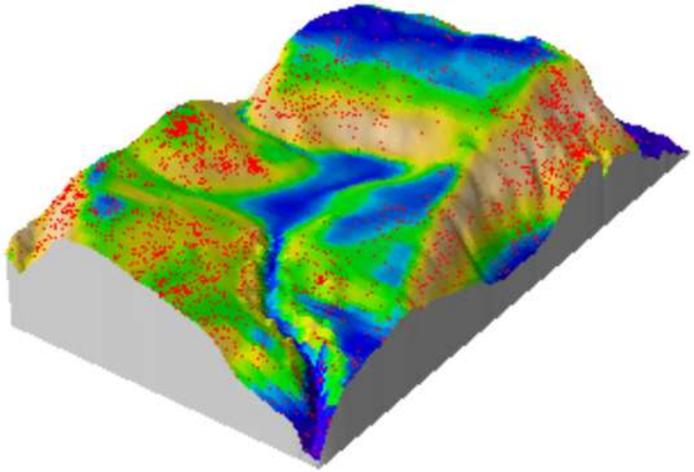


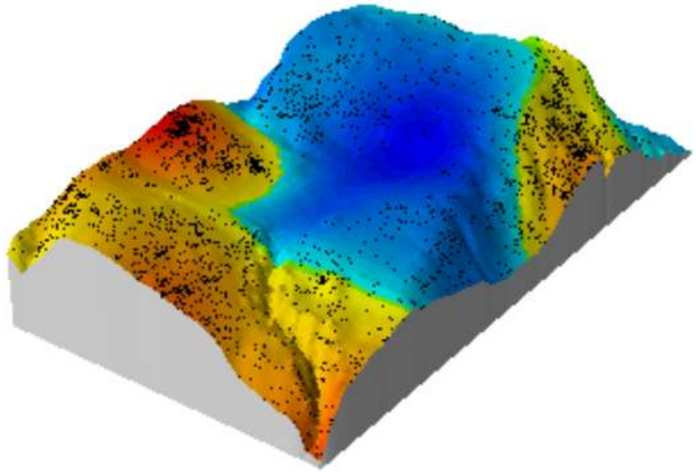


Stochastic Geometry: Sophisticated Statistical Toolboxes

spatstat analysing spatial point patterns

Home News Download Resources Book Help FAQ About





Welcome to the `spatstat` website

Stochastic Geometry: Sophisticated Statistical Toolboxes

Package 'spatstat'

July 8, 2016

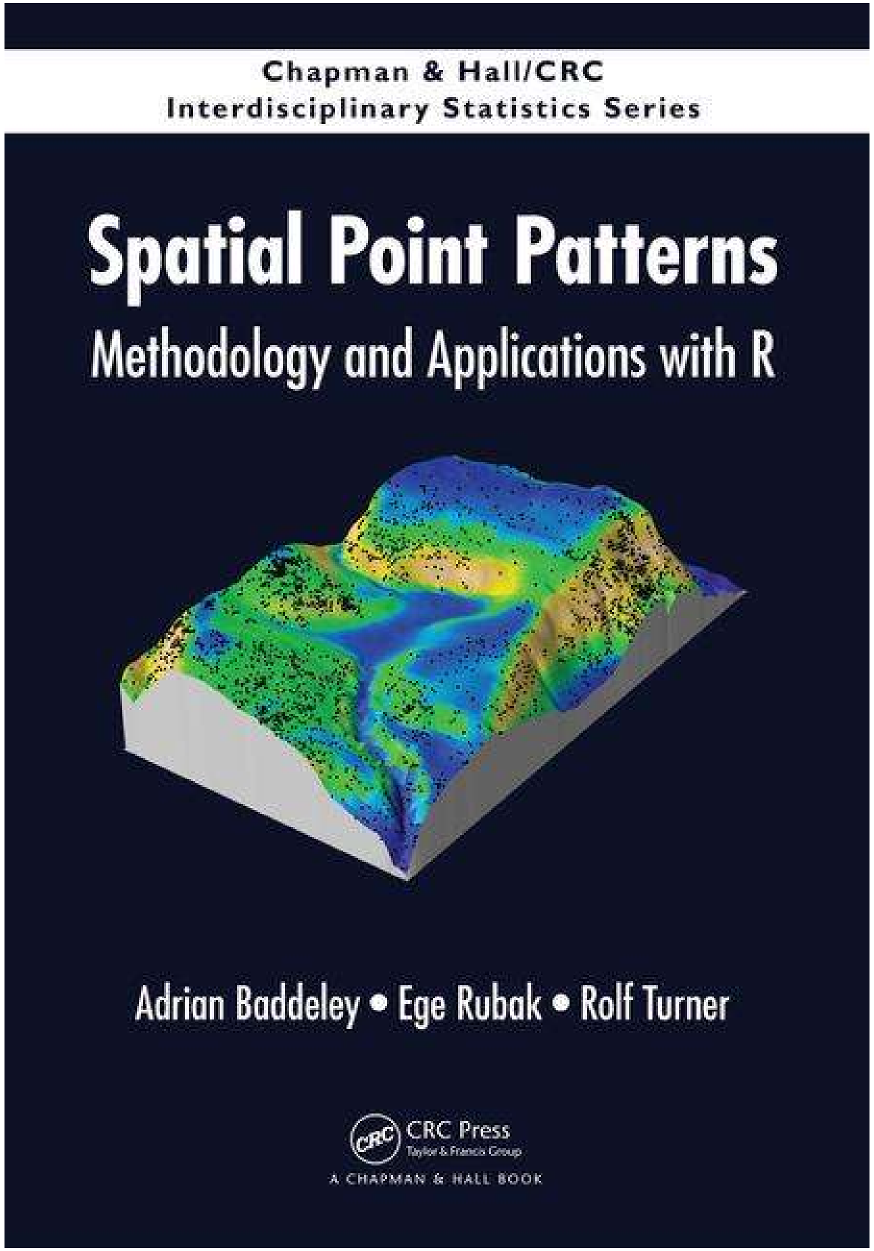
Version 1.46-1

Nickname Spoiler Alert

Date 2016-07-08

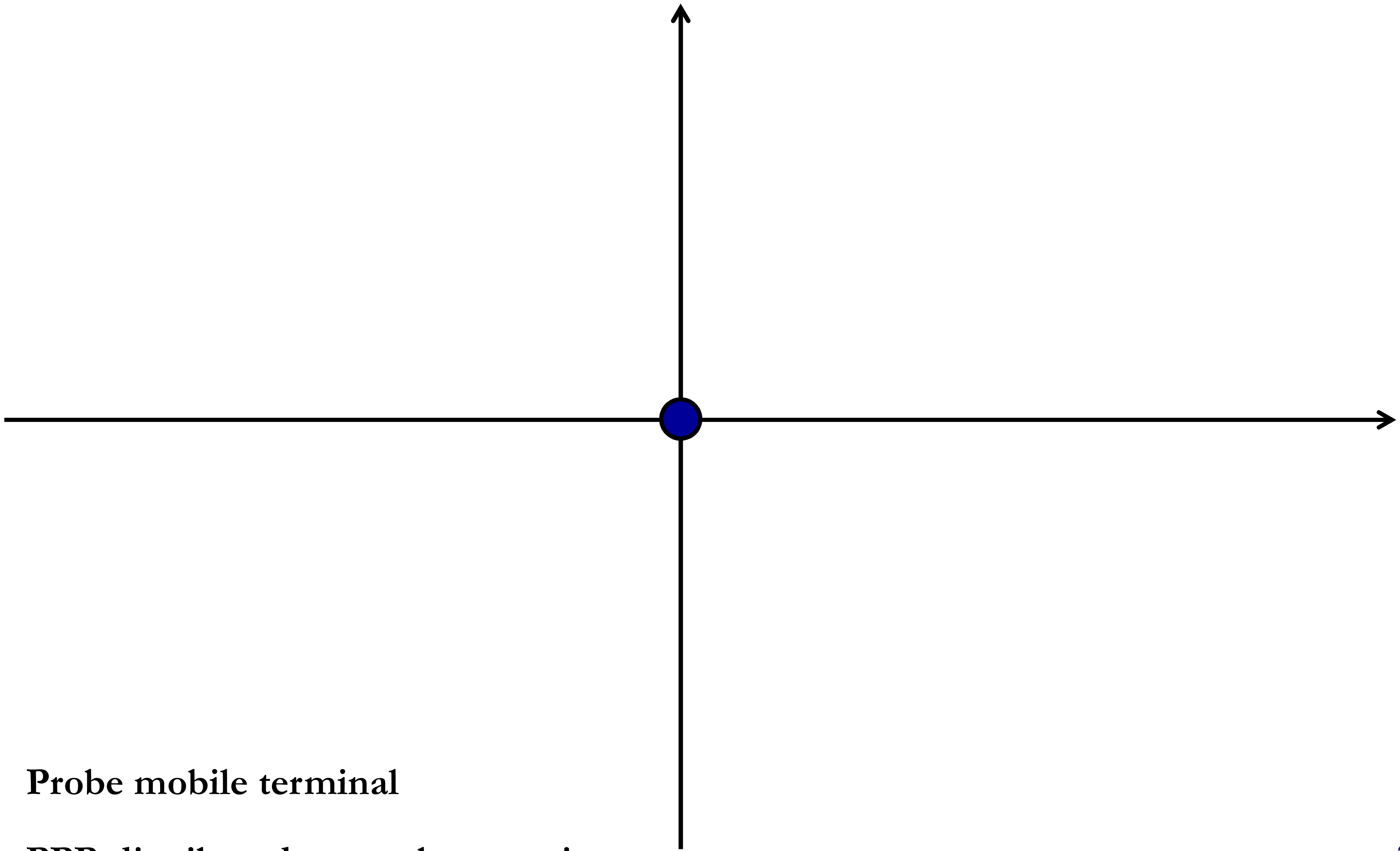
Title Spatial Point Pattern Analysis, Model-Fitting, Simulation, Tests

Author Adrian Baddeley <Adrian.Baddeley@curtin.edu.au>,
Rolf Turner <r.turner@auckland.ac.nz>
and Ege Rubak <rubak@math.aau.dk>,
with substantial contributions of code by
Kasper Klitgaard Berthelsen;
Ottmar Cronie;
Ute Hahn;
Abdollah Jalilian;
Marie-Colette van Lieshout;
Tuomas Rajala;
Dominic Schuhmacher;
and



PPP-Based Abstraction

How It Works (Downlink – 1-tier)

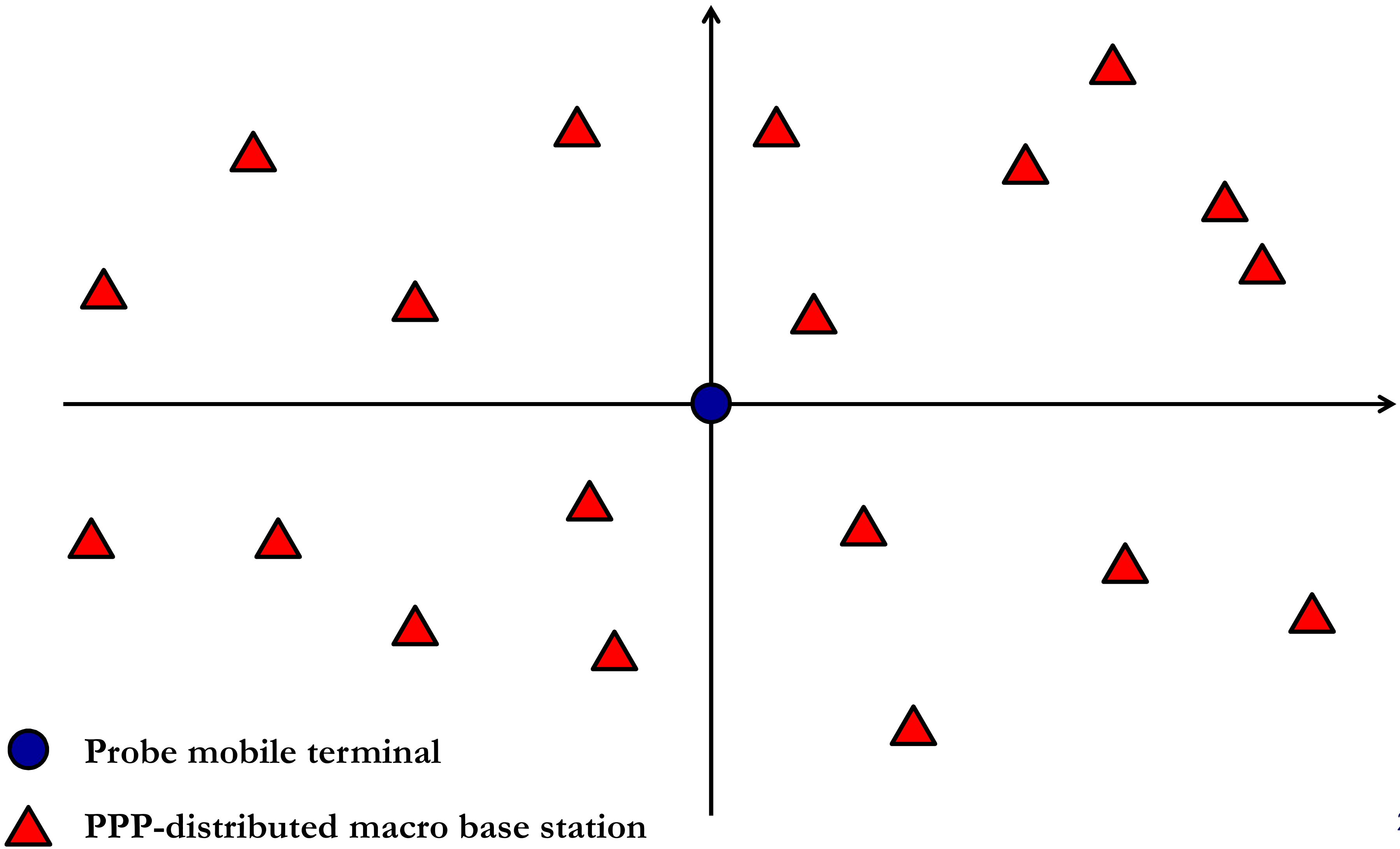


Probe mobile terminal

PPP-distributed macro base station

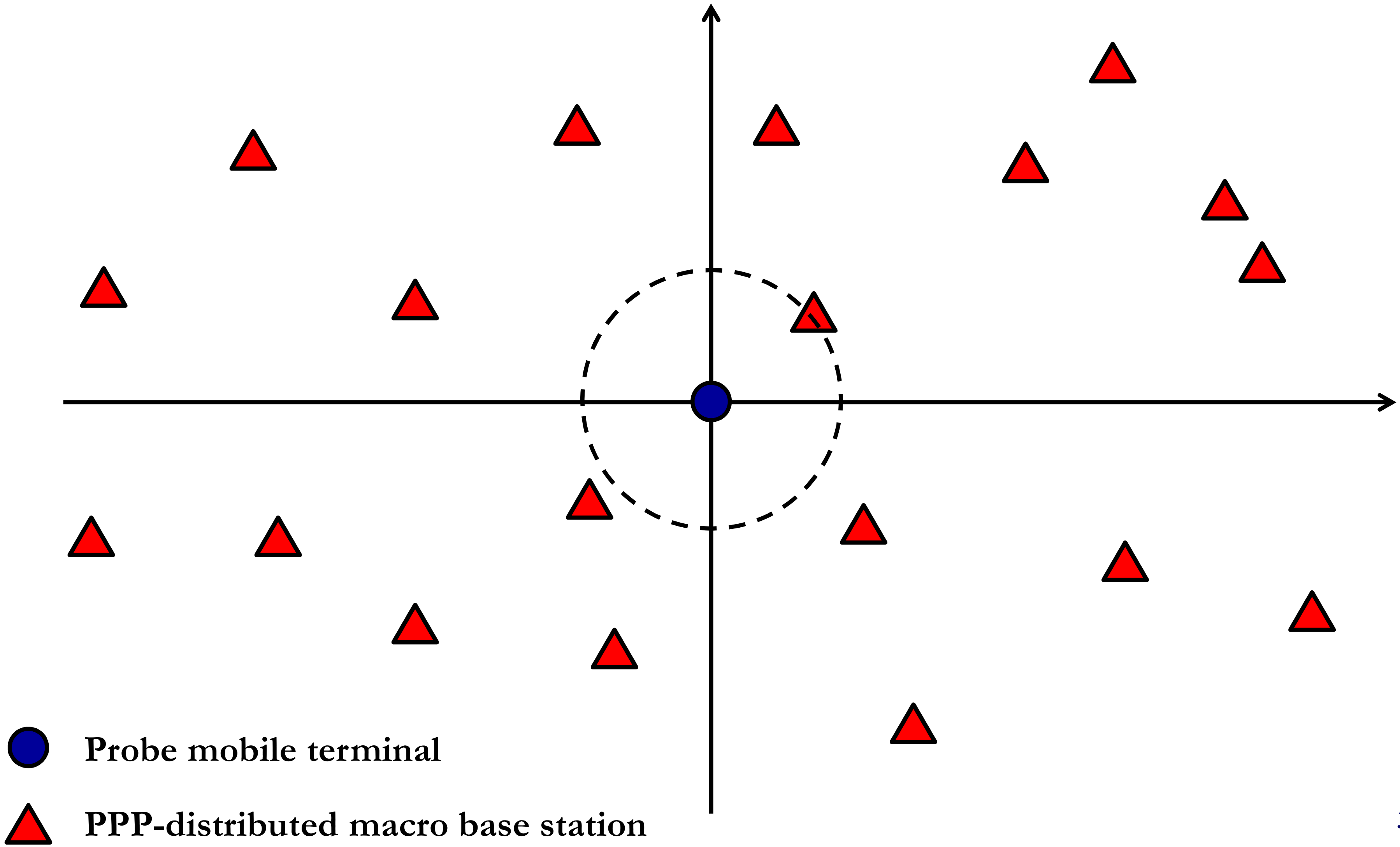
PPP-Based Abstraction

How It Works (Downlink – 1-tier)



PPP-Based Abstraction

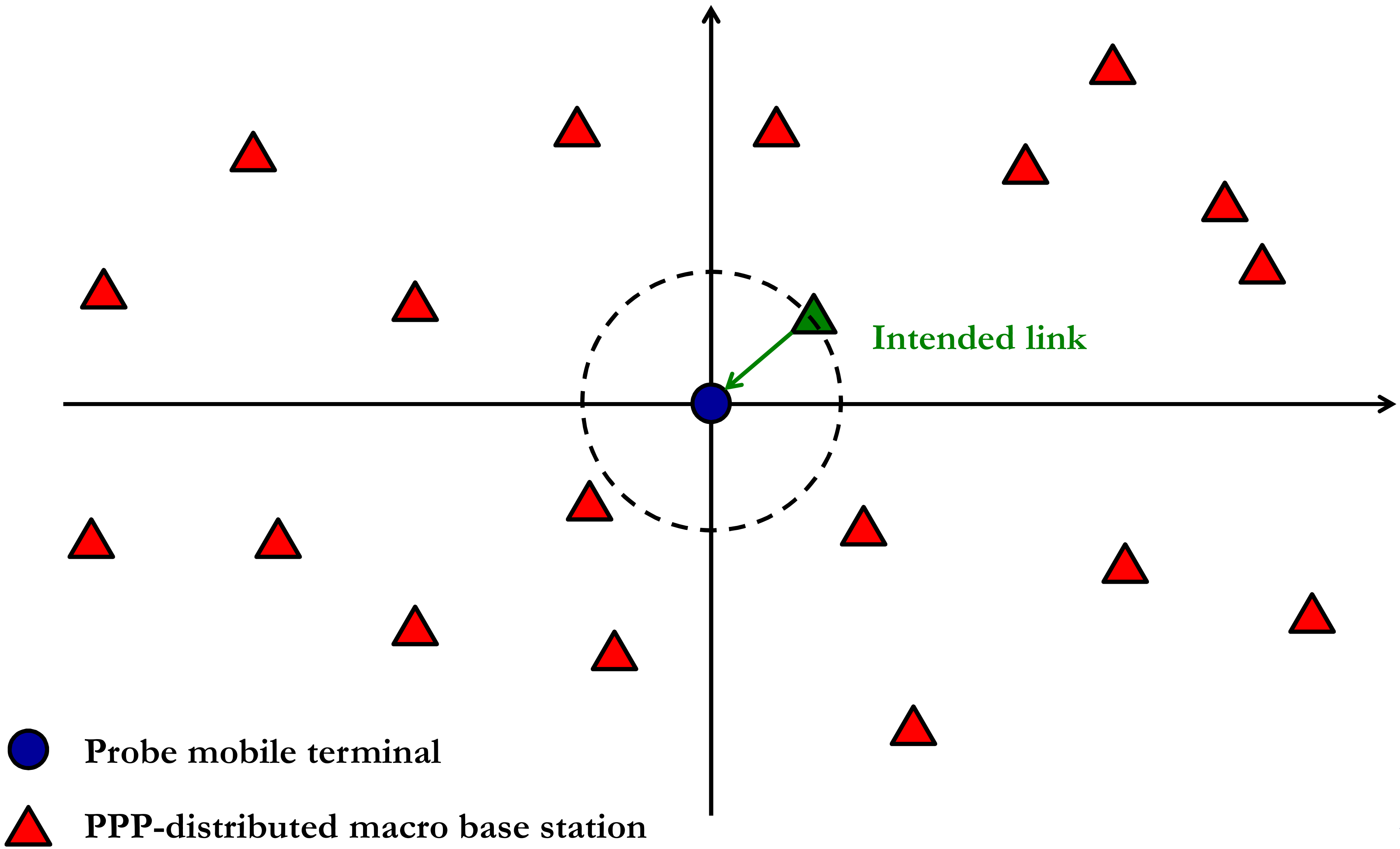
How It Works (Downlink – 1-tier)



PPP-Based Abstraction

$$C\left(r_0^{(1)},\left\{r_i^{(1)}\right\}\right)=\mathrm{B_w}\log_2\left(1+\mathrm{SINR}\left(r_0^{(1)},\left\{r_i^{(1)}\right\}\right)\right)$$

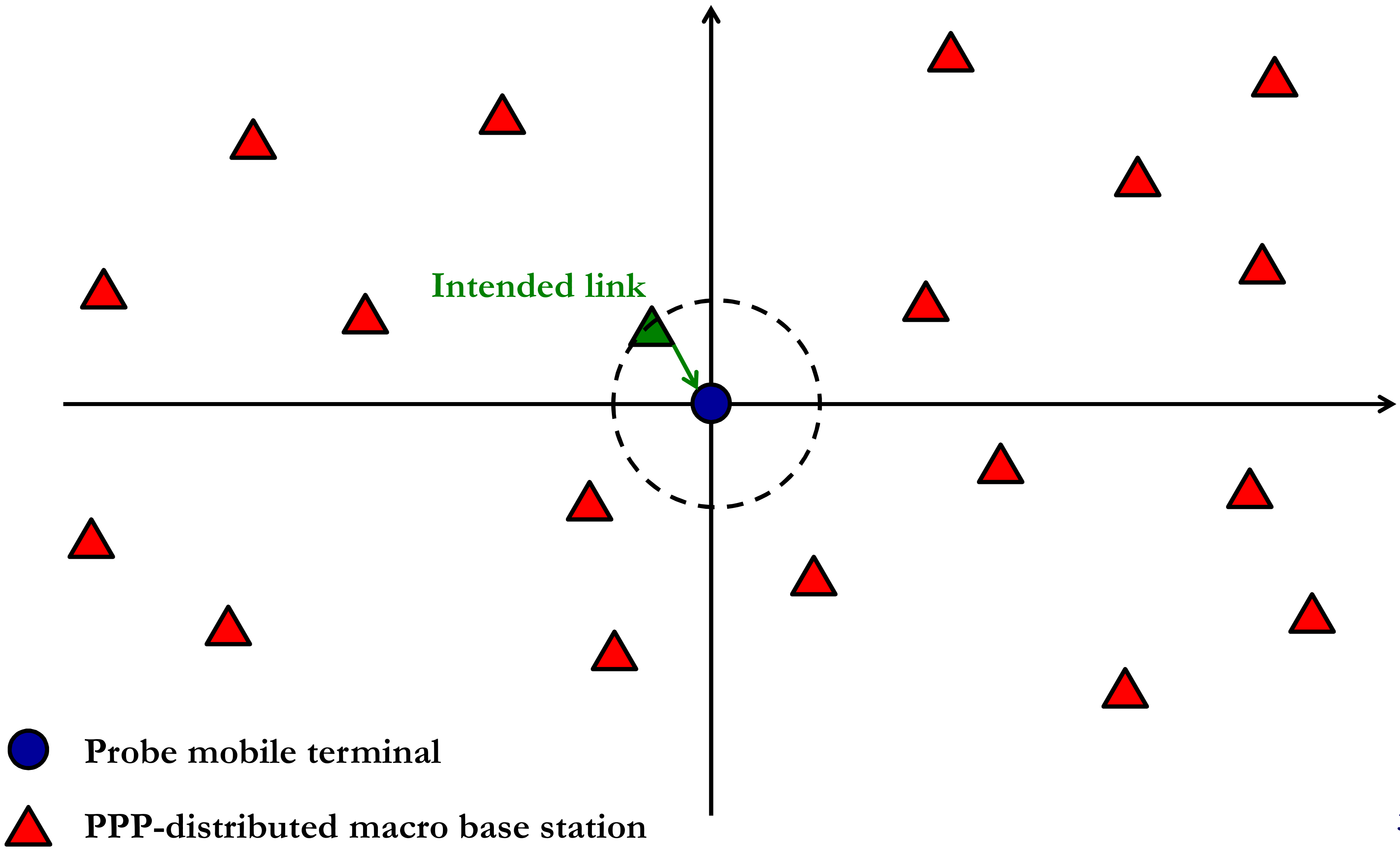
How It Works (Downlink – 1-tier)



PPP-Based Abstraction

$$C\left(r_0^{(2)},\left\{r_i^{(2)}\right\}\right)=\mathrm{B_w}\log_2\left(1+\mathrm{SINR}\left(r_0^{(2)},\left\{r_i^{(2)}\right\}\right)\right)$$

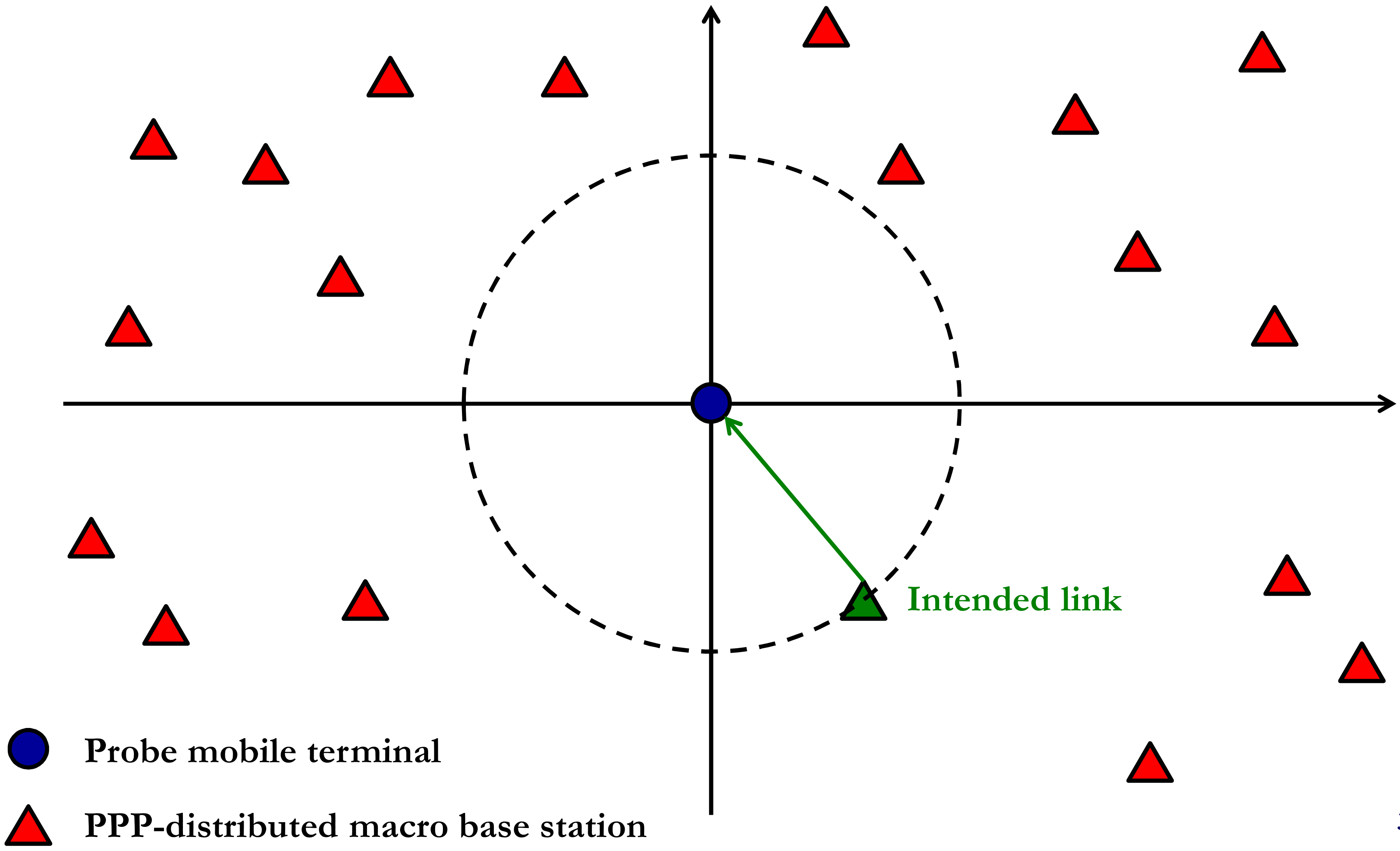
How It Works (Downlink – 1-tier)



PPP-Based Abstraction

$$C\left(r_0^{(3)},\left\{r_i^{(3)}\right\}\right)=\mathrm{B_w}\log_2\left(1+\mathrm{SINR}\left(r_0^{(3)},\left\{r_i^{(3)}\right\}\right)\right)$$

How It Works (Downlink – 1-tier)



Are you kidding me? … What makes it different?

$$\bar{C} = \mathrm{E}_{r_0,\{r_i\}}\left\{C\left(r_0,\left\{r_i\right\}\right)\right\}$$

$$\downarrow$$

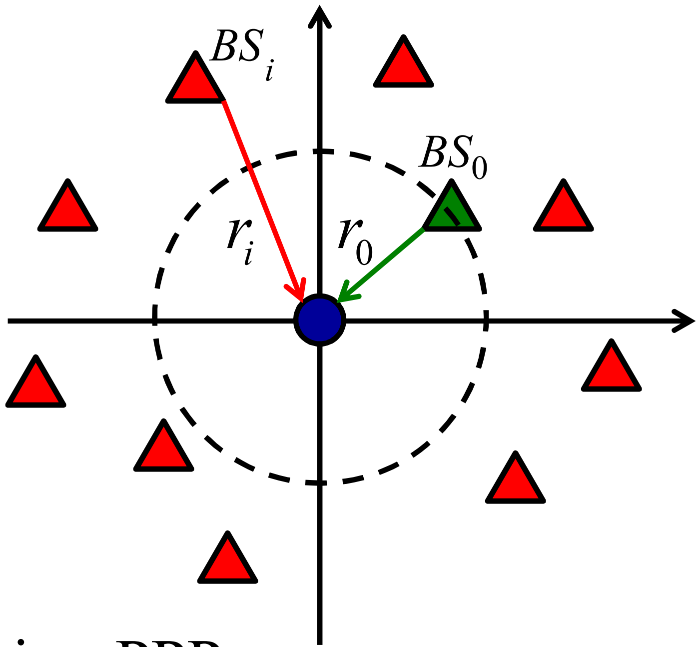
the following is not needed anymore

$$\approx \frac{1}{N}\sum_{n=1}^{N} C\left(r_0^{(n)},\left\{r_i^{(n)}\right\}\right)$$

$$= \frac{1}{N}\sum_{n=1}^{N} \mathrm{B_w}\log_2\left(1+\mathrm{SINR}\left(r_0^{(n)},\left\{r_i^{(n)}\right\}\right)\right)$$

How It Works: The Magic of Stochastic Geometry (1/6)

… understanding the basic math …



Φ is a PPP

$$P_{\text{cov}} = \Pr\{\text{SINR} > T\}$$

$$\text{SINR} = \frac{P|h_o|^2 r_o^{-\alpha}}{\sigma^2 + I_{agg}(r_0)}$$

$$I_{agg}(r_0) = \sum_{i \in \Phi \setminus BS_0} P|h_i|^2 r_i^{-\alpha}$$

$$P_{\text{cov}} = \Pr\left\{\frac{P|h_o|^2 r_o^{-\alpha}}{\sigma^2 + I_{agg}(r_0)} > T\right\} = \ldots$$

How It Works: The Magic of Stochastic Geometry (2/6)

… understanding the basic math …

$$\mathrm{P_{cov}} = \Pr\left\{ \frac{P\left|h_o\right|^2 r_o^{-\alpha}}{\sigma^2 + I_{agg}\left(r_0\right)} > T \right\}$$

$$= \Pr\left\{ \left|h_o\right|^2 > \left(\sigma^2 + I_{agg}\left(r_0\right)\right) P^{-1} T r_o^{\alpha} \right\}$$

$$\left(\left|h_o\right|^2 \sim \exp \Rightarrow \right) = \mathrm{E}_{I_{agg}\left(r_0\right), r_0} \left\{ \exp\left(-\left(\sigma^2 + I_{agg}\left(r_0\right)\right) P^{-1} T r_o^{\alpha} \right) \right\}$$

$$\begin{pmatrix} \mathrm{MGF}_X\left(s\right) = \\ \mathrm{E}_X\left\{e^{-sX}\right\} \Rightarrow \end{pmatrix} = \mathrm{E}_{r_0} \left\{ \exp\left(-\sigma^2 P^{-1} T r_o^{\alpha} \right) \mathrm{MGF}_{I_{agg}\left(r_0\right)}\left(P^{-1} T r_o^{\alpha} \right) \right\}$$

How It Works: The Magic of Stochastic Geometry (3/6)

… understanding the basic math …

$$\mathrm{P_{cov}} = \mathrm{E}_{r_0}\left\{\exp\left(-T\sigma^2 P^{-1} r_o^{\alpha}\right)\mathrm{MGF}_{I_{agg}(r_0)}\left(P^{-1} T r_o^{\alpha}\right)\right\}$$

$$= \int_0^{+\infty} \exp\left(-T\sigma^2 P^{-1}\xi^{\alpha}\right)\mathrm{MGF}_{I_{agg}(r_0)}\left(P^{-1}T\xi^{\alpha}\right)\mathrm{PDF}_{r_0}\left(\xi\right)d\xi$$

How It Works: The Magic of Stochastic Geometry (3/6)

… understanding the basic math …

$$
\begin{aligned}
\mathrm{P_{cov}} &= \mathrm{E}_{r_0}\left\{\exp\left(-T\sigma^2 P^{-1} r_o^{\alpha}\right)\mathrm{MGF}_{I_{agg}(r_0)}\left(P^{-1} T r_o^{\alpha}\right)\right\} \\
&= \int_0^{+\infty} \exp\left(-T\sigma^2 P^{-1}\xi^{\alpha}\right)\mathrm{MGF}_{I_{agg}(r_0)}\left(P^{-1} T\xi^{\alpha}\right)\mathrm{PDF}_{r_0}(\xi)\, d\xi
\end{aligned}
$$

Trivial so far… where is the magic?

How It Works: The Magic of Stochastic Geometry (3/6)

… understanding the basic math …

$$\mathrm{P}_{\mathrm{cov}} = \mathrm{E}_{r_0}\left\{\exp\left(-T\sigma^2 P^{-1} r_o^{\alpha}\right)\mathrm{MGF}_{I_{agg}(r_0)}\left(P^{-1} T r_o^{\alpha}\right)\right\}$$

$$= \int_0^{+\infty} \exp\left(-T\sigma^2 P^{-1} \xi^{\alpha}\right)\mathrm{MGF}_{I_{agg}(r_0)}\left(P^{-1} T \xi^{\alpha}\right)\mathrm{PDF}_{r_0}\left(\xi\right) d\xi$$

Trivial so far… where is the magic?

Stochastic Geometry provides us with the mathematical tools for computing, in closed-form, the MGF and the PDF of the equation above

How It Works: The Magic of Stochastic Geometry (4/6)

… understanding the basic math …

$$I_{agg}(r_0) = \sum_{i \in \Phi \backslash BS_0} P |h_i|^2 r_i^{-\alpha}$$

The aggregate other-cell interference constitues a Marked PPP, where the marks are the channel power gains

$$\text{PDF}_{r_0}(\xi) = 2\pi\lambda\xi \exp(-\pi\lambda\xi^2)$$

The PDF of the closest-distance follows from the null probability of spatial PPPs

$$\text{MGF}_{I_{agg}(r_0)}(s) = \ldots$$

The MGF of the aggregate other-cell interference follows from the Probability Generating Functional (PGFL) of Marked PPPs

How It Works: The Magic of Stochastic Geometry (5/6)

… understanding the basic math …

$$\mathrm{MGF}_{I_{agg}(r_0)}(s) = \mathrm{E}_{\Phi,\{|h_i|^2\}}\left\{\exp\left(-s\sum_{i\in\Phi\backslash BS_0} P|h_i|^2 r_i^{-\alpha}\right)\right\}$$

$$= \mathrm{E}_{\Phi}\left\{\prod_{i\in\Phi\backslash BS_0} \mathrm{E}_{\{|h_i|^2\}}\left\{\exp\left(-sP|h_i|^2 r_i^{-\alpha}\right)\right\}\right\}$$

$$(\mathrm{PGFL} \Rightarrow) = \exp\left(-2\pi\lambda\int_{r_0}^{+\infty}\left(1-\mathrm{E}_{|h_i|^2}\left\{\exp\left(-sP|h_i|^2 \xi_i^{-\alpha}\right)\right\}\right)\xi_i d\xi_i\right)$$

$$= \exp\left(\pi\lambda r_0^2\left(1-{}_2F_1\left(1,-\frac{2}{\alpha},1-\frac{1}{\alpha},-\frac{sP}{r_0^{\alpha}}\right)\right)\right)$$

How It Works: The Magic of Stochastic Geometry (6/6)

… a relevant example: Interference-Limited regime …

Coverage: $$\mathrm{P_{cov}} = \mathrm{P_{cov}}(T) = \frac{1}{{}_2F_1(1,-2/\alpha,1-2/\alpha,-T)}$$

Rate: $$\bar{C} = \frac{\mathrm{B_W}}{\ln(2)} \int_0^{+\infty} \frac{\mathrm{P_{cov}}(x)}{1+x} dx \quad [\text{bit/sec}]$$

Area Spectral Efficiency: $$\mathrm{ASE} = \lambda \bar{C} \quad [\text{bit/sec/m}^2]$$

Potential Spectral Efficiency: $$\mathrm{PSE}(T) = \lambda \mathrm{B_W} \log_2(1+T)\mathrm{P_{cov}}(T) \quad [\text{bit/sec/m}^2]$$

So Powerful and Just Two Lemmas Need To Be Used…

Sums over PPP

Lemma (Campbells theorem)

Let Φ be a PPP of density λ and $f(x) : \mathbb{R}^2 \to \mathbb{R}^+$.

$$\mathbb{E}[\sum_{x \in \Phi} f(x)] = \lambda \int_{\mathbb{R}^2} f(x) \mathrm{d}x$$

Products over PPP

Lemma (Probability generating functional (PGFL))

Let Φ be a PPP of density λ and $f(x) : \mathbb{R}^2 \to [0, 1]$ be a real valued function. Then

$$\mathbb{E}\left[\prod_{x \in \Phi} f(x)\right] = \exp\left(-\lambda \int_{\mathbb{R}^2} (1 - f(x)) \mathrm{d}x\right).$$

… On Abstraction Modeling …

George Edward Pelham Box
(18 October 1919 – 28 March 2013)
Statistician
Fellow of the Royal Society (UK)
Director of the Statistical Research Group
(Princeton University)
Emeritus Professor
(University of Wisconsin-Madison)

"…all models are wrong, but some are useful…"

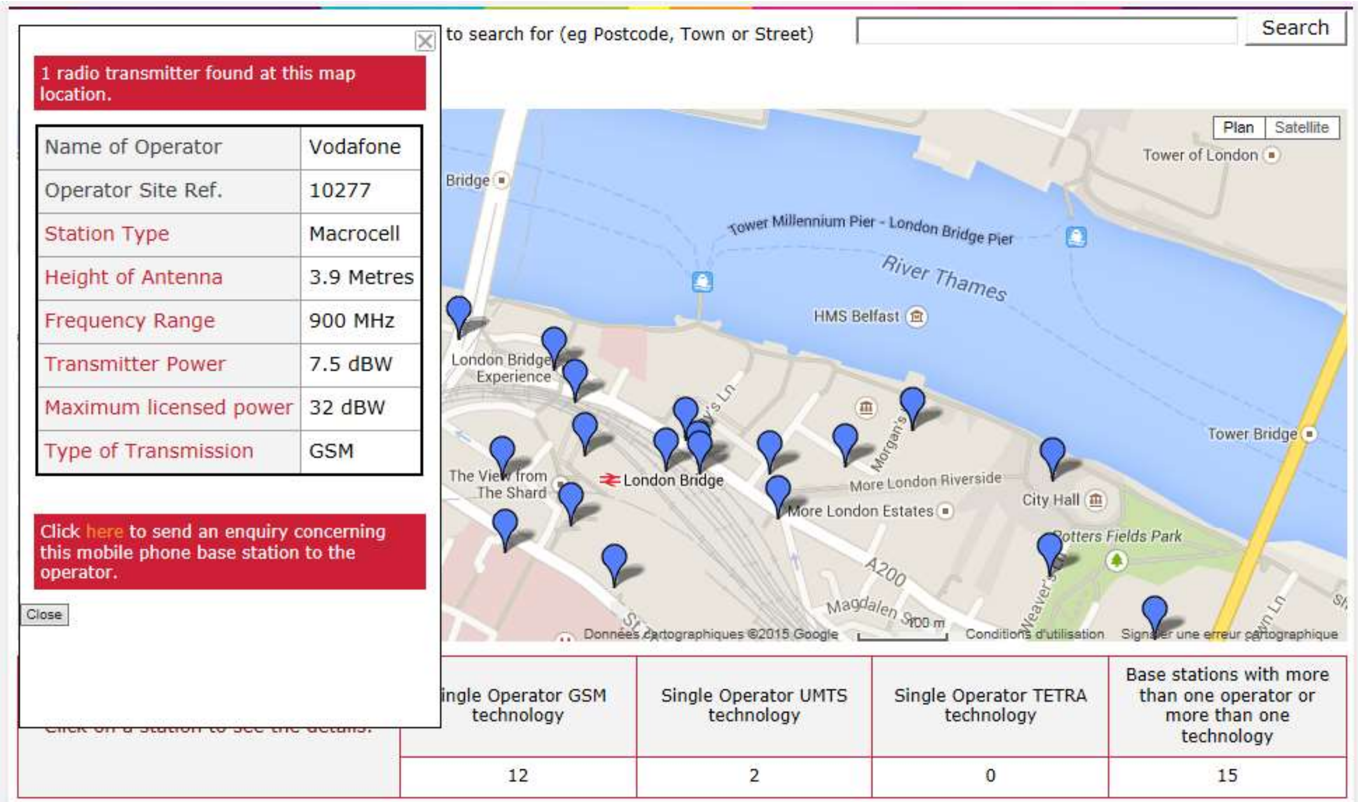
Is This Abstraction Model Accurate?

- **Methodology:**

Is This Abstraction Model Accurate?

- **Methodology:**
  - **Actual base station locations from OFCOM (UK)**

OFCOM:
London
"London Bridge area"

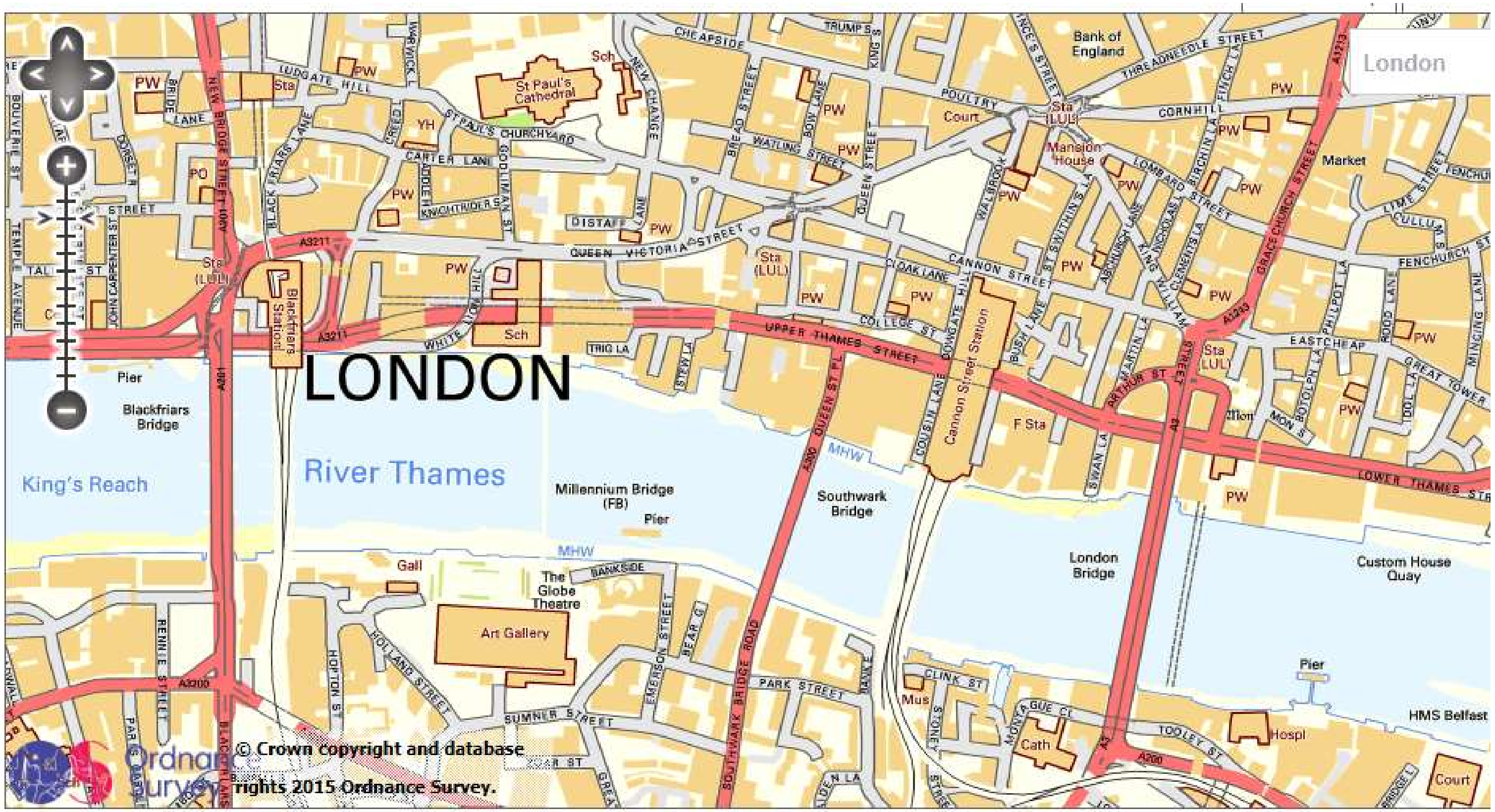


OFCOM: http://stakeholders.ofcom.org.uk/sitefinder/sitefinder-dataset/

Is This Abstraction Model Accurate?

- **Methodology:**
  - **Actual base station locations from OFCOM (UK)**
  - **Actual building footprints from ORDNANCE SURVEY (UK)**

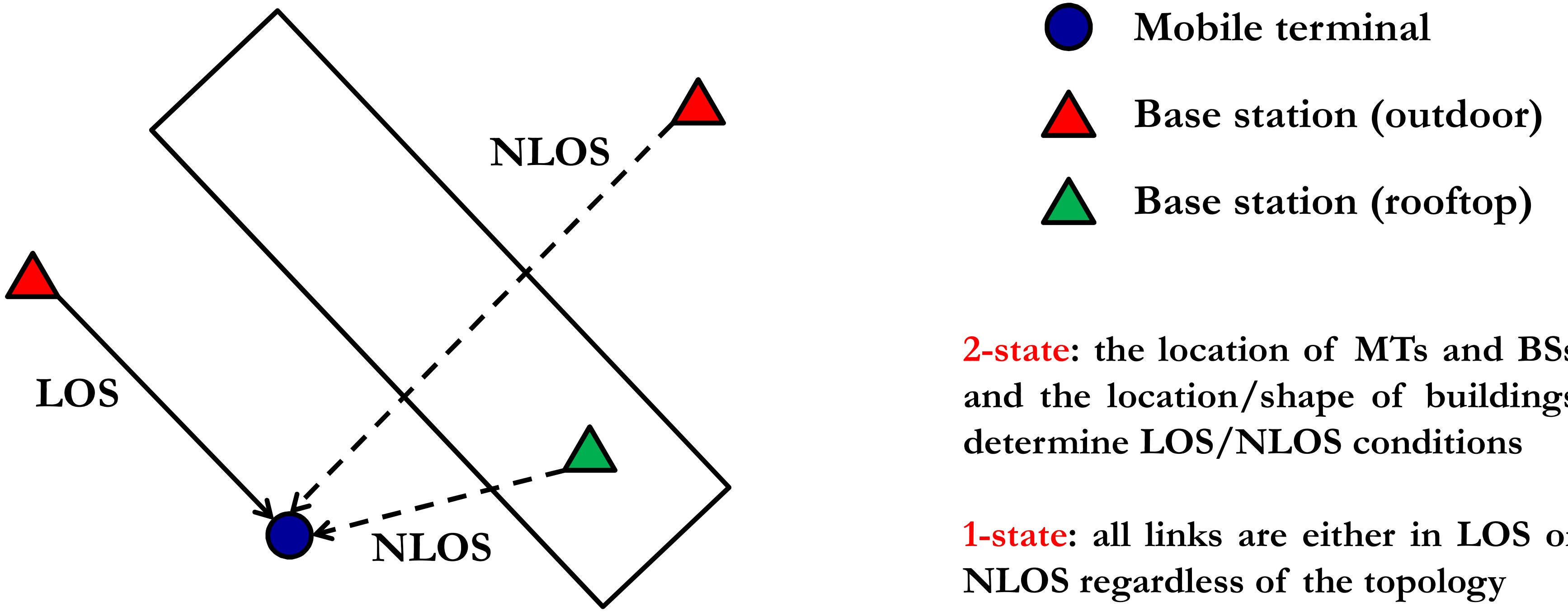
ORDNANCE SURVEY: London "London Bridge area"



OFCOM: http://stakeholders.ofcom.org.uk/sitefinder/sitefinder-dataset/
ORDNANCE SURVEY: https://www.ordnancesurvey.co.uk/opendatadownload/products.html

Is This Abstraction Model Accurate?

- **Methodology:**
  - Actual base station locations from **OFCOM** (UK)
  - Actual building footprints from **ORDNANCE SURVEY** (UK)
  - Channel model added on top (1-state and 2-state with LOS/NLOS)



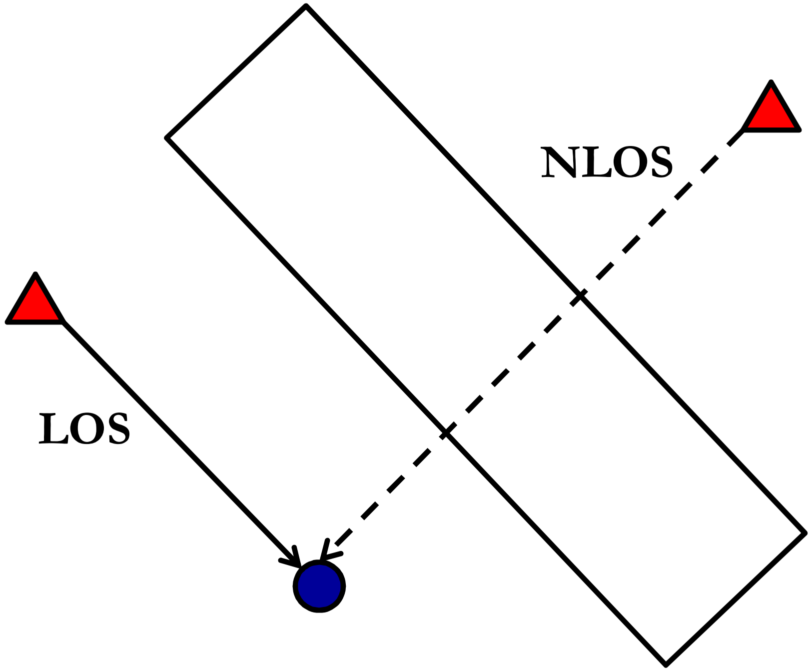
2-state: the location of MTs and BSs and the location/shape of buildings determine LOS/NLOS conditions

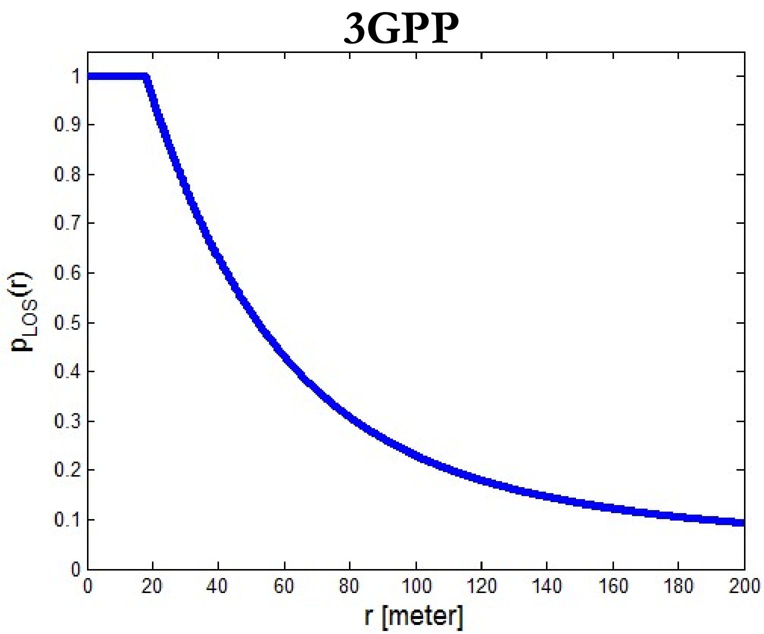
1-state: all links are either in LOS or NLOS regardless of the topology

OFCOM: http://stakeholders.ofcom.org.uk/sitefinder/sitefinder-dataset/
ORDNANCE SURVEY: https://www.ordnancesurvey.co.uk/opendatadownload/products.html

Practical Example of Blockage Model (3GPP)

... statistically modeling of blockages using LOS/NLOS links ...





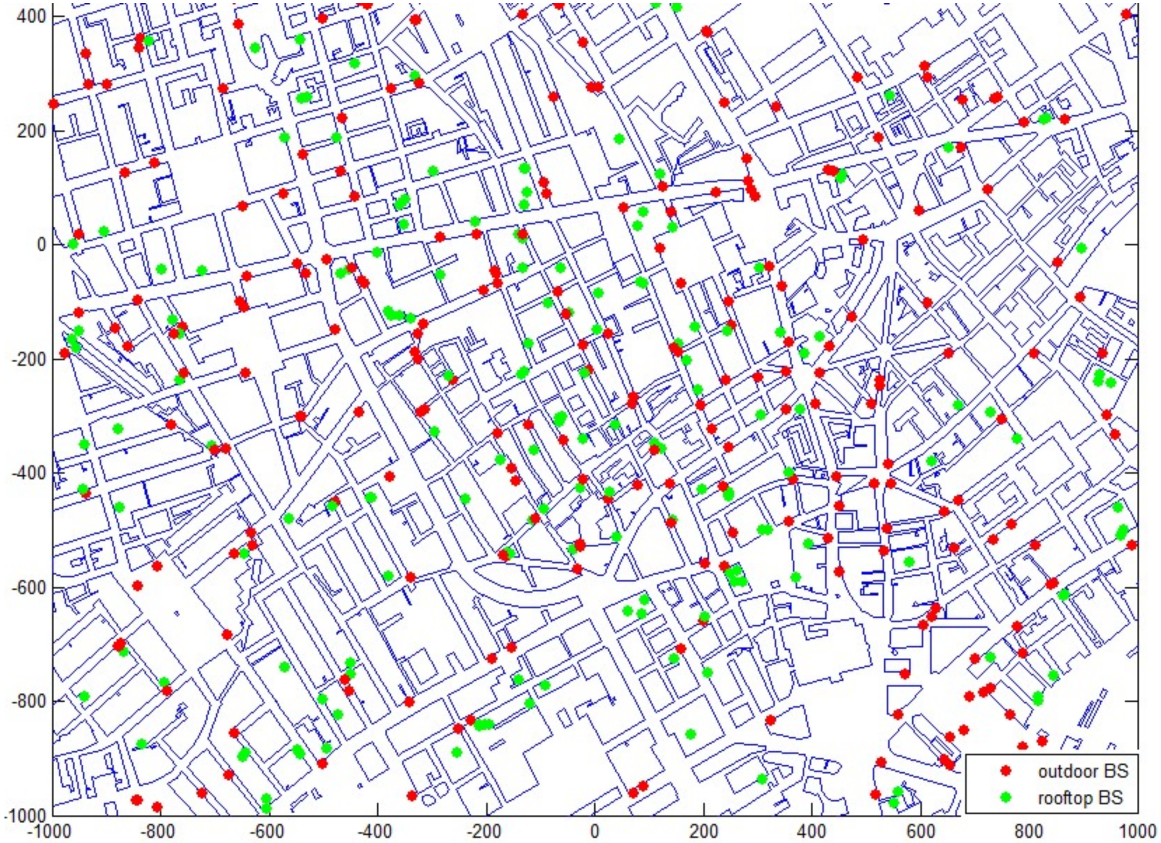
$$p_{\mathrm{LOS}}(r) = \min\left\{\frac{18}{r}, 1\right\}\left(1 - e^{-\frac{r}{36}}\right) + e^{-\frac{r}{36}}$$

Mobile terminal

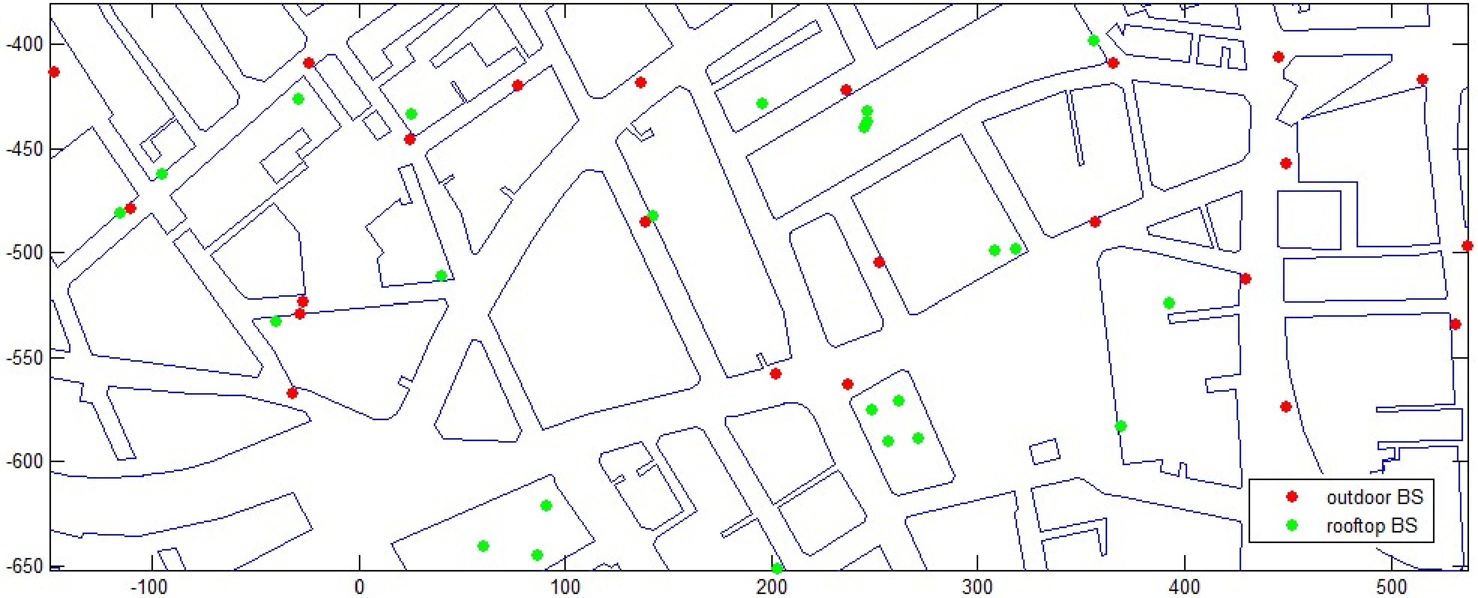
Base station

The London Case Study

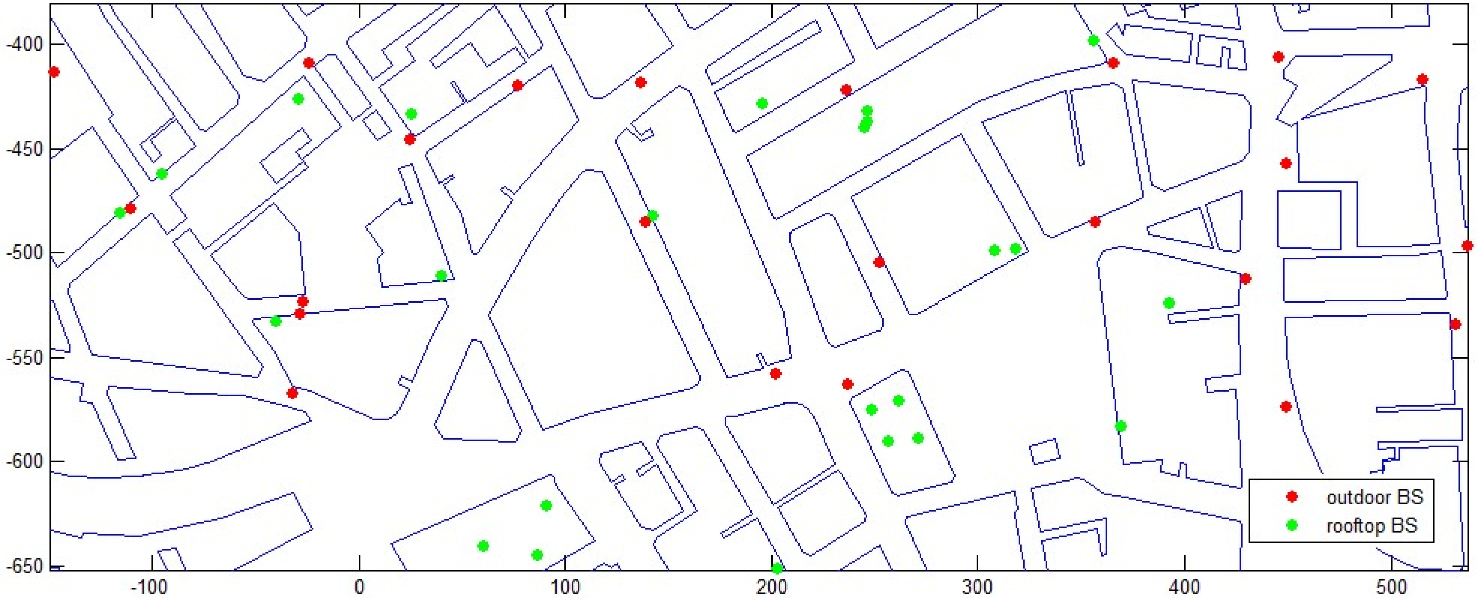
	O2 + Vodafone	O2	Vodafone
Number of BSs	319	183	136
Number of rooftop BSs	95	62	33
Number of outdoor BSs	224	121	103
Average cell radius (m)	63.1771	83.4122	96.7577



The London Case Study



The London Case Study

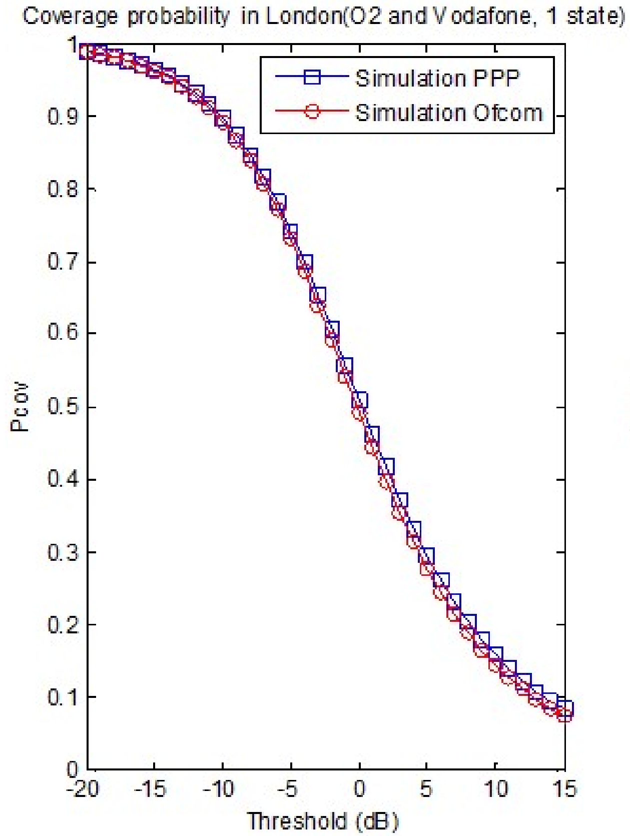


The London Case Study

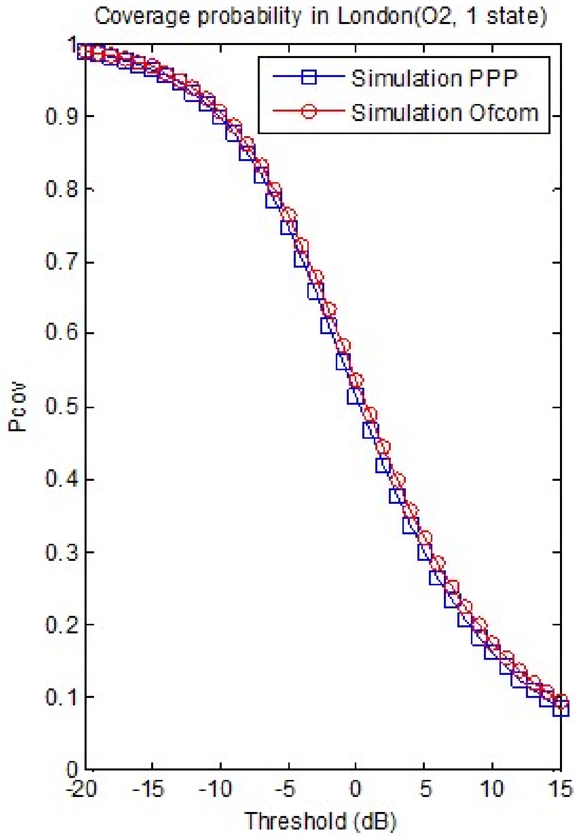
PPP Accuracy: 1-State Channel Model

- **OFCOM**: **Actual base station locations**, **(actual building footprints), actual channels**
- **PPP**: **Random base station locations**, **(actual building footprints), actual channels**

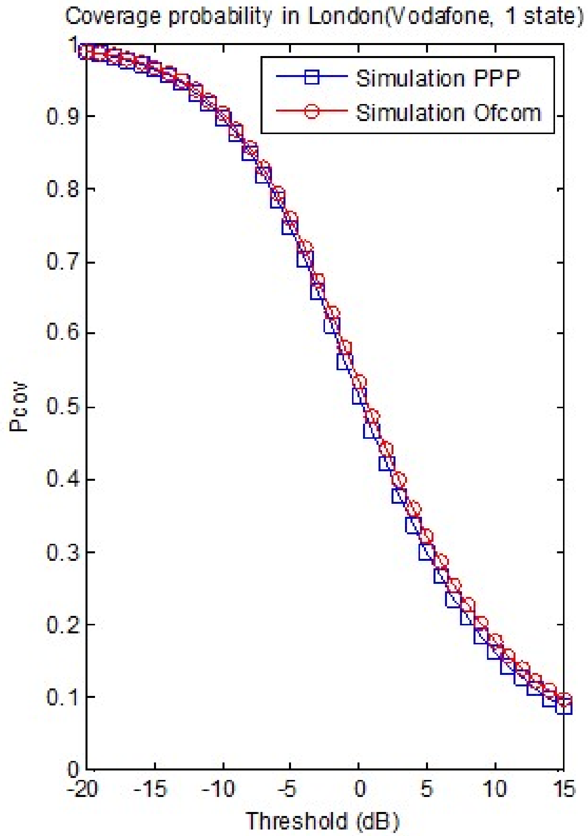
O2+VODAFONE



O2



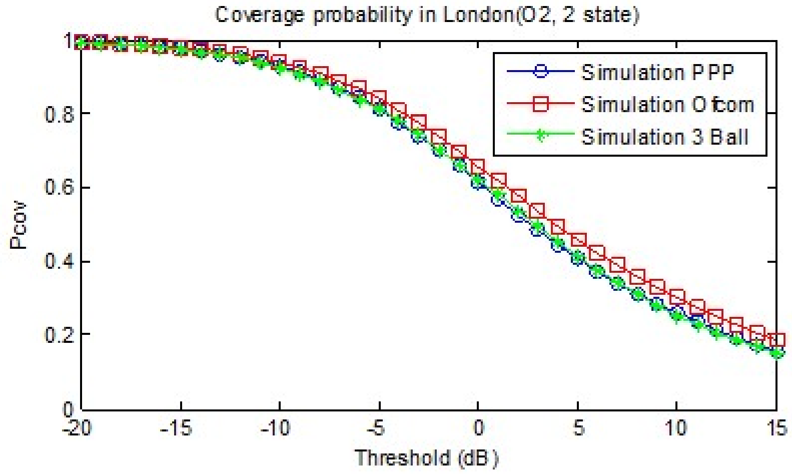
VODAFONE



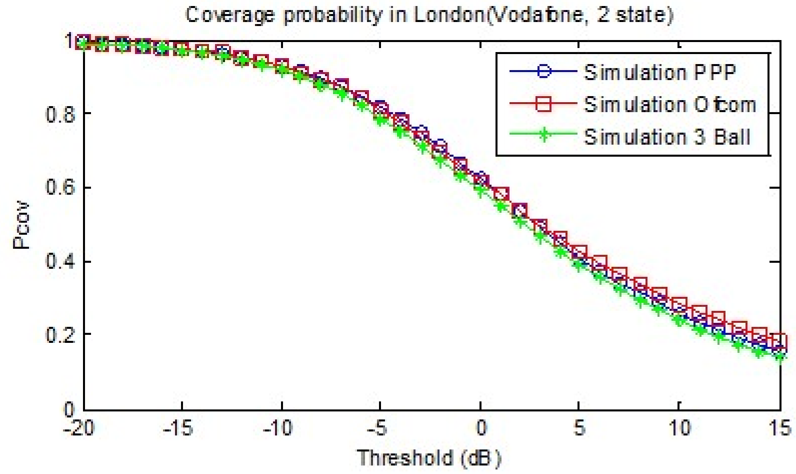
The London Case Study

PPP Accuracy: 2-State Channel Model

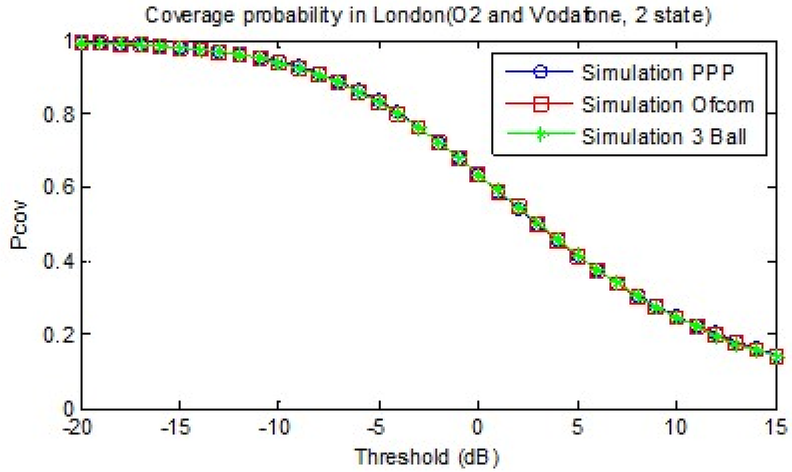
O2



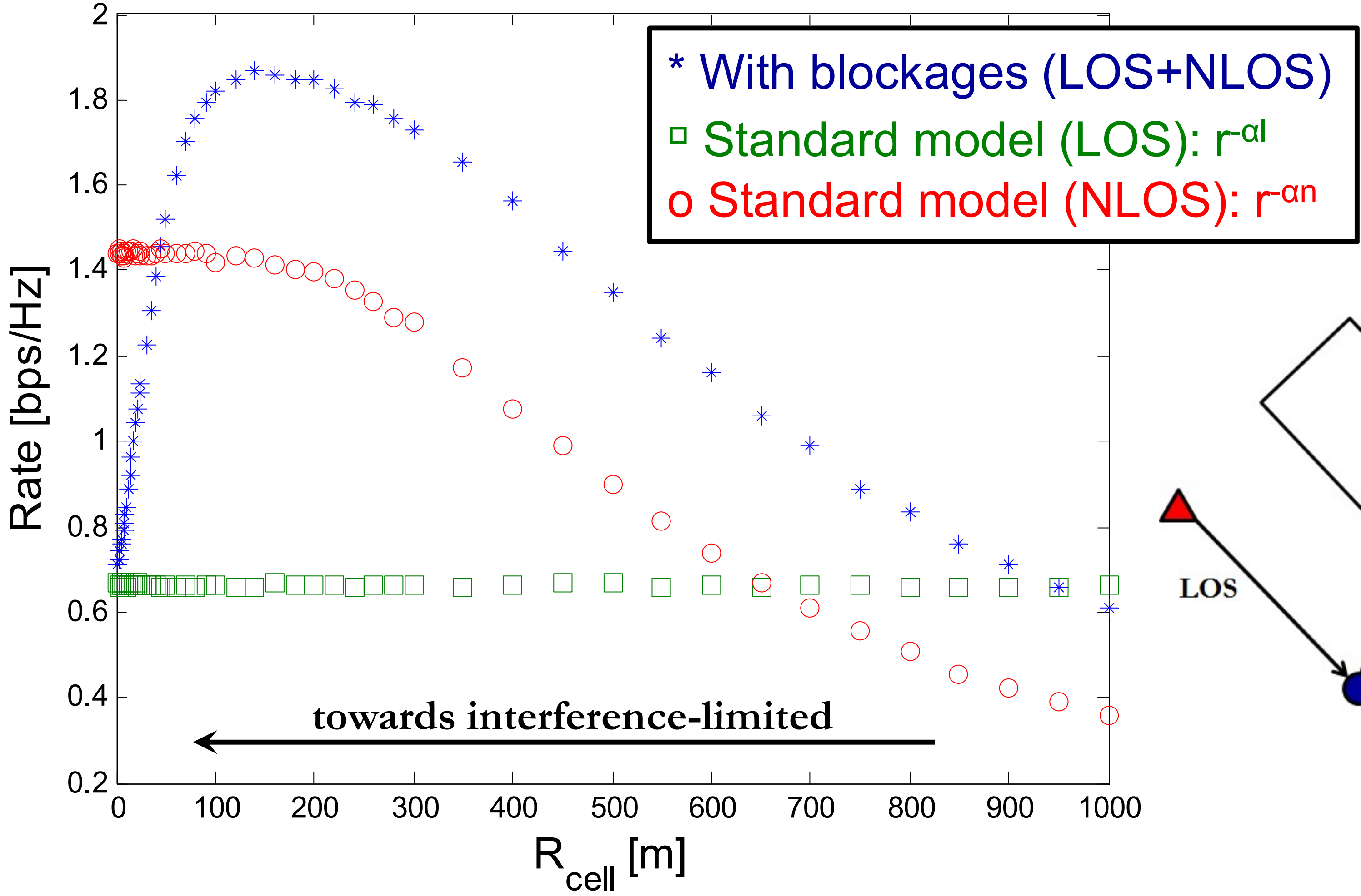
VODAFONE

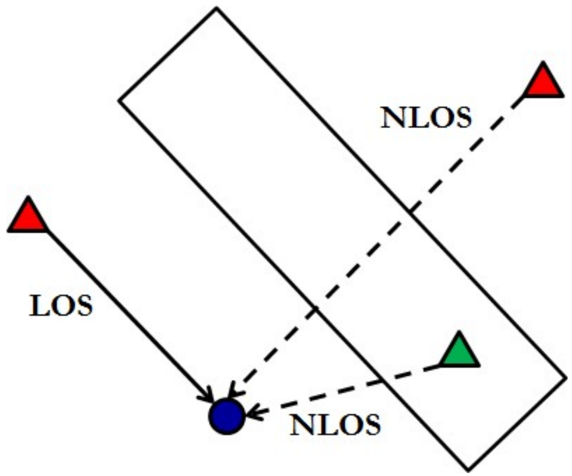


O2+VODAFONE



The London Case Study





M. Di Renzo, W. Lu, and P. Guan, "The Intensity Matching Approach: A Tractable Stochastic Geometry Approximation to System-Level Analysis of Cellular Networks", IEEE Trans. Wireless Commun., Sep. 2016.

Intrigued Enough? On Experimental Validation…

Stochastic Geometry Modeling of Cellular Networks: Analysis, Simulation and Experimental Validation

Wei Lu
Paris-Saclay University
Laboratory of Signals and Systems (UMR-8506)
CNRS-CentraleSupelec-University Paris-Sud XI
3, rue Joliot-Curie
91192 Gif-sur-Yvette (Paris), France
wei.lu@l2s.centralesupelec.fr

Marco Di Renzo
Paris-Saclay University
Laboratory of Signals and Systems (UMR-8506)
CNRS-CentraleSupelec-University Paris-Sud XI
3, rue Joliot-Curie
91192 Gif-sur-Yvette (Paris), France
marco.direnzo@l2s.centralesupelec.fr

ABSTRACT

Due to the increasing heterogeneity and deployment density of emerging cellular networks, new flexible and scalable approaches for their modeling, simulation, analysis and optimization are needed. Recently, a new approach has been proposed: it is based on the theory of point processes and it leverages tools from stochastic geometry for tractable system-level modeling, performance evaluation and optimization. In this paper, we investigate the accuracy of this emerging abstraction for modeling cellular networks, by explicitly taking realistic base station locations, building footprints, spatial blockages and antenna radiation patterns into account. More specifically, the base station locations and the building footprints are taken from two publicly available databases from the United Kingdom. Our study confirms that the abstraction model based on stochastic geometry is capable of accurately modeling the communication performance of cellular networks in dense urban environments.

pected to provide [1]. Modeling, simulating, analyzing and optimizing such networks is, however, a non-trivial problem. This is due to the large number of access points that are expected to be deployed and their dissimilar characteristics, which encompass deployment density, transmit power, access technology, etc. Motivated by these considerations, several researchers are investigating different options for modeling, simulating, mathematically analyzing and optimizing these networks. The general consensus is, in fact, that the methods used in the past for modeling cellular networks, e.g., the hexagonal grid-based model [2], are not sufficiently scalable and flexible for taking the ultra-dense and irregular deployments of emerging cellular topologies into account.

Recently, a new approach for overcoming these limitations has been proposed. It is based on the theory of point processes (PP) and leverages tools from stochastic geometry for system-level modeling, performance evaluation and optimization of cellular networks [3]. In this paper, it is referred

W. Lu and M. Di Renzo, "Stochastic Geometry Modeling of Cellular Networks: Analysis, Simulation and Experimental Validation", ACM Int. Conf. Modeling, Analysis and Simulation of Wireless and Mobile Systems, Nov. 2015. [Online]. Available: http://arxiv.org/pdf/1506.03857.pdf.

W. Lu and M. Di Renzo, "Stochastic Geometry Modeling of mmWave Cellular Networks: Analysis and Experimental Validation", IEEE Int. Workshop on Measurement and Networking (M&N) – Special Session on Advances in 5G Wireless Networks, Oct. 12-13, 2015.

Intrigued Enough? On Mathematical Modeling…

The Intensity Matching Approach: A Tractable Stochastic Geometry Approximation to System-Level Analysis of Cellular Networks

Marco Di Renzo, *Senior Member, IEEE*, Wei Lu, *Student Member, IEEE*, and Peng Guan, *Student Member, IEEE*

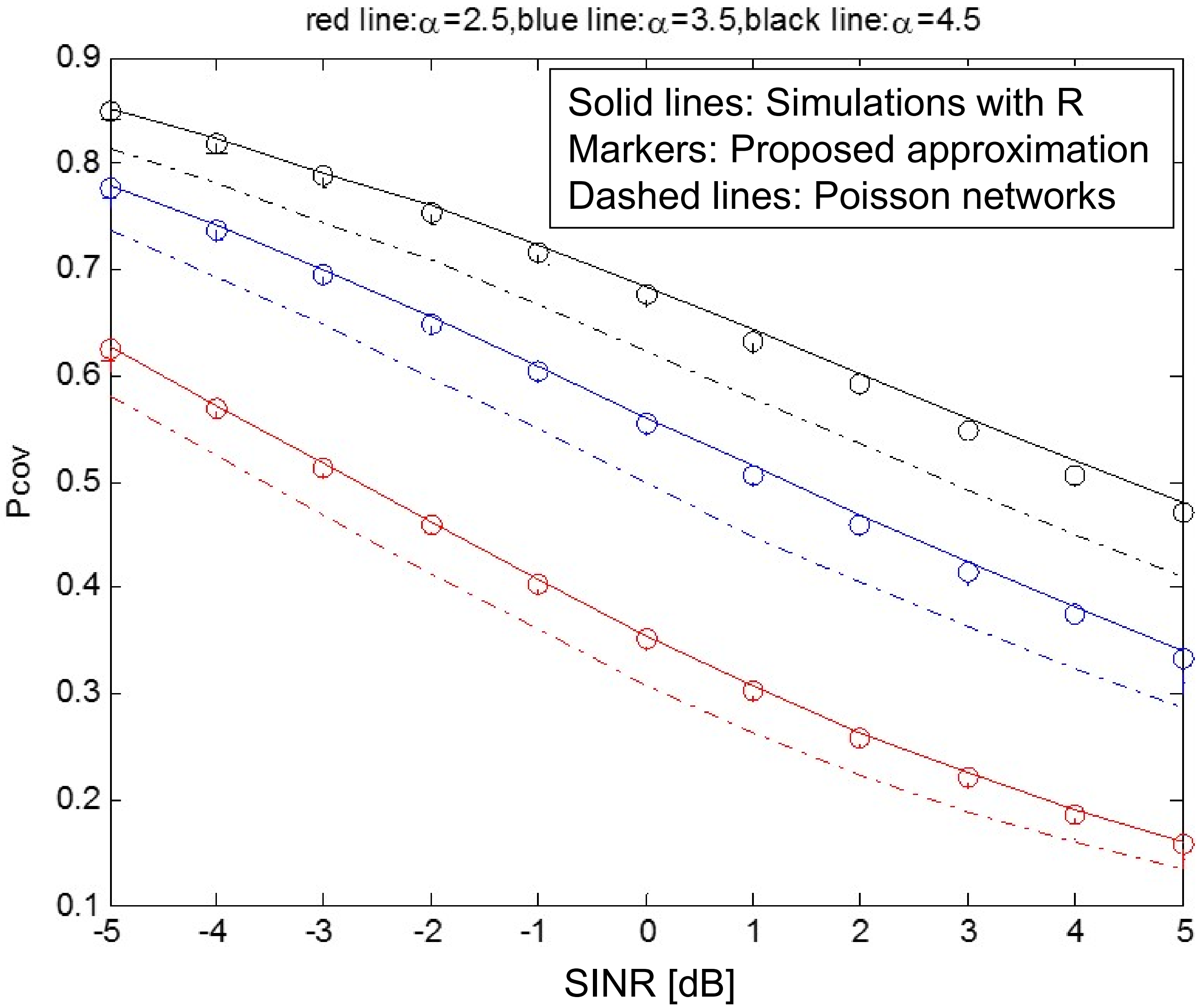
Abstract

The intensity matching approach for tractable performance evaluation and optimization of cellular networks is introduced. It assumes that the base stations are modeled as points of a Poisson point process and leverages stochastic geometry for system-level analysis. Its rationale relies on observing that system-level performance is determined by the intensity measure of transformations of the underlaying spatial Poisson point process. By approximating the original system model with a simplified one, whose performance is determined by a mathematically convenient intensity measure, tractable yet accurate integral expressions for computing area spectral efficiency and potential throughput are provided. The considered system model accounts for many practical aspects that, for tractability, are typically neglected, *e.g.*, line-of-sight and non-line-of-sight propagation, antenna radiation patterns, traffic load, practical cell associations, general fading channels. The proposed approach, more importantly, is conveniently formulated for unveiling the impact of several system parameters, *e.g.*, the density of base stations and blockages. The effectiveness of this novel and general methodology is validated with the aid of empirical data for the locations of base stations and for the footprints of buildings in a dense urban environment.

M. Di Renzo, W. Lu, and P. Guan, "The Intensity Matching Approach: A Tractable Stochastic Geometry Approximation to System-Level Analysis of Cellular Networks", IEEE Trans. Wireless Commun., Sep 2016.

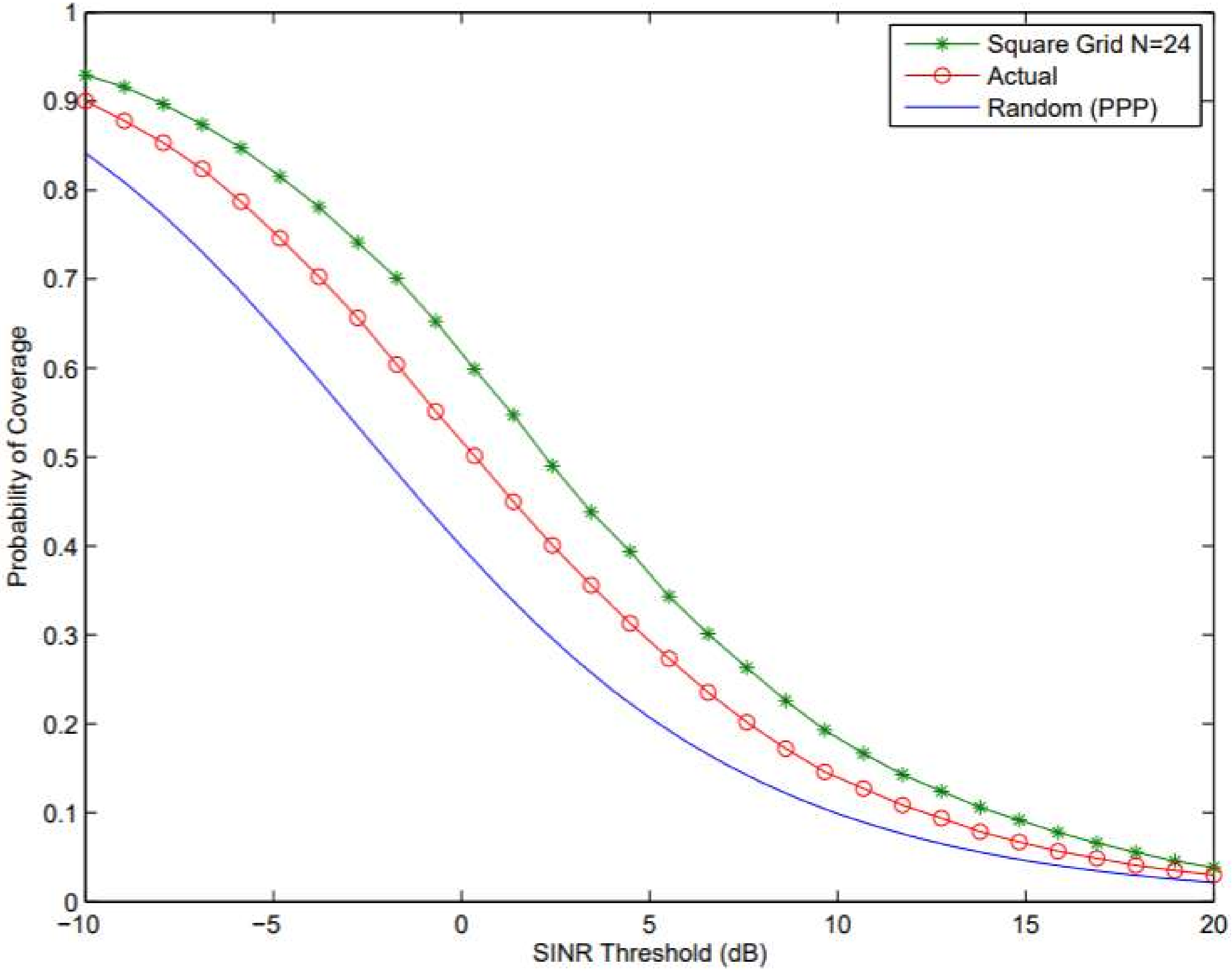
But, The PPP Is Not Always Accurate

… Cauchy determinantal point process (spatially-repulsive) …



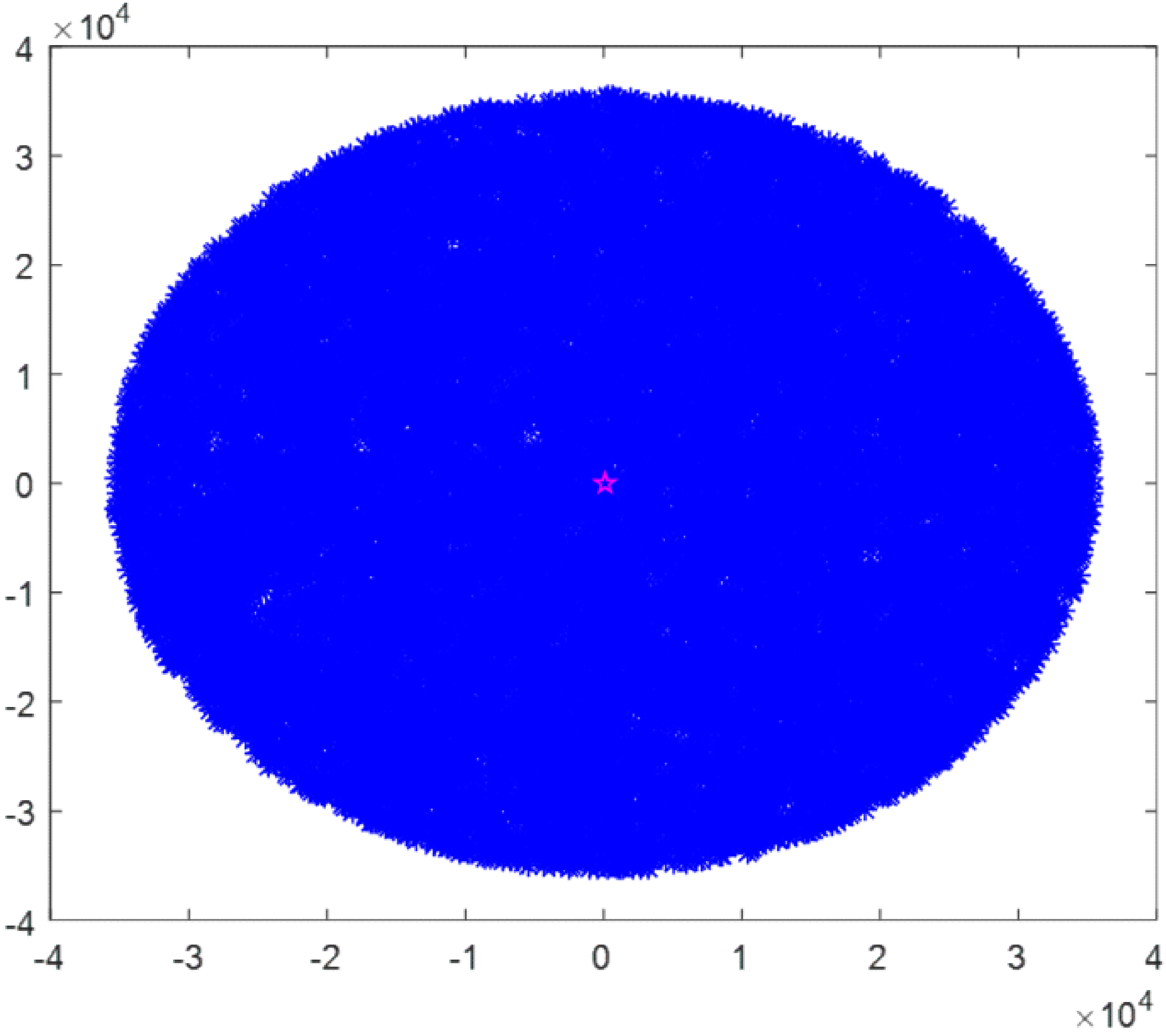
But, The PPP Is Not Always Accurate: Rule of Thumb

… PPP: Lower-Bound – Grid: Upper-Bound …



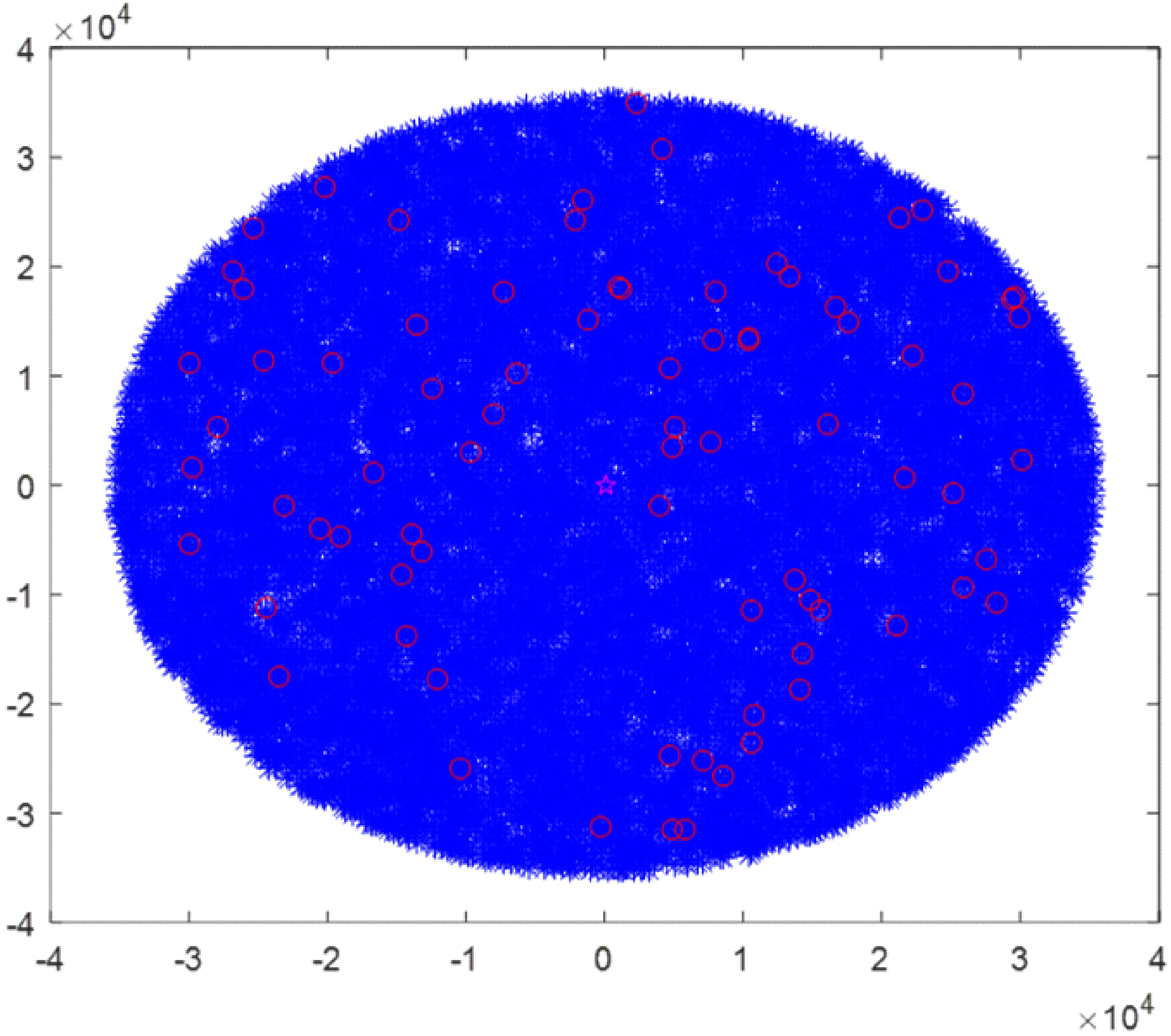
Or, It May Not Necessarily Be The Best Choice

Just an Example: Fully-Operational Cellular Networks



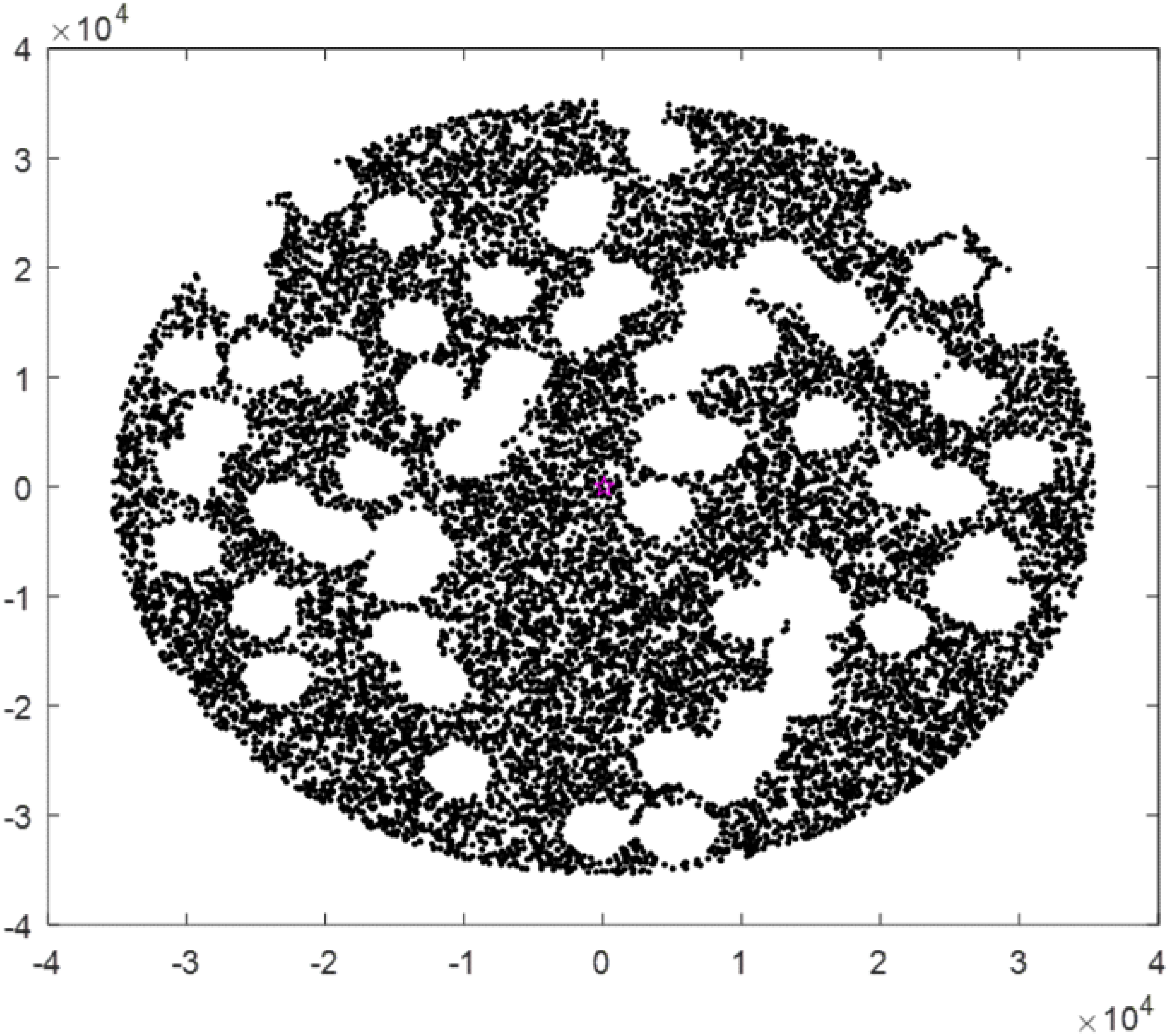
Or, It May Not Necessarily Be The Best Choice

Just an Example: Epicenters of a Natural Disaster



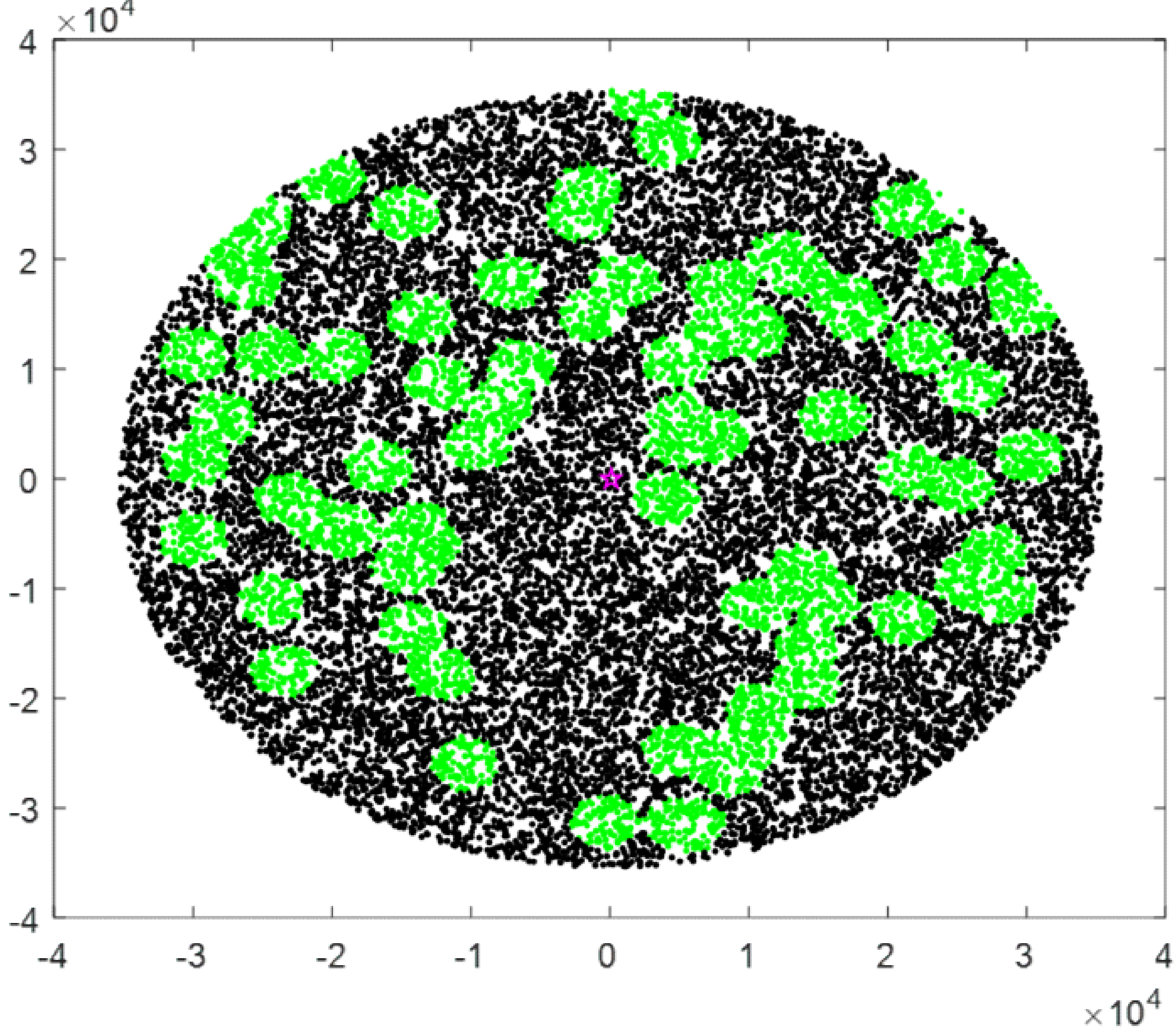
Or, It May Not Necessarily Be The Best Choice

Just an Example: Partially-Operational Cellular Networks



Or, It May Not Necessarily Be The Best Choice

Just an Example: Drone-Aided Cellular Networks



Or, It May Not Necessarily Be The Best Choice

Just a *Science Fiction* Example: Drone-Aided Cellular Networks

Or, We Just Love Math ☺

… The Ginibre Point Process …

Theorem 1 *Consider the cellular network model with a single tier such that the BSs are deployed according to the α-Ginibre point process with intensity λ. Then, the downlink coverage probability of a typical user is given by*

$$\mathsf{P}(\mathsf{SINR}_o > \theta) = \alpha \int_0^\infty e^{-s}\, \mathcal{L}_W\Bigl(\frac{\theta}{p\,c}\Bigl(\frac{\alpha\, s}{\pi\,\lambda}\Bigr)^{\beta/2}\Bigr)\, M(s,\theta)\, S(s,\theta)\, \mathrm{d}s, \tag{5}$$

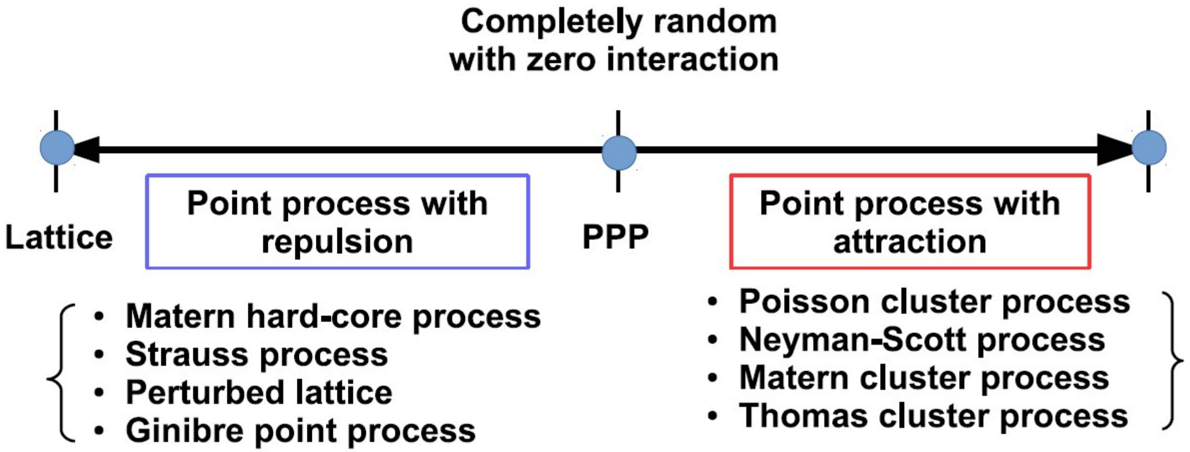
where $\mathcal{L}_W$ denotes the LST of W_o and

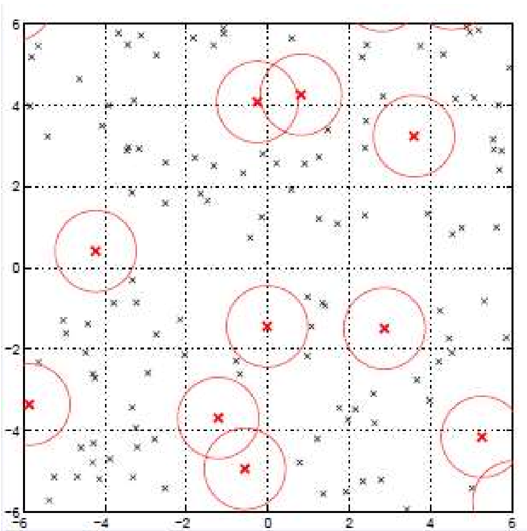
$$M(s,\theta) = \prod_{j=0}^{\infty}\Bigl(1-\alpha+\frac{\alpha}{j!}\int_s^\infty \frac{t^j\, e^{-t}}{1+\theta\,(s/t)^{\beta/2}}\,\mathrm{d}t\Bigr), \tag{6}$$

$$S(s,\theta) = \sum_{i=0}^{\infty} s^i \Bigl((1-\alpha)\, i! + \alpha \int_s^\infty \frac{t^i\, e^{-t}}{1+\theta\,(s/t)^{\beta/2}}\,\mathrm{d}t\Bigr)^{-1}. \tag{7}$$

I. Nakata and N. Miyoshi, "Spatial stochastic models for analysis of heterogeneous cellular networks with repulsively deployed base stations", Research Reports on Mathematical and Computing Sciences (ISSN 1342-2804), Oct. 2013, B-473.

We Are Lucky: Stochastic Geometry Is A Rich Toolbox





Matern Hard-Core PP

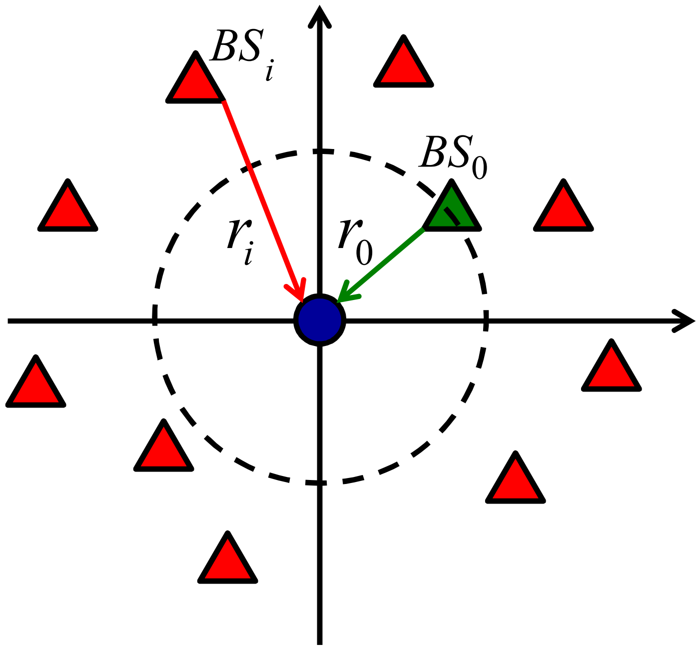
Take a homogeneous PPP and remove any pairs of points that are closer to each other than a predefined minimum distance R

Y. J. Chun, M. O. Hasna, A. Ghrayeb, and M. Di Renzo, "On modeling heterogeneous wireless networks using non-Poisson point processes", IEEE Commun. Mag., submitted. [Online]. Available: http://arxiv.org/pdf/1506.06296.pdf.

… Part II …

(Beyond The PPP)

Non-PPPs: The Problem Statement Is The Same…



Φ is NOT a PPP

$$\mathrm{P_{cov}} = \Pr\left\{\mathrm{SINR} > T\right\}$$

$$\mathrm{SINR} = \frac{P\left|h_o\right|^2 r_o^{-\alpha}}{\sigma^2 + I_{agg}\left(r_0\right)}$$

$$I_{agg}\left(r_0\right) = \sum_{i \in \Phi \backslash BS_0} P\left|h_i\right|^2 r_i^{-\alpha}$$

$$\mathrm{P_{cov}} = \Pr\left\{\frac{P\left|h_o\right|^2 r_o^{-\alpha}}{\sigma^2 + I_{agg}\left(r_0\right)} > T\right\} = \ldots$$

And The Following Still Applies

$$\mathrm{P}_{\mathrm{cov}} = \mathrm{E}_{r_0}\left\{\exp\left(-T\sigma^2 P^{-1} r_o^{\alpha}\right)\mathrm{MGF}_{I_{agg}(r_0)}\left(P^{-1} T r_o^{\alpha}\right)\right\}$$

$$= \int_0^{+\infty} \exp\left(-T\sigma^2 P^{-1}\xi^{\alpha}\right)\mathrm{MGF}_{I_{agg}(r_0)}\left(P^{-1} T \xi^{\alpha}\right)\mathrm{PDF}_{r_0}(\xi)\, d\xi$$

And The Following Still Applies

$$\mathrm{P}_{\mathrm{cov}} = \mathrm{E}_{r_0}\left\{\exp\left(-T\sigma^2 P^{-1} r_o^{\alpha}\right)\mathrm{MGF}_{I_{agg}(r_0)}\left(P^{-1}T r_o^{\alpha}\right)\right\}$$

$$= \int_0^{+\infty} \exp\left(-T\sigma^2 P^{-1}\xi^{\alpha}\right)\mathrm{MGF}_{I_{agg}(r_0)}\left(P^{-1}T\xi^{\alpha}\right)\mathrm{PDF}_{r_0}\left(\xi\right)d\xi$$

What About the MGF and the PDF Now ?

And The Following Still Applies

$$\mathrm{P_{cov}} = \mathrm{E}_{r_0}\left\{\exp\left(-T\sigma^2 P^{-1} r_o^{\alpha}\right)\mathrm{MGF}_{I_{agg}(r_0)}\left(P^{-1}T r_o^{\alpha}\right)\right\}$$

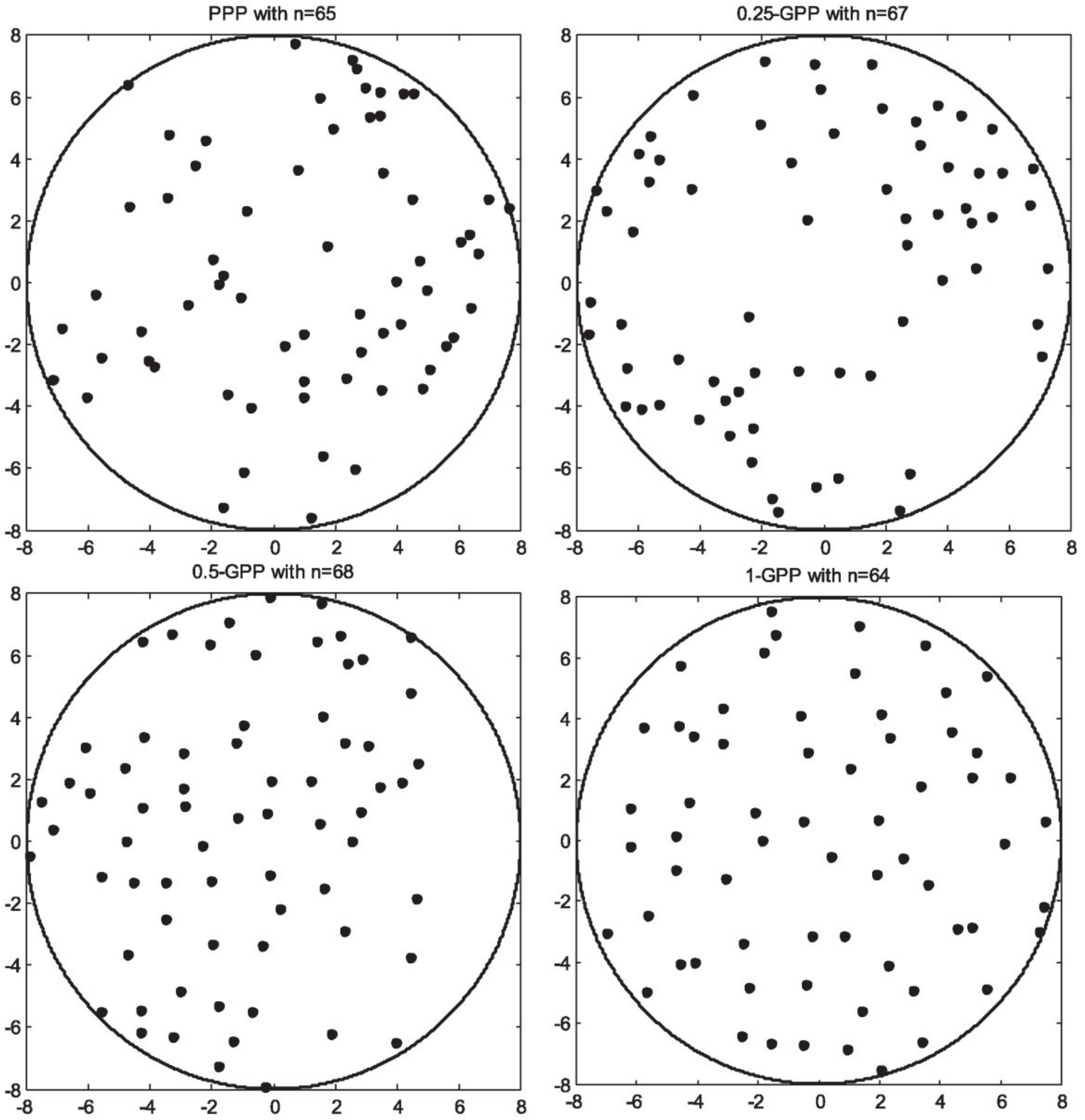
$$= \int_0^{+\infty} \exp\left(-T\sigma^2 P^{-1}\xi^{\alpha}\right)\mathrm{MGF}_{I_{agg}(r_0)}\left(P^{-1}T\xi^{\alpha}\right)\mathrm{PDF}_{r_0}\left(\xi\right)d\xi$$

What About the MGF and the PDF Now ?

- PDF → Contact Distance Distribution of the PP

- MGF → Reduced Palm Distribution of the PP

Example 1: The Ginibre Point Process (GPP)



Example 1: The Ginibre Point Process (GPP)

It is known that the moduli (on the complex plane) of the points of the GPP have the same distribution as independent gamma random variables [16]. For the β-GPP, from Theorem 4.7.1 in [17], we have the following result:

Proposition 1: Let $\Phi_c = \{X_i\}_{i \in \mathbb{N}}$ be a scaled β-GPP. For $k \in \mathbb{N}$, let Q_k be a random variable with probability density function

$$f_{Q_k}(q) = \frac{q^{k-1} e^{-\frac{c}{\beta} q}}{\left(\frac{\beta}{c}\right)^k \Gamma(k)}, \tag{8}$$

i.e., $Q_k \sim \text{gamma}(k, \beta/c)$, with Q_k independent of Q_j if $k \neq j$. Then the set $\{|X_i|^2\}_{i \in \mathbb{N}}$ has the same distribution as the set Ξ obtained by retaining from $\{Q_k\}_{k \in \mathbb{N}}$ each Q_k with probability β independently of everything else.[2]

Example 1: The Ginibre Point Process (GPP)

From Theorem 1 and Remark 24 in [15], we know that there exists a version of the GPP Φ_c such that the Palm measure of Φ_c is the law of the process obtained by removing from Φ_c a Gaussian-distributed point and then adding the origin. Thus, we have the following proposition.

Proposition 2. (The Palm Measure of the Scaled β-Ginibre Point Process): For a scaled β-GPP Φ_c, the Palm measure of Φ_c is the law of the process obtained by adding the origin and deleting the point X if it belongs (which occurs with probability β) to the process Φ_c, where $|X|^2 = Q_1$.

From Propositions 1 and 2, we observe that the Palm distribution of the squared moduli Q_k is closely related to the non-Palm version, the only difference being that Q_1 is removed if it is included in Ξ.

Example 1: The Ginibre Point Process (GPP)

$$\mathbb{P}(\text{SINR} > \theta) = \sum_{i=1}^{\infty} \mathbb{P}(\text{SINR} > \theta, B_o = i)$$

$$= \sum_{i=1}^{\infty} \mathbb{P}(\text{SINR} > \theta, B_o = i \mid T_i = 1)\mathbb{P}(T_i = 1)$$

$$= \sum_{i=1}^{\infty} \beta \mathbb{P} \left\{ h_i > \frac{\theta \left(\sigma^2 + \sum_{k \in \mathbb{N} \setminus \{i\}} h_k Q_k^{-\alpha/2} T_k \right)}{Q_i^{-\alpha/2}}, B_o = i \right\}$$

$$= \sum_{i=1}^{\infty} \beta \mathbb{E} \left\{ e^{-\theta \sigma^2 Q_i^{\frac{\alpha}{2}}} \right.$$

$$\left. \times \exp\left(-\theta \sum_{k \in \mathbb{N} \setminus \{i\}} h_k \left(\frac{Q_i}{Q_k} \right)^{\frac{\alpha}{2}} T_k \right), B_o = i \right\}$$

Example 1: The Ginibre Point Process (GPP)

Theorem 2: For an SINR threshold θ, the coverage probability of the typical user in the β-Ginibre wireless network is given by

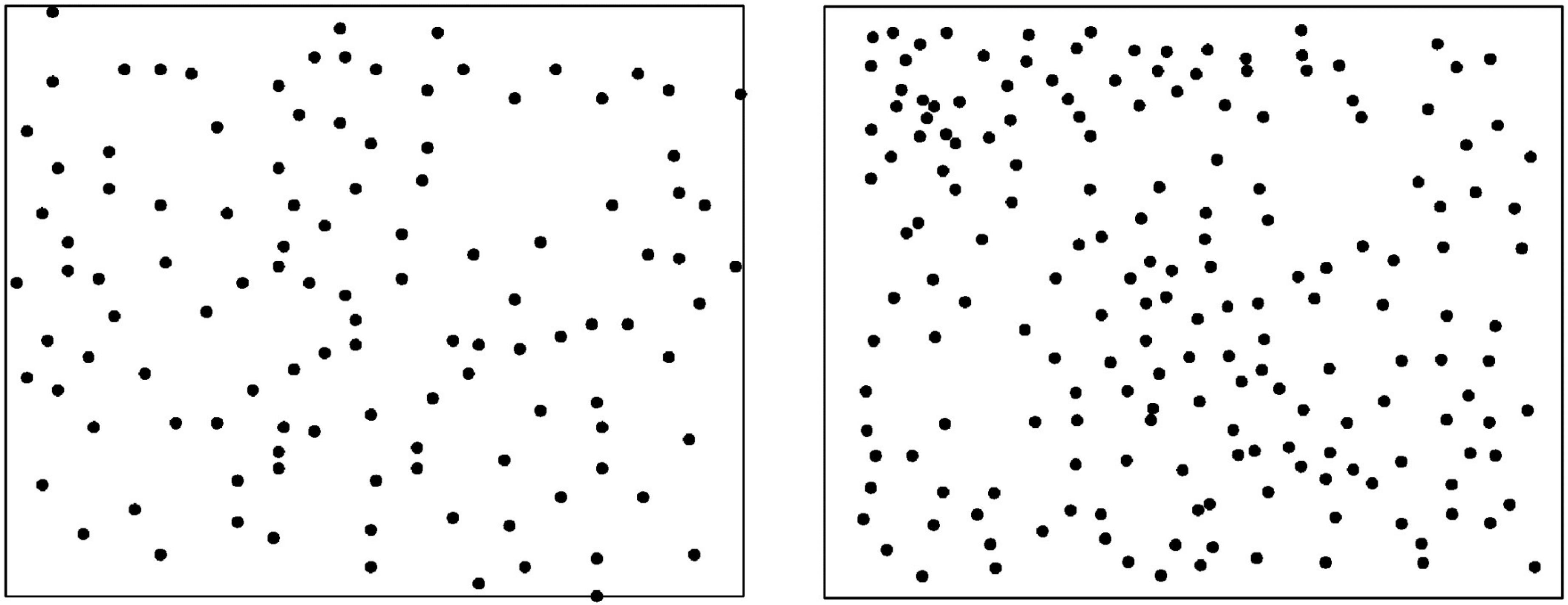
$$p(\theta,\alpha,\beta)=\beta\int_0^\infty e^{-s}e^{-\mu\theta\sigma^2\left(\frac{\beta s}{c}\right)^{\frac{\alpha}{2}}}M(\theta,s,\alpha,\beta)S(\theta,s,\alpha,\beta)\,ds, \tag{42}$$

where

$$M(\theta,s,\alpha,\beta)=\prod_{k=1}^{\infty}\left(\int_s^\infty \frac{v^{k-1}e^{-v}}{(k-1)!}\frac{\beta}{1+\theta\left(\frac{s}{v}\right)^{\frac{\alpha}{2}}}dv+1-\beta\right), \tag{43}$$

$$S(\theta,s,\alpha,\beta)=\sum_{i=1}^{\infty}s^{i-1}\left(\int_s^\infty v^{i-1}e^{-v}\frac{\beta}{1+\theta\left(\frac{s}{v}\right)^{\frac{\alpha}{2}}}dv+(1-\beta)(i-1)!\right)^{-1}. \tag{44}$$

Example 2: The Determinantal Point Process (DPP)



(a) (b)

Fig. 1. Real macro BS deployments. (a) Houston data set; (b) LA data set.

Example 2: The Determinantal Point Process (DPP)

Model	λ	α	ν
Gauss DPP	0.4492	0.8417	–
Cauchy DPP	0.4492	1.558	3.424
Generalized Gamma DPP	0.4492	2.539	2.63

TABLE II
DPP PARAMETERS FOR THE LA DATA SET

Model	λ	α	ν
Gauss DPP	0.2347	1.165	–
Cauchy DPP	0.2347	2.13	3.344
Generalized Gamma DPP	0.2347	3.446	2.505

Example 2: The Determinantal Point Process (DPP)

Lemma 5: For any $\Phi \sim \mathrm{DPP}(K)$, the empty space function $F(r)$ for $r \geq 0$ is given by:

$$F(r) = \sum_{n=1}^{+\infty} \frac{(-1)^{n-1}}{n!} \int_{(B(0,r))^n} \det\left(K(x_i, x_j)\right)_{1 \leq i,j \leq n} \mathrm{d}x_1 \ldots \mathrm{d}x_n. \tag{11}$$

Example 2: The Determinantal Point Process (DPP)

Lemma 4 (Shirai et al. [21]): Consider $\Phi \sim \text{DPP}(K)$, where the kernel K guarantees the existence of Φ. Then under the reduced Palm distribution at $x_0 \in \mathbb{R}^2$, Φ coincides with another DPP associated with kernel $K^!_{x_0}$ for Lebesgue almost all x_0 with $K(x_0, x_0) > 0$, where:

$$K^!_{x_0}(x, y) = \frac{1}{K(x_0, x_0)} \det \begin{pmatrix} K(x, y) & K(x, x_0) \\ K(x_0, y) & K(x_0, x_0) \end{pmatrix}. \qquad (9)$$

This property shows that DPPs are closed under the reduced Palm distribution, which provides a tool similar to Slyvniak's theorem for Poisson processes [29]. In cellular networks, when x_0 is chosen as the serving base station to the typical user, this property shows that all other interferers will form another DPP with the modified kernel provided in (9).

Example 2: The Determinantal Point Process (DPP)

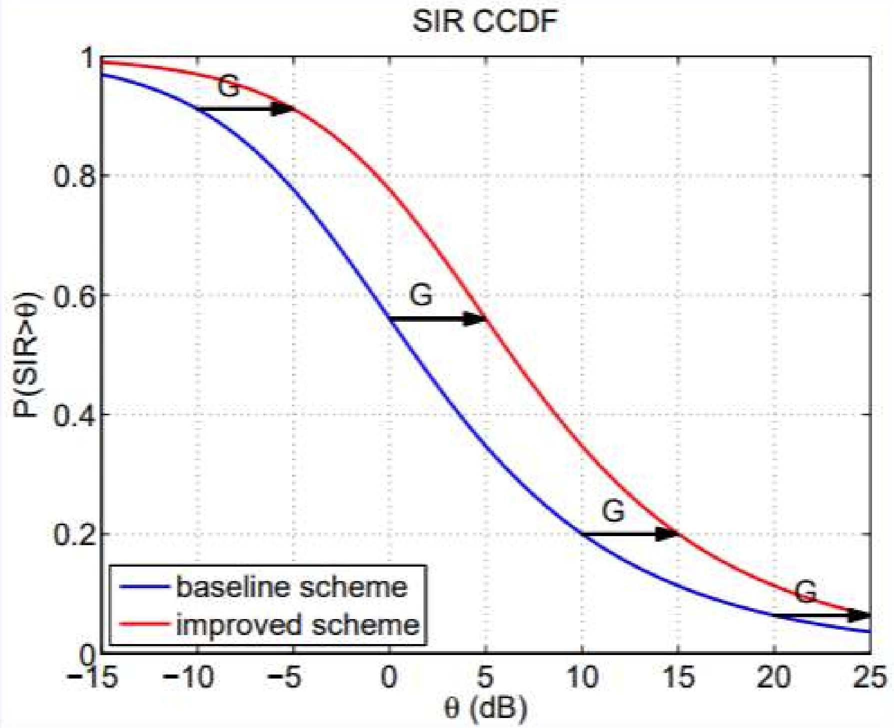
Theorem 2: The SIR distribution of the typical user at the origin is given by:

$$\mathbb{P}\left(\mathrm{SIR}(0,\Phi) > T\right)$$

$$= \int_0^{+\infty} \lambda 2\pi \left[\sum_{n=0}^{+\infty} \frac{(-1)^n}{n!} \int_{(\mathbb{R}^2)^n} \det\left(K^!_{x_0}(x_i, x_j)_{1\le i,j\le n} \right) \right.$$

$$\left. \times \prod_{i=1}^{n} \left[1 - \frac{\mathbb{1}_{|x_i|\ge r_0}}{1 + Tl(x_i)/l(x_0)} \right] \Bigg|_{x_0=(r_0,0)} \mathrm{d}x_1 \ldots \mathrm{d}x_n \right] r_0 \mathrm{d}r_0. \tag{22}$$

Example 3: The As-A-PPP (ASAPPP) Approach



This SIR gain is nearly constant over θ in many cases.

$$p_s(\theta) = \mathbb{P}(\text{SIR} > \theta) \quad \Rightarrow \quad p_s(\theta) = \mathbb{P}(\text{SIR} > \theta/G).$$

Can we quantify this gain?

Example 3: The As-A-PPP (ASAPPP) Approach

Horizontal gap at probability p

The horizontal gap between two SIR ccdfs is

$$G(p) \triangleq \frac{\bar{F}^{-1}_{\mathrm{SIR}_2}(p)}{\bar{F}^{-1}_{\mathrm{SIR}_1}(p)}, \quad p \in (0,1),$$

where $\bar{F}^{-1}_{\mathrm{SIR}}$ is the inverse of the ccdf of the SIR, and p is the target success probability.
We also define the asymptotic gain (whenever the limit exists) as

$$G = \lim_{p \to 1} G(p).$$

Example 3: The As-A-PPP (ASAPPP) Approach

The ISR and the MISR

Definition ($\mathrm{I\bar{S}R}$)

The interference-to-average-signal ratio is

$$\mathrm{I\bar{S}R} \triangleq \frac{I}{\mathbb{E}_h(S)},$$

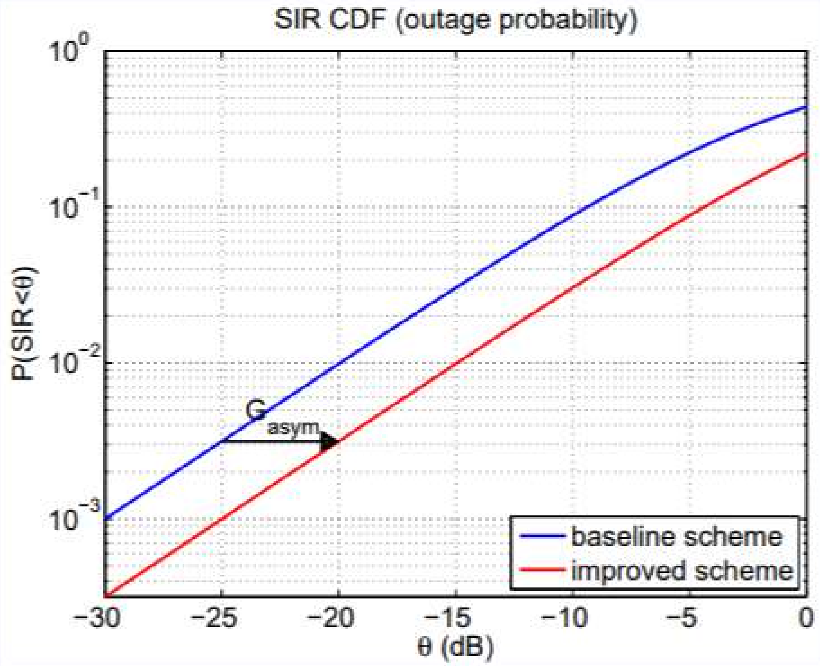
where $\mathbb{E}_h(S)$ is the desired signal power averaged over the fading.

Comments

- The $\mathrm{I\bar{S}R}$ is a random variable due to the random positions of BSs and users. Its mean MISR is a function of the network geometry only.
- If the desired signal comes from a BS at distance R, $(\mathbb{E}_h(S))^{-1} = R^\alpha$.
- If the interferers are located at distances R_k,

$$\mathrm{MISR} \triangleq \mathbb{E}(\mathrm{I\bar{S}R}) = \mathbb{E}\left(R^\alpha \sum h_k R_k^{-\alpha}\right) = \sum \mathbb{E}\left(\frac{R}{R_k}\right)^\alpha.$$

Example 3: The As-A-PPP (ASAPPP) Approach



Outage probability:

$$p_{\text{out}}(\theta) = \mathbb{P}(hR^{-\alpha} < \theta I)$$
$$= \mathbb{P}(h < \theta\,\mathsf{I\bar{S}R})$$

For exponential h:

$$= 1 - e^{-\theta\,\mathsf{I\bar{S}R}}$$
$$\sim \theta\,\mathrm{MISR}, \quad \theta \to 0.$$

So $\quad F_{\mathrm{SIR}}(\theta) \sim \theta\,\mathrm{MISR} \quad \Longrightarrow \quad \bar{F}_{\mathrm{SIR}}^{-1}(p) \sim (1-p)/\mathrm{MISR},\ (p \to 1).$

So the asymptotic gain is the ratio of the two MISRs: $\quad G = \dfrac{\mathrm{MISR}_1}{\mathrm{MISR}_2}$

We need to find a reference MISR_1 that is easy to calculate...

Example 3: The As-A-PPP (ASAPPP) Approach

The MISR for the HIP model

For the (single-tier) HIP model,

$$\mathrm{MISR} = \mathbb{E}\left(R_1^\alpha \sum_{k=2}^{\infty} R_k^{-\alpha}\right) = \sum_{k=2}^{\infty} \mathbb{E}\left(\frac{R_1}{R_k}\right)^\alpha ,$$

where R_k is the distance to the k-th nearest BS.
The distribution of $\nu_k = R_1/R_k$ is

$$F_{\nu_k}(x) = 1 - (1 - x^2)^{k-1}, \quad x \in [0, 1].$$

Summing up the α-th moments $\mathbb{E}(\nu_k^\alpha)$, we obtain (remarkably) [Hae14]

$$\mathrm{MISR} = \frac{2}{\alpha - 2}.$$

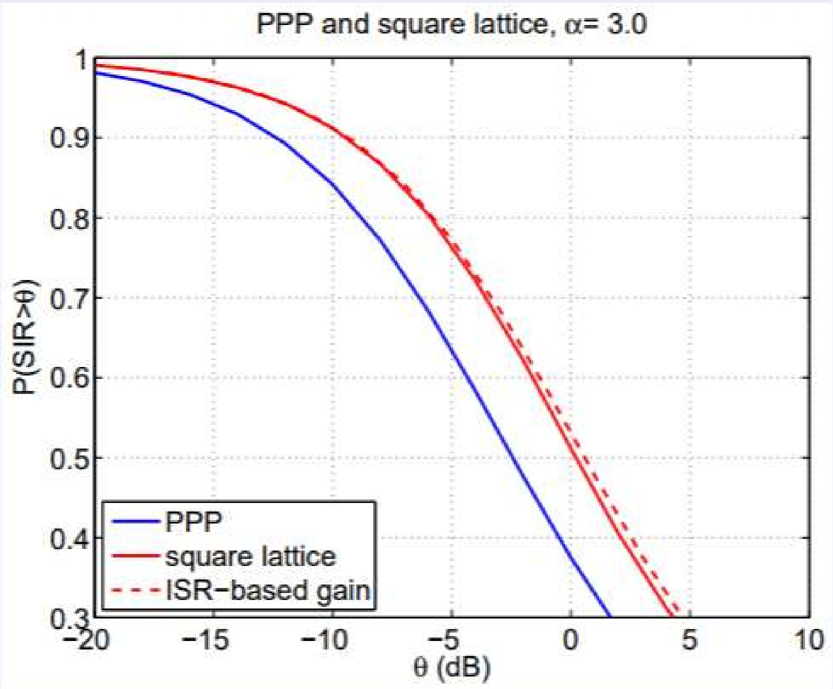
This is the baseline MISR relative to which we can measure the gain G.
For $\alpha = 4$, it is 1.

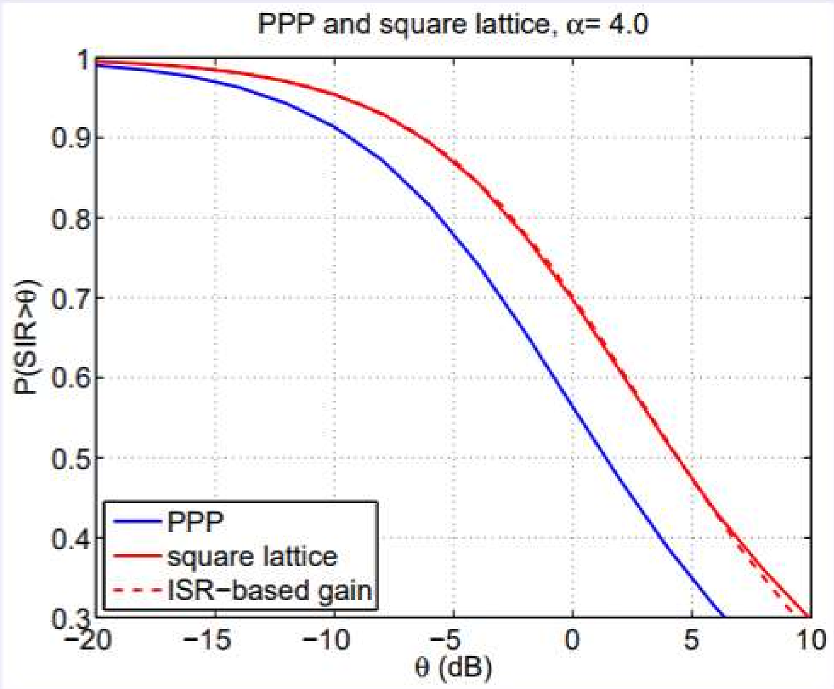
Example 3: The As-A-PPP (ASAPPP) Approach

Gain relative to HIP

We can approximate the SIR distribution of arbitrary point processes and transmission schemes by shifting the Poisson curve:

$$p_s(\theta) = p_s^{\mathrm{HIP}}\left(\theta \frac{\mathrm{MISR}}{\mathrm{MISR}_{\mathrm{HIP}}}\right)$$





Example 3: The As-A-PPP (ASAPPP) Approach

A. The MISR for General Point Processes

The first result gives an expression for the MISR for a general point process.

Theorem 1: The MISR of a motion-invariant point process Φ is given by

$$\mathsf{MISR} = 2\int_0^1 t^{\alpha-3}\beta_1(t)\mathrm{d}t,$$

$$\beta_n(t_1,\ldots,t_n) \triangleq \frac{1}{n!2^n}\int_{\mathbb{R}^2}\|y_0\|^{2n}\int_{[0,2\pi]^n}\mathbb{E}^!_{y_0,\left(\frac{\|y_0\|}{t_1},\varphi_1\right),\ldots,\left(\frac{\|y_0\|}{t_n},\varphi_n\right)}[g(\Phi,y_0)] \cdot \rho_\Phi^{(n+1)}\left(y_0,\left(\frac{\|y_0\|}{t_1},\varphi_1\right),\ldots,\left(\frac{\|y_0\|}{t_n},\varphi_n\right)\right)\mathrm{d}\varphi_1\cdots\mathrm{d}\varphi_n\mathrm{d}y_0.$$

Let's Step Back… And The Following Still Applies

$$\mathrm{P}_{\mathrm{cov}} = \mathrm{E}_{r_0}\left\{\exp\left(-T\sigma^2 P^{-1} r_o^{\alpha}\right)\mathrm{MGF}_{I_{agg}(r_0)}\left(P^{-1}T r_o^{\alpha}\right)\right\}$$

$$= \int_0^{+\infty} \exp\left(-T\sigma^2 P^{-1}\xi^{\alpha}\right)\mathrm{MGF}_{I_{agg}(r_0)}\left(P^{-1}T\xi^{\alpha}\right)\mathrm{PDF}_{r_0}(\xi)\,d\xi$$

What About the MGF and the PDF Now ?

And The Following Still Applies → I-PPP

… Inhomogeneous Poisson Point Processes …

$$\mathrm{P}_{\mathrm{cov}} = \mathrm{E}_{r_0}\left\{ \exp\left(-T\sigma^2 P^{-1} r_o^{\alpha}\right) \mathrm{MGF}_{I_{agg}(r_0)}\left(P^{-1} T r_o^{\alpha}\right) \right\}$$

$$= \int_0^{+\infty} \exp\left(-T\sigma^2 P^{-1} \xi^{\alpha}\right) \mathrm{MGF}_{I_{agg}(r_0)}\left(P^{-1} T \xi^{\alpha}\right) \mathrm{PDF}_{r_0}(\xi)\, d\xi$$

$$\mathrm{PDF} \overset{\text{Void Prob}}{\leftrightarrow} \Pr\left\{\Phi\left(B[0,r_0)\right) = 0\right\} = \exp\left(-\Lambda\left([0,r_0)\right)\right)$$

$$\mathrm{MGF} \overset{\text{PGFL}}{\leftrightarrow} \mathrm{E}\left\{ \prod_{\substack{x\in\Phi \\ r=\|x\|\geq r_0}} f(x) \right\} = \exp\left(-\int_{r_0}^{+\infty} \left(1 - f(r)\right) \Lambda^{(1)}\left([0,r)\right) r dr \right)$$

$$\Lambda\left([0,r_0)\right) = \mathrm{E}\left\{\Phi\left(B[0,r_0)\right)\right\} = 2\pi \int_0^{r_0} \lambda(r)\, r dr = 2\pi \int_0^{r_0} \lambda t(r)\, r dr$$

Why Inhomogeneous Poisson Point Processes ?

- **Mathematical tractability**
- **I-PPP are "*__unavoidable__*" for system-level analysis of HetNets**
  - **The PP of the path-losses is a I-PPP [1]**
  - **Modeling LOS and NLOS [2]**
  - **Modeling the uplink [3]**
  - **Repulsive ? Clustered ? Maybe just Inhomogeneous ?**

    … "it may be difficult to disentangle" … [4]

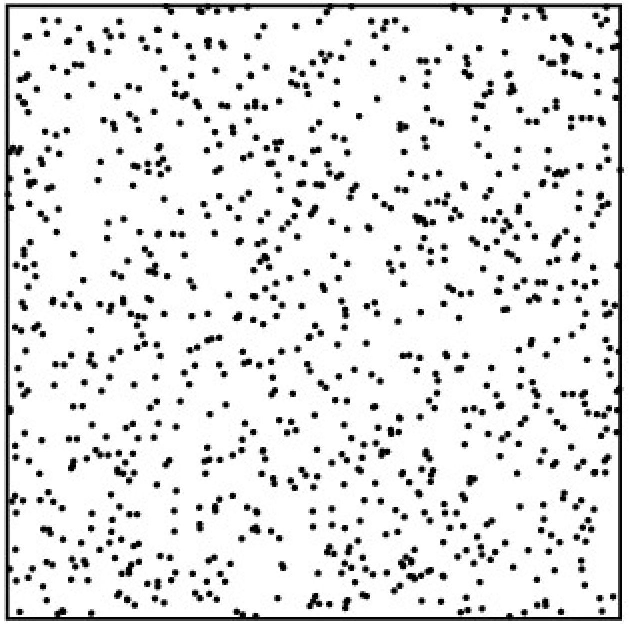
This local aggregation is not simply the result of random point density fluctuations. There exists a "fundamental ambiguity between heterogeneity and clustering, the first corresponding to spatial variation of the intensity function $\lambda(x)$, the second to stochastic dependence amongst the points of the process...[and these are]...difficult to disentangle" Diggle (2007).

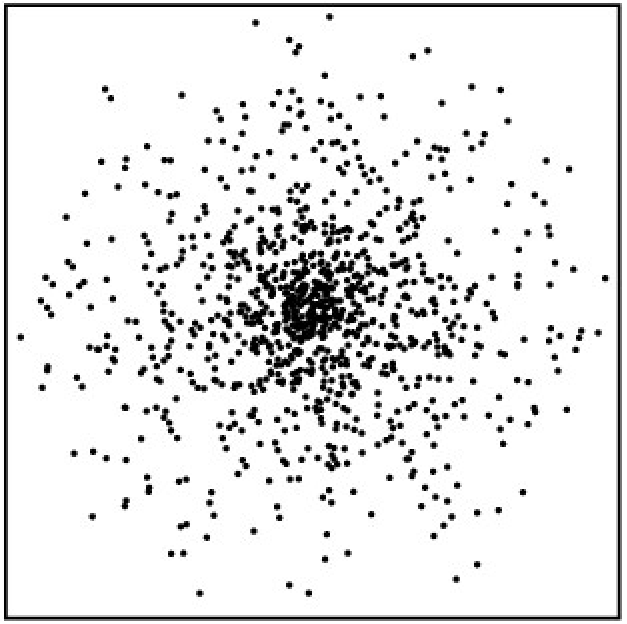
[1] B Blaszczyszyn et al., "Using Poisson Processes to Model Lattice Cellular Networks", INFOCOM 2013.
[2] M. Di Renzo et al., "The Intensity Matching Approach: A Tractable Stochastic Geometry Approx. to System-Level Analysis of Cellular Networks", IEEE Trans. Wireless Commun., Sep. 2016.
[3] M. Haenggi, "User point processes in cellular networks", IEEE Wireless Commun. Lett., Feb. 2017.
[4] A. Baddeley et al., Spatial Point Patterns – Methodology and Applications with R, Nov. 2015.

But, How To Define The Typical User ?

- **From empirical data, the BSs are modeled with stationary PPs**
  - **Ginibre PP (see Martin's paper)**
  - **Determinantal PP (see Jeff's and Francois's paper)**
  - **…**

- **I-PPPs are non-stationary PPs**
  - **How to define the typical user ?**





But, How To Define The Typical User ?

- From empirical data, the BSs are modeled with stationary PPs
  - Ginibre PP (see Martin's paper)
  - Determinantal PP (see Jeff's and Francois's paper)
  - …
- I-PPPs are non-stationary PPs
  - How to define the typical user ?

Approach and Interpretation

- We still assume that the ***original*** PP is stationary
- We "create" the inhomogeneity based on how the typical user of the ***original*** PP "sees" the network – "*the user's panorama*"

How Does The Typical User "See" The Network ?

- **Let us try to understand it from the Homogeneous PPP (H-PPP)**

$$\mathrm{P}_{\mathrm{cov}} = \int_{0}^{+\infty} \exp\left(-T\sigma^{2}P^{-1}\xi^{\alpha}\right)\mathrm{MGF}_{I_{agg}(r_0)}\left(P^{-1}T\xi^{\alpha}\right)\mathrm{PDF}_{r_0}(\xi)\,d\xi$$

- **The typical user sees the network through:**
  - **The distribution of the distance from the serving BS**
  - **The distribution of the aggregate interference from the other BSs**

How Does The Typical User "See" The Network ?

- **Let us try to understand it from the Homogeneous PPP (H-PPP)**

$$P_{\text{cov}} = \int_0^{+\infty} \exp\left(-T\sigma^2 P^{-1}\xi^{\alpha}\right) \text{MGF}_{I_{agg}(r_0)}\left(P^{-1}T\xi^{\alpha}\right) \text{PDF}_{r_0}(\xi)\, d\xi$$

- **The typical user sees the network through:**
  - **The distribution of the distance from the serving BS**
  - **The distribution of the aggregate interference from the other BSs**

Proposed Approach

We introduce a new I-PPP such that the network "seen" by a user located at the origin is the same as the network "seen" by the typical user of the original stationary PP, where "seen" means in terms of PDF and MGF (coverage, in general…)

How Does The Typical User "See" The Network ?

- **Let us try to understand it from the Homogeneous PPP (H-PPP)**

$$\mathrm{P_{cov}} = \int_0^{+\infty} \exp\left(-T\sigma^2 P^{-1}\xi^{\alpha}\right) \mathrm{MGF}_{I_{agg}(r_0)}\left(P^{-1}T\xi^{\alpha}\right) \mathrm{PDF}_{r_0}(\xi)\, d\xi$$

- **The typical user sees the network through:**
  - **The distribution of the distance from the serving BS**
  - **The distribution of the aggregate interference from the other BSs**

Proposed Approach

$$\mathrm{P_{cov}} \approx \int_0^{+\infty} \exp\left(-T\sigma^2 P^{-1}\xi^{\alpha}\right) \overline{\mathrm{MGF}}_{I_{agg}(r_0)}\left(P^{-1}T\xi^{\alpha}\right) \overline{\mathrm{PDF}}_{r_0}(\xi)\, d\xi$$

Given a PP, What Determines The PDF And The MGF ?

- **The PDF is determined by the Contact Distance Distribution (or F-Function):**

$$F(r) = \Pr\{\Phi(B[u,r]) > 0\} = \Pr\{\|u - \Phi\| < r\}$$

- **The MGF is determined, "*at the first order*", by the Ripley's K-Function:**

$$\lambda K(r) = \mathrm{E}^{!x}\{\Phi(B[x,r])\} = \mathrm{E}^{!x}\{\|x - \Phi\| < r\}$$

Given a PP, What Determines The PDF And The MGF ?

- **The PDF is determined by the Contact Distance Distribution (or F-Function):**

$$F(r) = \Pr\{\Phi(B[u,r]) > 0\} = \Pr\{\|u - \Phi\| < r\}$$

- **The MGF is determined, "*at the first order*", by the Ripley's K-Function:**

$$\lambda K(r) = \mathrm{E}^{!x}\{\Phi(B[x,r])\} = \mathrm{E}^{!x}\{\|x - \Phi\| < r\}$$

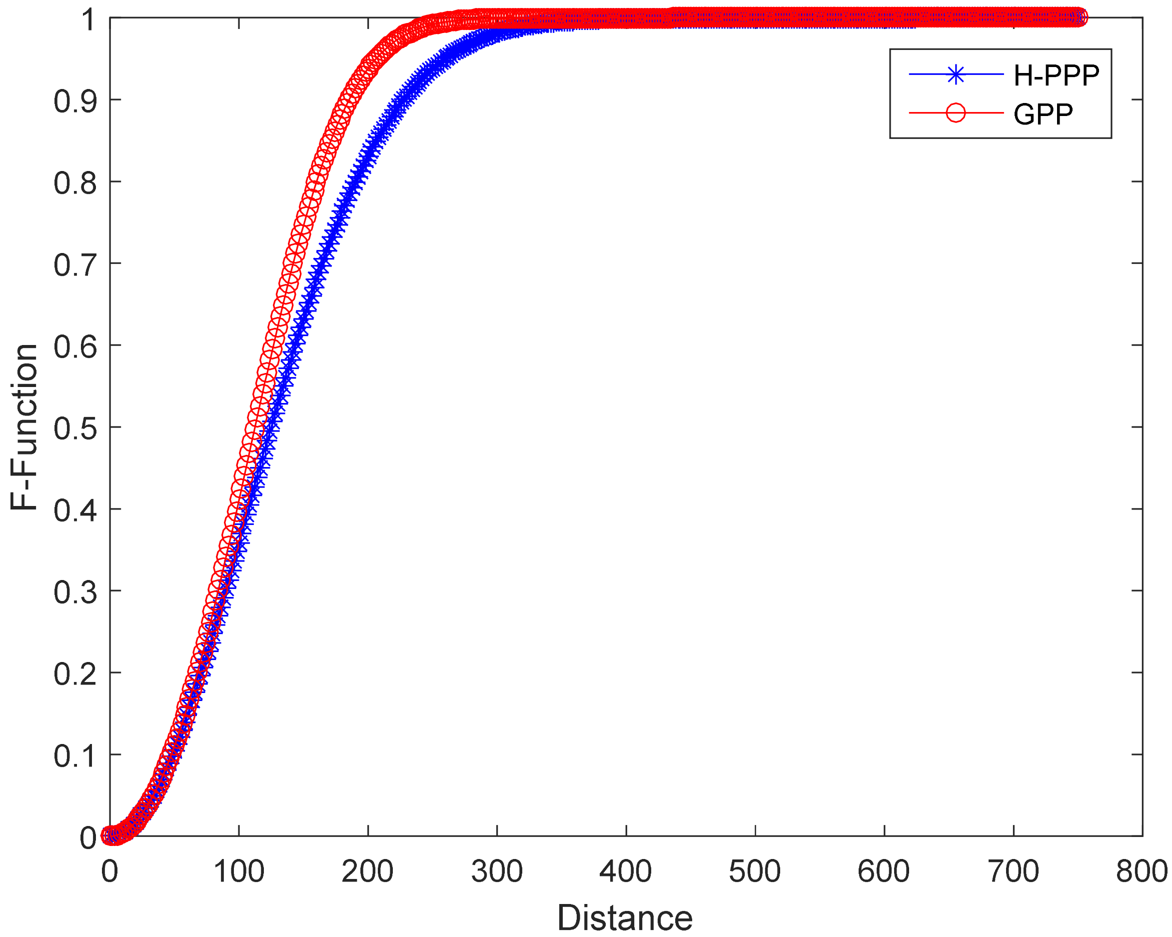
Proposed Approach

We introduce a new I-PPP, i.e., we create the inhomogeneity, such that the F-Function and K-Function of the original PP "are the same" (in the mean square error sense) as those of the approximating I-PPP

Is One I-PPP Sufficient ? Conflicting Trends…

… The Ginibre Point Process as an Example of Repulsive PP …

❑ F-Function



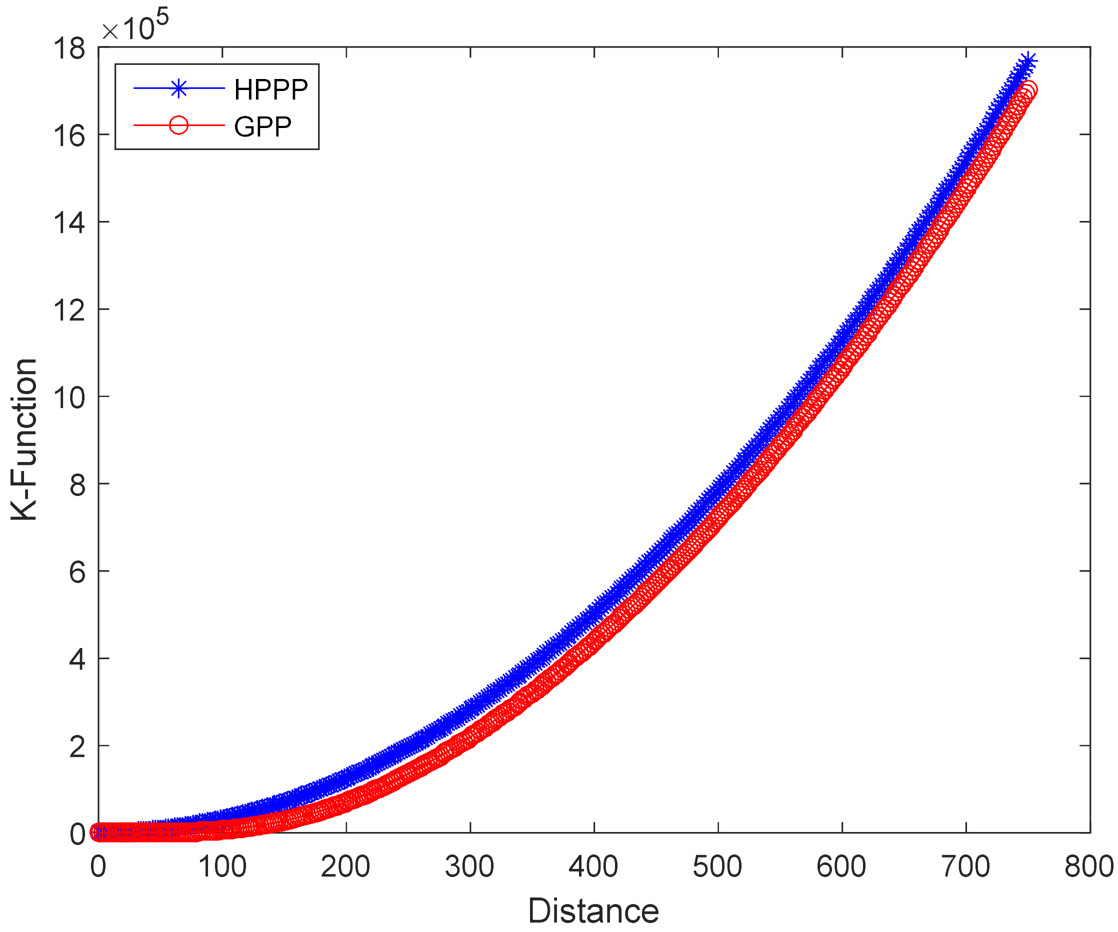
$$F_{\text{H-PPP}}\left(r\right) = 1 - \exp\left(-\lambda \pi r^2\right)$$

$$F_{\text{GPP}}\left(r\right) = 1 - \prod_{k=1}^{\infty}\left(1 - \beta \tilde{\gamma}\left(k, \frac{c}{\beta} r^2\right)\right)$$

Is One I-PPP Sufficient ? Conflicting Trends…

… The Ginibre Point Process as an Example of Repulsive PP …

❑ K-Function



$$K_{\text{H-PPP}}(r) = \pi r^2$$

$$K_{\text{GPP}}(r) = \pi r^2 - \frac{\beta\pi}{c}\left(1-\exp\left(-\frac{c}{\beta}r^2\right)\right)$$

Is One I-PPP Sufficient ? Conflicting Trends…

… The Ginibre Point Process as an Example of Repulsive PP …

- **F-Function**

$$F_{\mathrm{GPP}}(r) \geq F_{\mathrm{H\text{-}PPP}}(r)$$

- **K-Function:**

$$K_{\mathrm{GPP}}(r) \leq K_{\mathrm{H\text{-}PPP}}(r)$$

Is One I-PPP Sufficient ? Conflicting Trends…

… The Ginibre Point Process as an Example of Repulsive PP …

- **F-Function**

$$F_{\text{GPP}}\left(r\right) \geq F_{\text{H-PPP}}\left(r\right)$$

- **K-Function:**

$$K_{\text{GPP}}\left(r\right) \leq K_{\text{H-PPP}}\left(r\right)$$

Proposed Approach

We introduce TWO CONDITIONALLY-INDEPENDENT I-PPPs, the first one for approximating the F-Function and the second one for approximating, *<u>conditioned on the location of the serving BS</u>*, the K-Function

The Inhomogeneous Double Thinning (IDT) Approach

… In Simple Formulas …

- **The 1st I-PPP determines the location of the serving BS:**

$$\lambda_F(r) = \lambda t_F(r) \quad \text{such that} \quad \left\| F_{\text{GPP}}(r) - F_{\text{H-PPP}}(r) \right\| \to 0$$

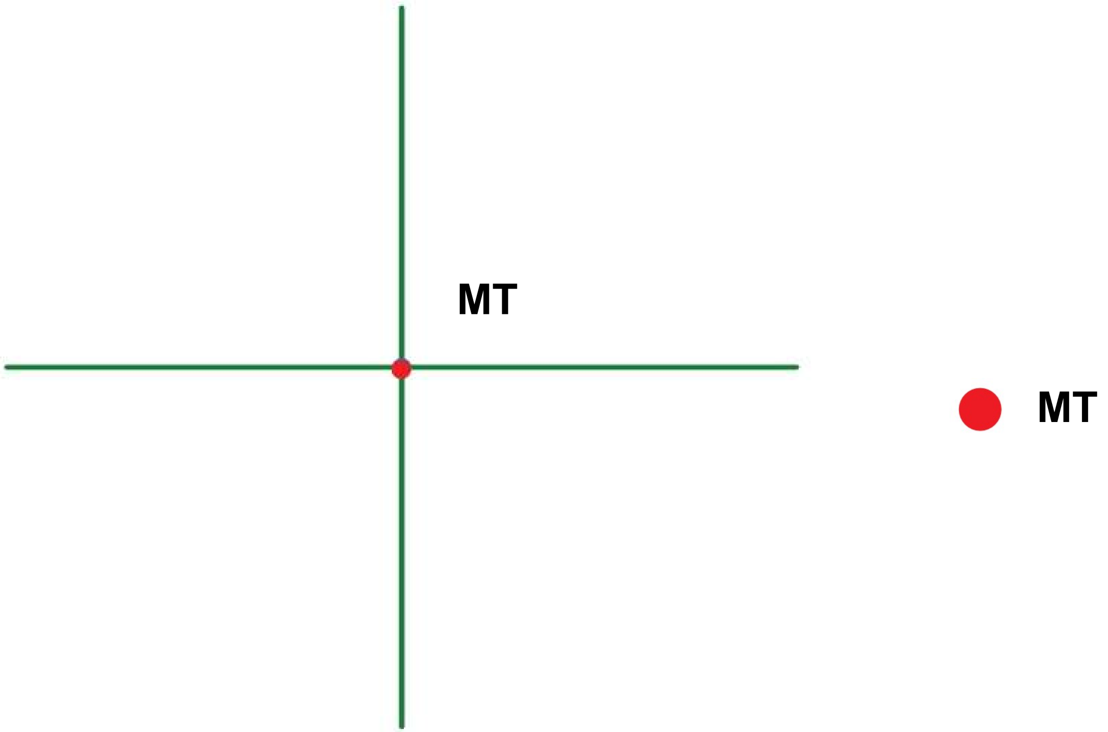
- **The 2nd I-PPP determines the locations of the interfering BSs:**

$$\lambda_K(r) = \lambda t_K(r) \quad \text{such that} \quad \left\| K_{\text{GPP}}(r) - K_{\text{H-PPP}}(r) \right\| \to 0$$

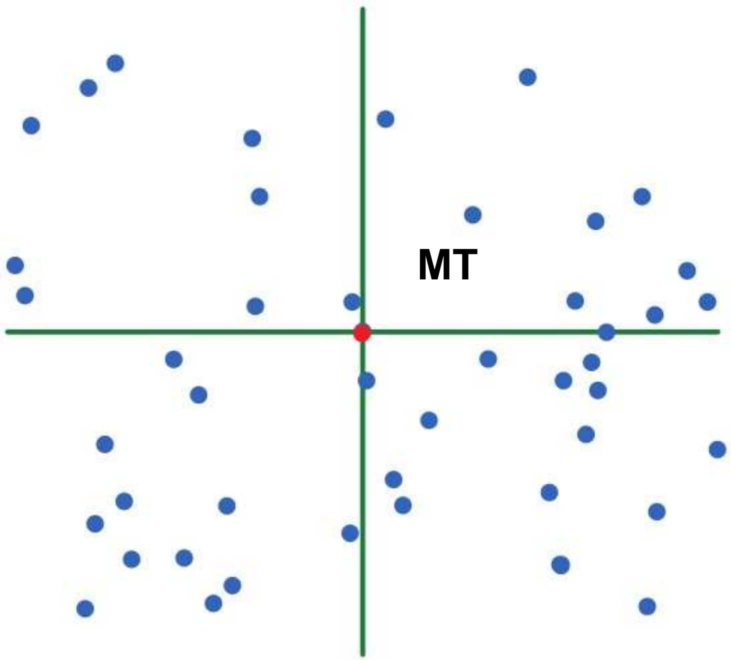
The Essence of the Proposed Approach

… To identify the two thinning functions reported above that provide accuracy and mathematical tractability …

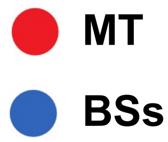
IDT – How It Works



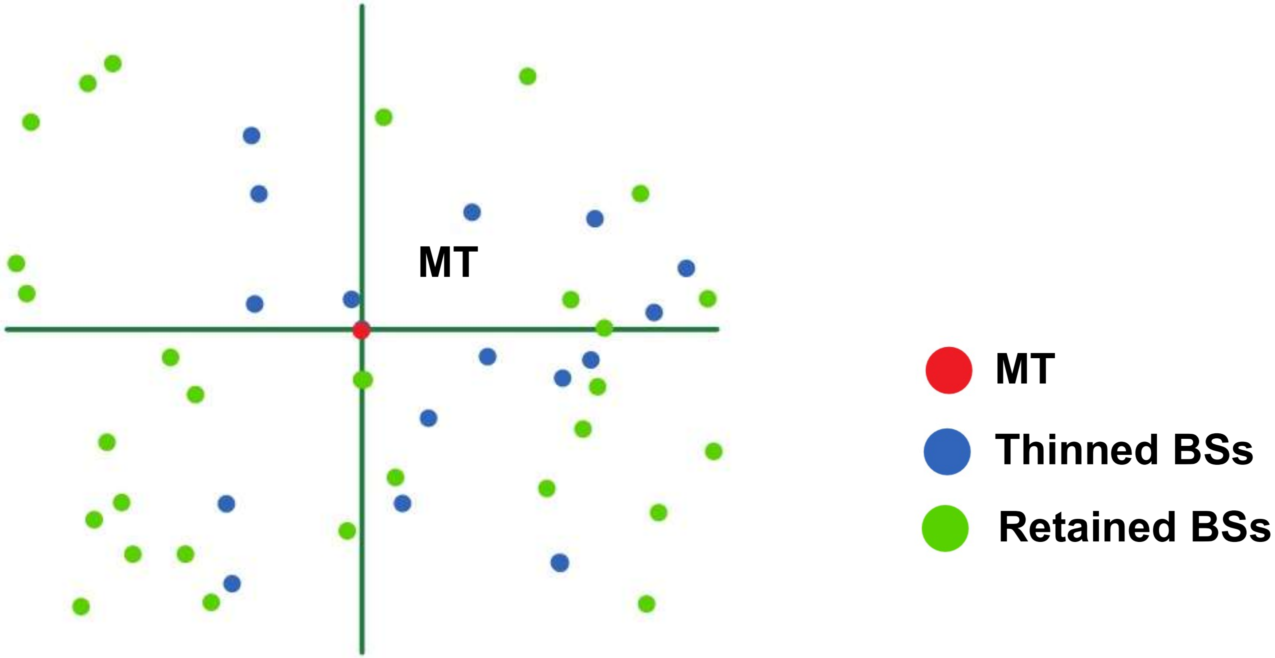
IDT – How It Works



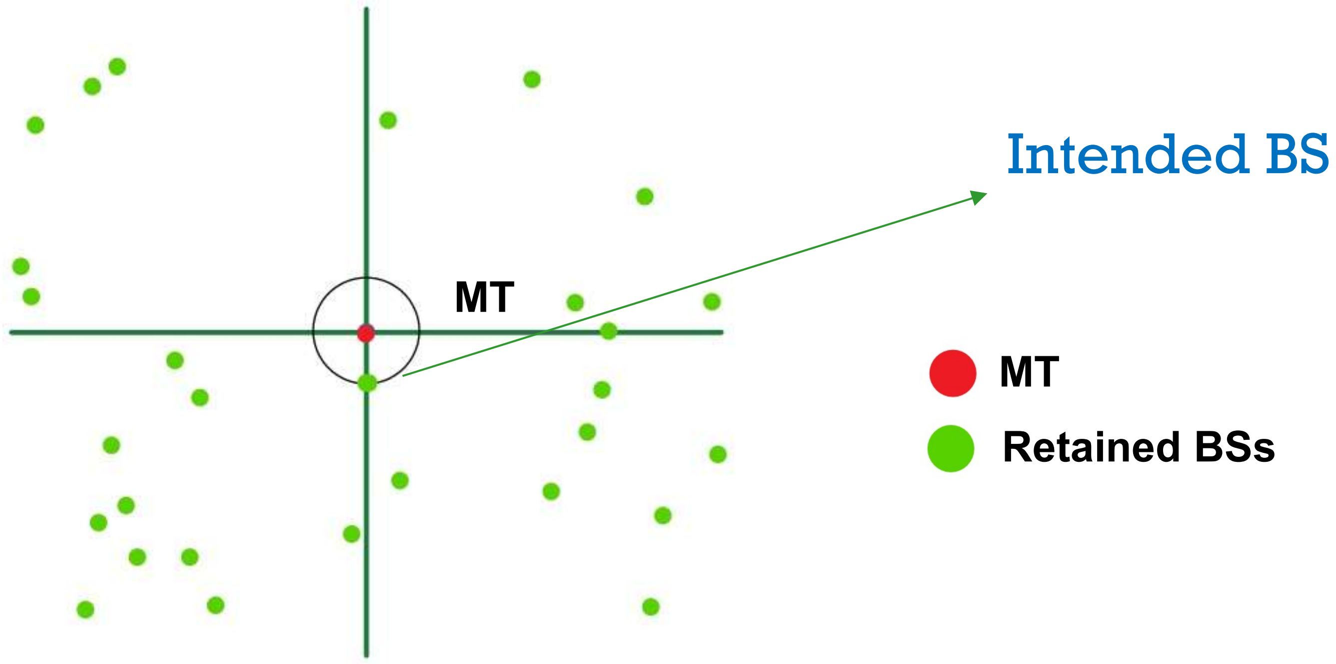
Homogeneous PPP with density λ_1



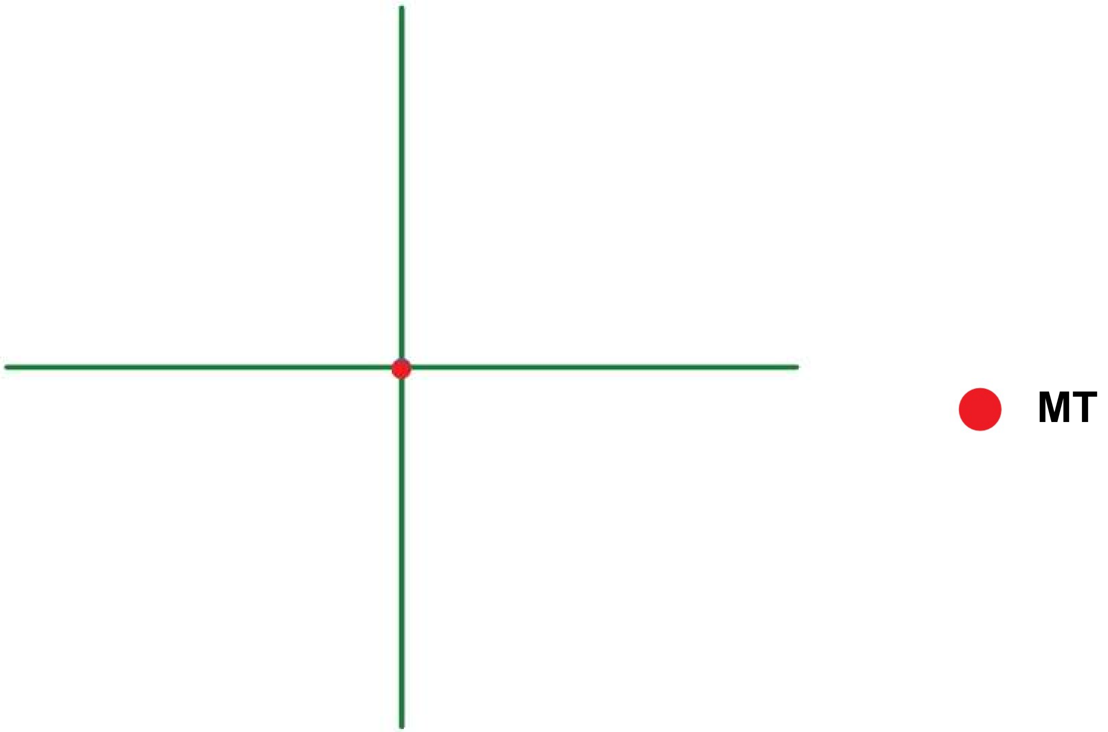
IDT – How It Works



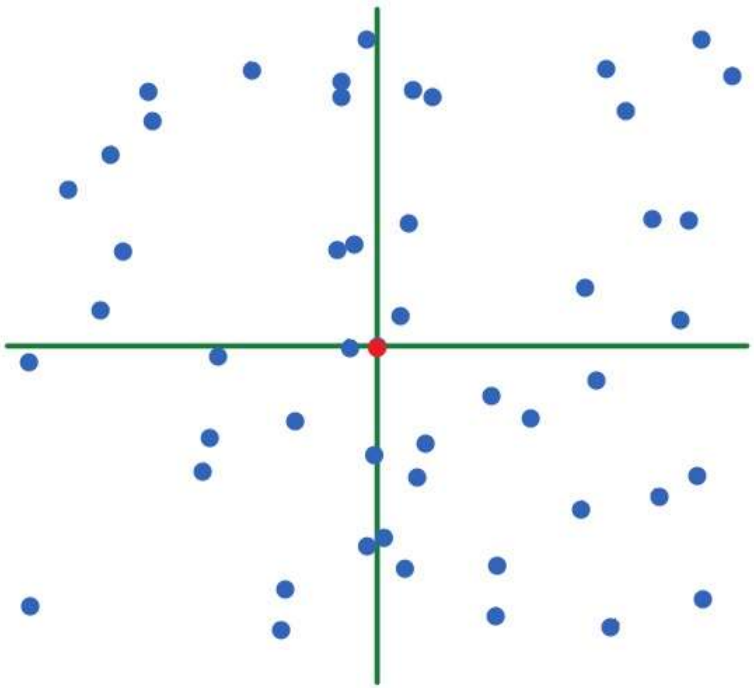
IDT – How It Works



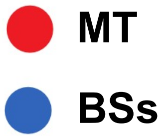
IDT – How It Works



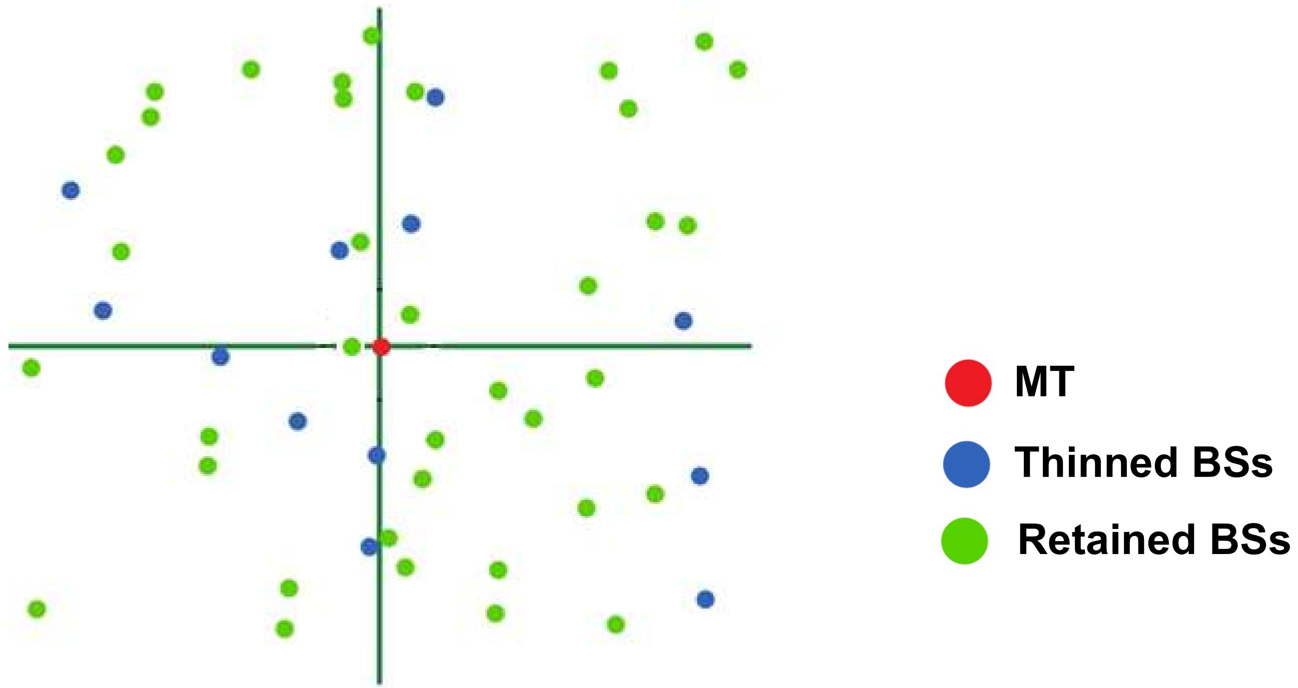
IDT – How It Works



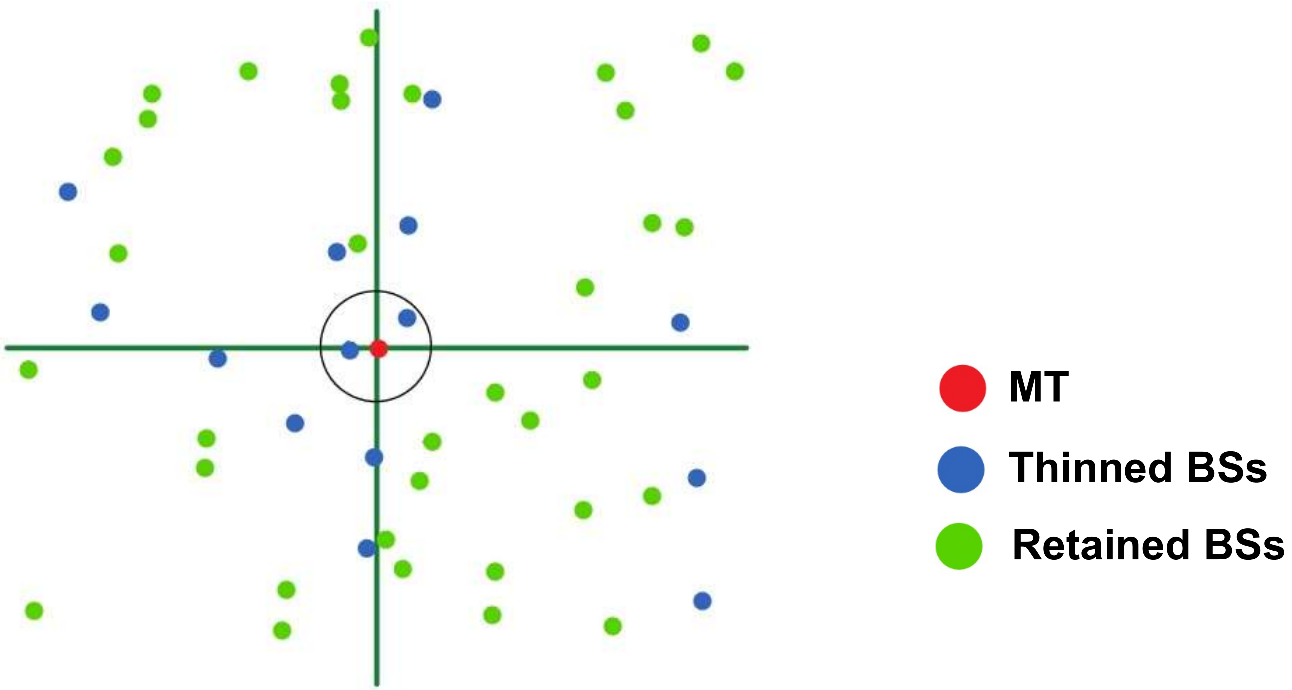
Homogeneous PPP
with density λ_2



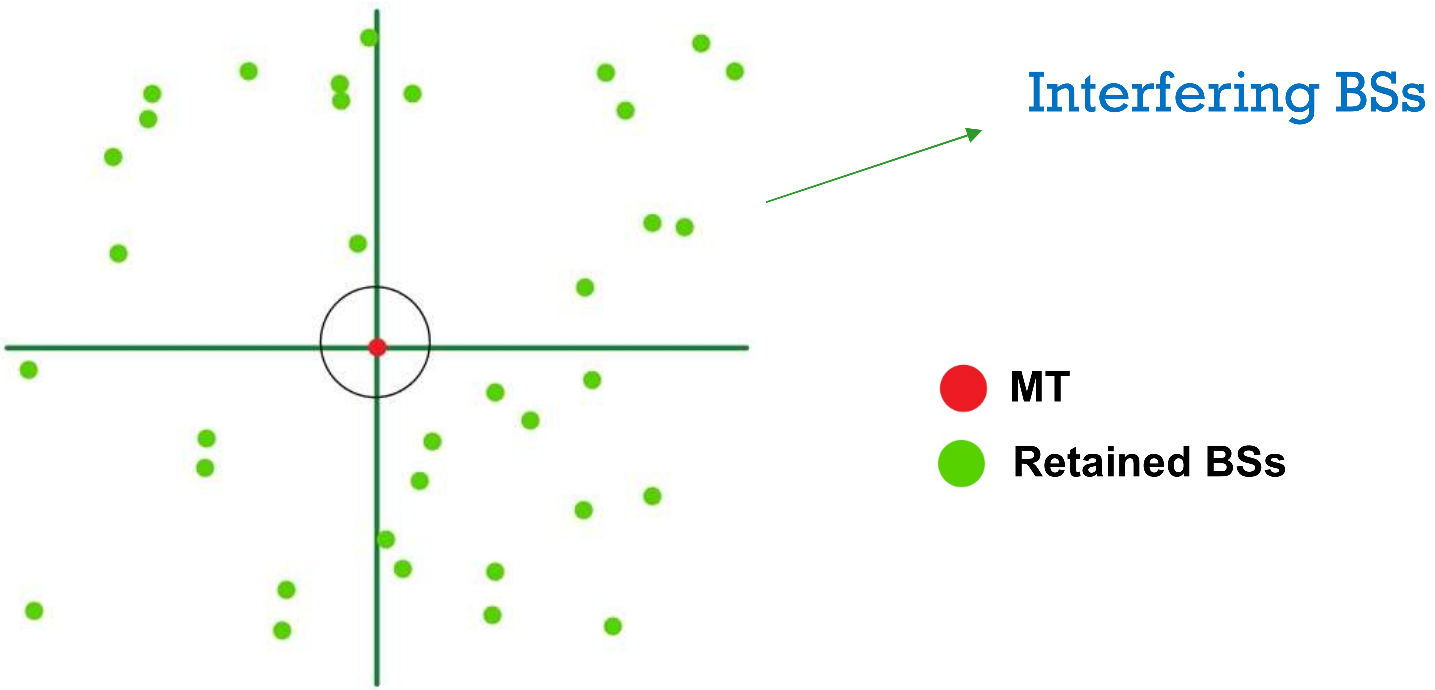
IDT – How It Works



IDT – How It Works



IDT – How It Works



What Is The "Magic" Thinning Function ?

- **F-Function and K-Function**

$$t_F(r) = f(r; a_F, b_F, c_F)$$

$$t_K(r) = f(r; a_K, b_K, c_K)$$

What Is The "Magic" Thinning Function ?

- **F-Function and K-Function**

$$t_F(r) = f(r; a_F, b_F, c_F)$$

$$t_K(r) = f(r; a_K, b_K, c_K)$$

… read the paper …

What Is The "Magic" Thinning Function ?

- **F-Function and K-Function**

$$t_F(r) = f(r; a_F, b_F, c_F)$$

$$t_K(r) = f(r; a_K, b_K, c_K)$$

… read the paper …

BUT

… the final result is …

IDT Approximation Of The Coverage Probability

$$\mathrm{P}_{\mathrm{cov}}(\tau;a_F,b_F,c_F;a_K,b_K,c_K)=\int_0^{\infty}\exp\left(-\frac{x^{\alpha}\tau\sigma_N^2}{P}\right)\exp\left(T_K(x)\right)S_F(x)dx$$

$$T_K(x)=2\pi\lambda\begin{pmatrix}-\frac{a_K}{3}\left(\frac{c_K-b_K}{a_K}\right)^3{}_2F_1\left(1,\frac{3}{\alpha},1+\frac{3}{\alpha},-\frac{k}{x\tau}\left(\frac{c_K-b_K}{a_K}\right)^{\alpha}\right)\\-\frac{b_K}{2}\left(\frac{c_K-b_K}{a_K}\right)^2{}_2F_1\left(1,\frac{2}{\alpha},1+\frac{2}{\alpha},-\frac{k}{x\tau}\left(\frac{c_K-b_K}{a_K}\right)^{\alpha}\right)\\+x^{\frac{3}{\alpha}}\frac{a_K}{3}k^{-\frac{3}{\alpha}}{}_2F_1\left(1,\frac{3}{\alpha},1+\frac{3}{\alpha},-\frac{1}{\tau}\right)+x^{\frac{2}{\alpha}}\frac{b_K}{2}k^{-\frac{2}{\alpha}}{}_2F_1\left(1,\frac{2}{\alpha},1+\frac{2}{\alpha},-\frac{1}{\tau}\right)\\+\frac{c_K}{2}\left(\frac{c_K-b_K}{a_K}\right)^2\left(1-{}_2F_1\left(-\frac{2}{\alpha},1,1-\frac{2}{\alpha},-\frac{x\tau}{k}\left(\frac{c_K-b_K}{a_K}\right)^{-\alpha}\right)\right)\end{pmatrix}H\left(\frac{c_K-b_K}{a_K}-\left(\frac{x}{k}\right)^{\frac{1}{\alpha}}\right)$$

$$+2\pi\lambda\left(x^{\frac{2}{\alpha}}\frac{c_K}{2}k^{-\frac{2}{\alpha}}\left(1-{}_2F_1\left(-\frac{2}{\alpha},1,1-\frac{2}{\alpha},-\tau\right)\right)\right)H\left(\left(\frac{x}{k}\right)^{\frac{1}{\alpha}}-\frac{c_K-b_K}{a_K}\right)$$

$$S_F(x)=\Lambda_F^{(1)}([0,x))\exp\left(-\Lambda_F([0,x))\right)$$

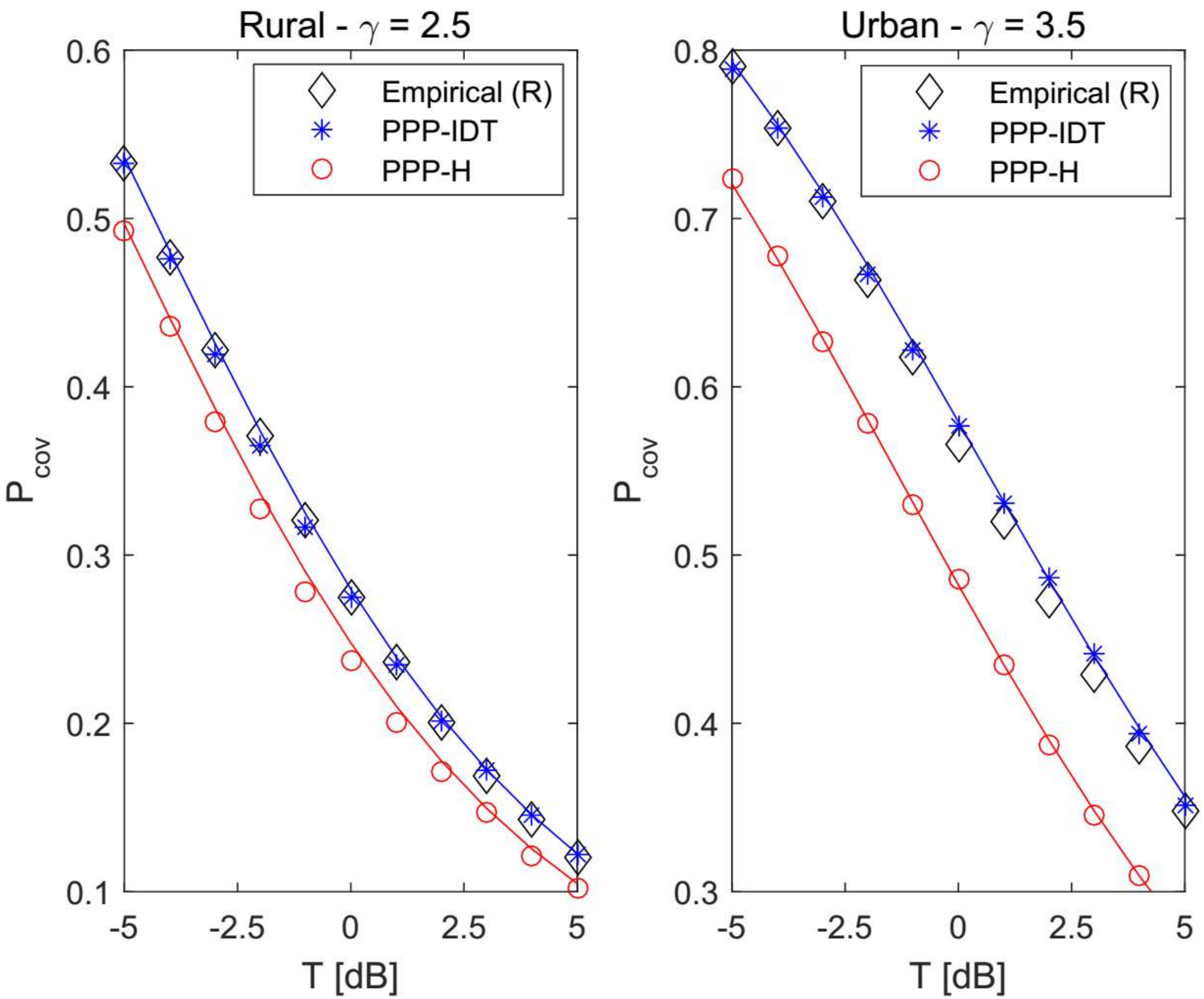
How Good Is The IDT Approach ?

Point Process	Parameters
DPP (Cauchy, Los Angeles)	$\lambda_{\mathrm{BS}} = 0.2346$ BS/km^2, $\alpha = 2.13$, $\mu = 3.344$, Area = 28×28 km^2
DPP (Cauchy, Houston)	$\lambda_{\mathrm{BS}} = 0.4490$ BS/km^2, $\alpha = 1.558$, $\mu = 3.424$, Area = 16×16 km^2
DPP (Gaussian, Los Angeles)	$\lambda_{\mathrm{BS}} = 0.2345$ BS/km^2, $\alpha = 1.165$, Area = 28×28 km^2
DPP (Gaussian, Houston)	$\lambda_{\mathrm{BS}} = 0.4492$ BS/km^2, $\alpha = 0.8417$, Area = 16×16 km^2
GPP (Urban, $\beta = 0.900$)	$\lambda_{\mathrm{BS}} = 31.56$ BS/km^2, Area = $3.784^2\pi$ km^2, $\gamma = 3.5$
GPP (Urban, $\beta = 0.925$)	$\lambda_{\mathrm{BS}} = 31.56$ BS/km^2, Area = $3.784^2\pi$ km^2, $\gamma = \{2.5, 4\}$
GPP (Urban, $\beta = 0.975$)	$\lambda_{\mathrm{BS}} = 31.56$ BS/km^2, Area = $3.784^2\pi$ km^2, $\gamma = 3$
GPP (Rural, $\beta = 0.200$)	$\lambda_{\mathrm{BS}} = 0.03056$ BS/km^2, Area = $124.578^2\pi$ km^2, $\gamma = 3.5$
GPP (Rural, $\beta = 0.225$)	$\lambda_{\mathrm{BS}} = 0.03056$ BS/km^2, Area = $124.578^2\pi$ km^2, $\gamma = \{3, 4\}$
GPP (Rural, $\beta = 0.375$)	$\lambda_{\mathrm{BS}} = 0.03056$ BS/km^2, Area = $124.578^2\pi$ km^2, $\gamma = 2.5$
Lattice PP	ISD = $\{100, 200, 300, 500\}$ m
Perturbed Lattice PP ($\mathrm{R_{cell}} = 100$ m)	$s = \{50, 80, 100, 200\}$ m
LGCP (Urban) (exponential covariance)	$\lambda_{\mathrm{BS}} = 4$ BS/km^2, $\beta = 0.03$, $\sigma^2 = 3.904$, $\mu = -0.5634$, Area = 20×20 km^2
LGCP (London) (exponential covariance)	$\lambda_{\mathrm{BS}} = 9.919$ BS/km^2, $\beta = 0.054$, $\sigma^2 = 2.0561$, $\mu = 1.2665$, Area = 6×6 km^2
LGCP (Warsaw) (exponential covariance)	$\lambda_{\mathrm{BS}} = 27.36$ BS/km^2, $\beta = 0.0288$, $\sigma^2 = 2.7228$, $\mu = 1.9477$, Area = 8×8 km^2
PHP	$\mathrm{R_{cell}} = 0.5$ km, $\lambda_{\mathrm{hole}} = 0.005\lambda_{\mathrm{BS}}$ BS/km^2, $\mathrm{R_{hole}} = 4$ km
PHP	$\mathrm{R_{cell}} = 0.1$ km, $\lambda_{\mathrm{hole}} = 0.005\lambda_{\mathrm{BS}}$ BS/km^2, $\mathrm{R_{hole}} = 0.8$ km

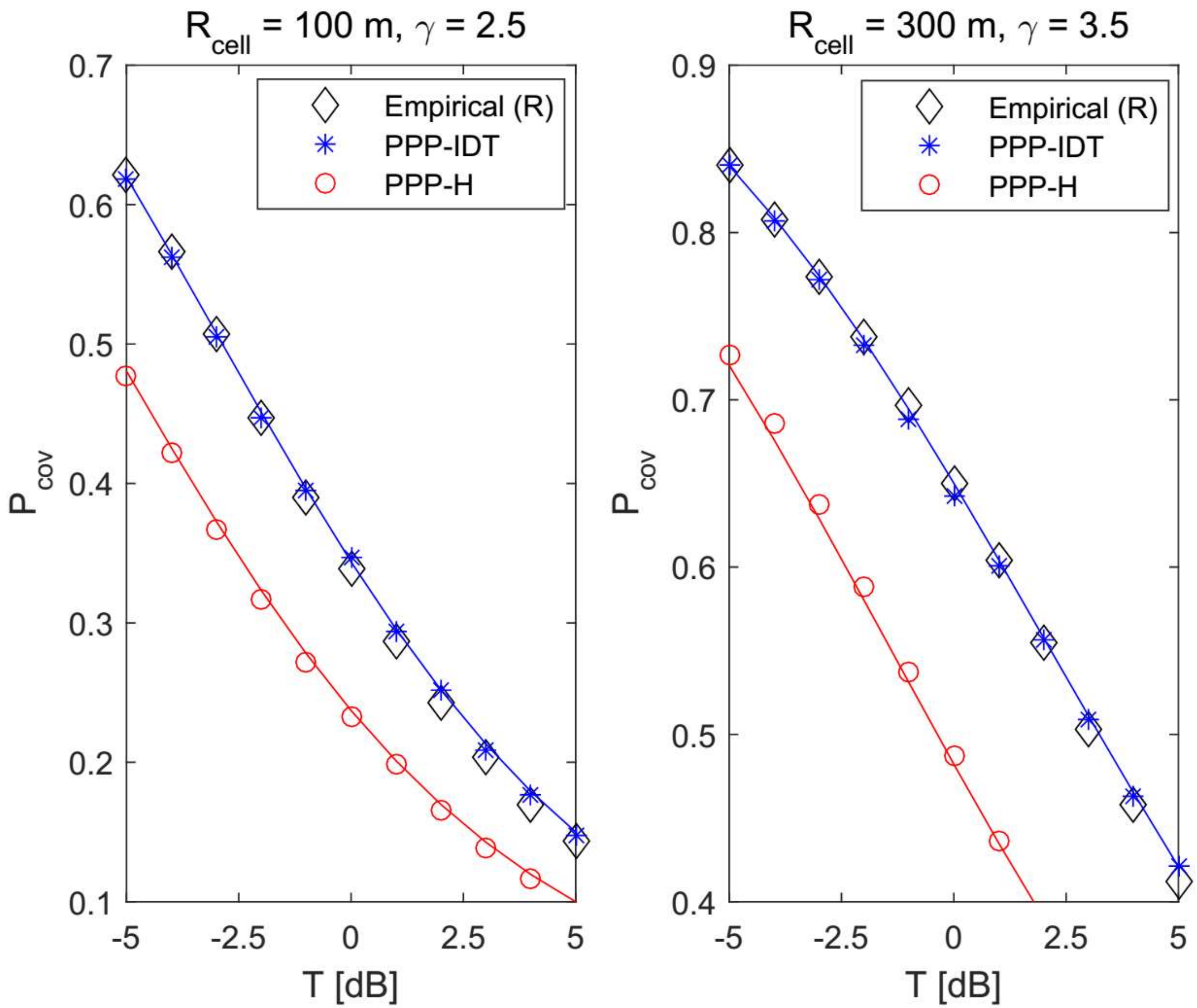
How Good Is The IDT Approach ?

Point Process	F-Function (a_F, b_F, c_F)	K-Function $(a_K,\ b_K,\ c_K)$
DPP (Cauchy, $\alpha = 2.13,\ \mu = 3.344$)	$a_F = 0.242792313440063 \cdot 10^{-3}$ $b_F = 1.00000000050633$ $c_F = 1.29043878627270$	$a_K = 0.665312376961223 \cdot 10^{-3}$ $b_K = 0.0800803505151663$ $c_K = 0.999966929758115$
DPP (Cauchy, $\alpha = 1.558,\ \mu = 3.424$)	$a_F = 0.329932369708525 \cdot 10^{-3}$ $b_F = 1.00000000203162$ $c_F = 1.31414585197489$	$a_K = 0.925771720753051 \cdot 10^{-3}$ $b_K = 0.0762137545180777$ $c_K = 0.999929848546426$
DPP (Gaussian, $\alpha = 1.165$)	$a_F = 0.257595475141932 \cdot 10^{-3}$ $b_F = 1.00000000000057$ $c_F = 1.46642395259731$	$a_K = 0.694526986147307 \cdot 10^{-3}$ $b_K = 0.00800453473629913$ $c_K = 0.999975490615518$
DPP (Gaussian, $\alpha = 0.8417$)	$a_F = 0.374139244964067 \cdot 10^{-3}$ $b_F = 1.00000000128277$ $c_F = 1.36923913017716$	$a_K = 0.963443744411944 \cdot 10^{-3}$ $b_K = 0.00642945511811224$ $c_K = 0.999947574776537$
GPP (Urban, $\beta = 0.900$)	$a_F = 0.00541280337683543$ $b_F = 1.00000000117948$ $c_F = 2.50742980678854$	$a_K = 0.00756610000002220$ $b_K = 0.0140800000000222$ $c_K = 0.999592878386863$
GPP (Urban, $\beta = 0.925$)	$a_F = 0.00556558536499347$ $b_F = 1.00000000213305$ $c_F = 2.52897621056288$	$a_K = 0.00839000000002220$ $b_K = 0.0200000000000222$ $c_K = 0.999432788402679$
GPP (Urban, $\beta = 0.975$)	$a_F = 0.00586932401892805$ $b_F = 1.00000000000032$ $c_F = 2.68047204883343$	$a_K = 0.0110000000000222$ $b_K = 0.0220000000000222$ $c_K = 0.999243424300274$
GPP (Rural, $\beta = 0.200$)	$a_F = 3.99946182077498 \cdot 10^{-5}$ $b_F = 1.01187371832462$ $c_F = 1.09948962377999$	$a_K = 0.000393029018145069$ $b_K = 0.0119099442149286$ $c_K = 0.999999841554118$
GPP (Rural, $\beta = 0.225$)	$a_F = 4.55473414133037 \cdot 10^{-5}$ $b_F = 1.01046879386340$ $c_F = 1.11306423054186$	$a_K = 0.000400570907629641$ $b_K = 0.0118898483733152$ $c_K = 0.999999810503409$
[illegible]	[illegible]	[illegible]

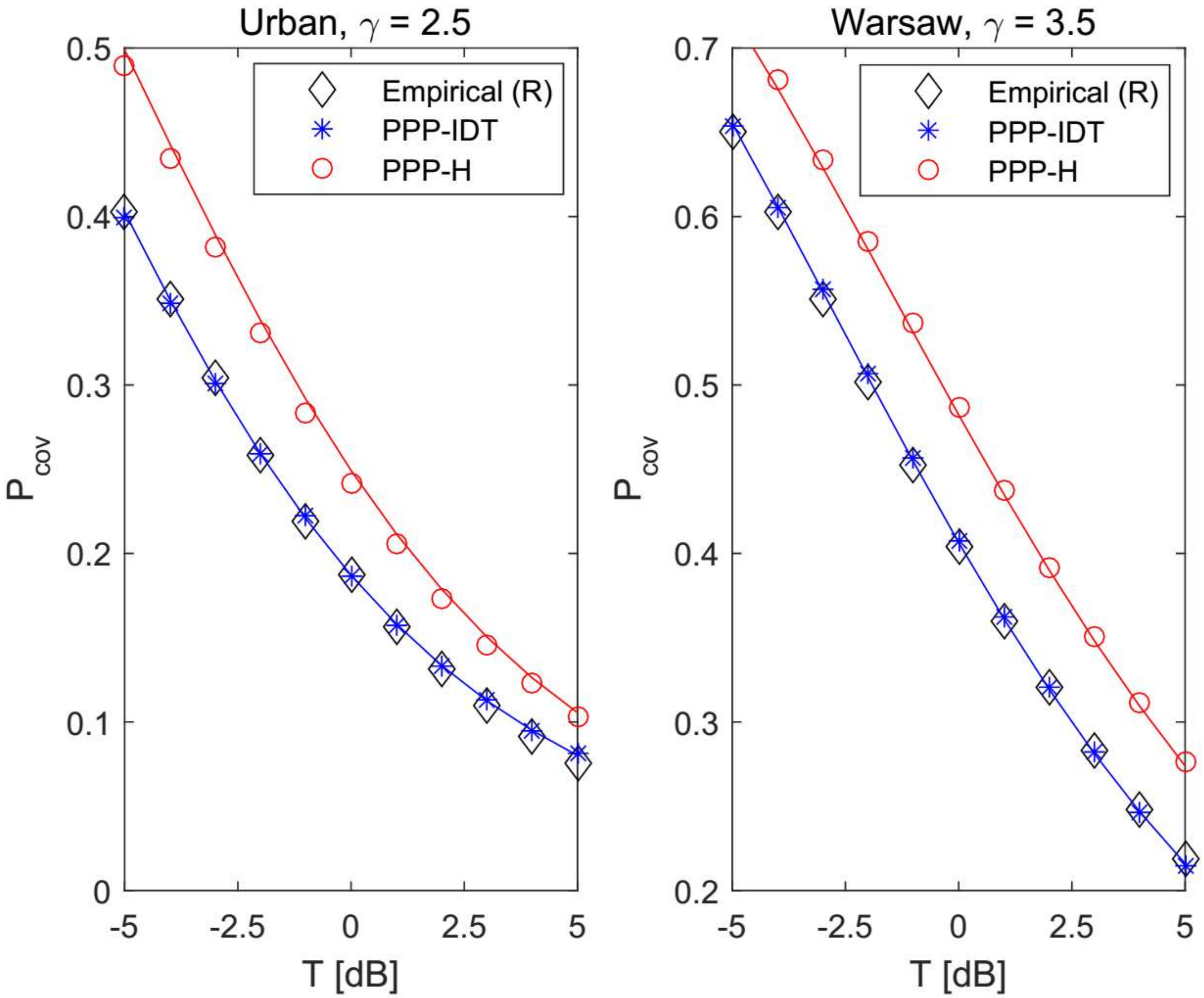
How Good Is The IDT Approach ? – GPP



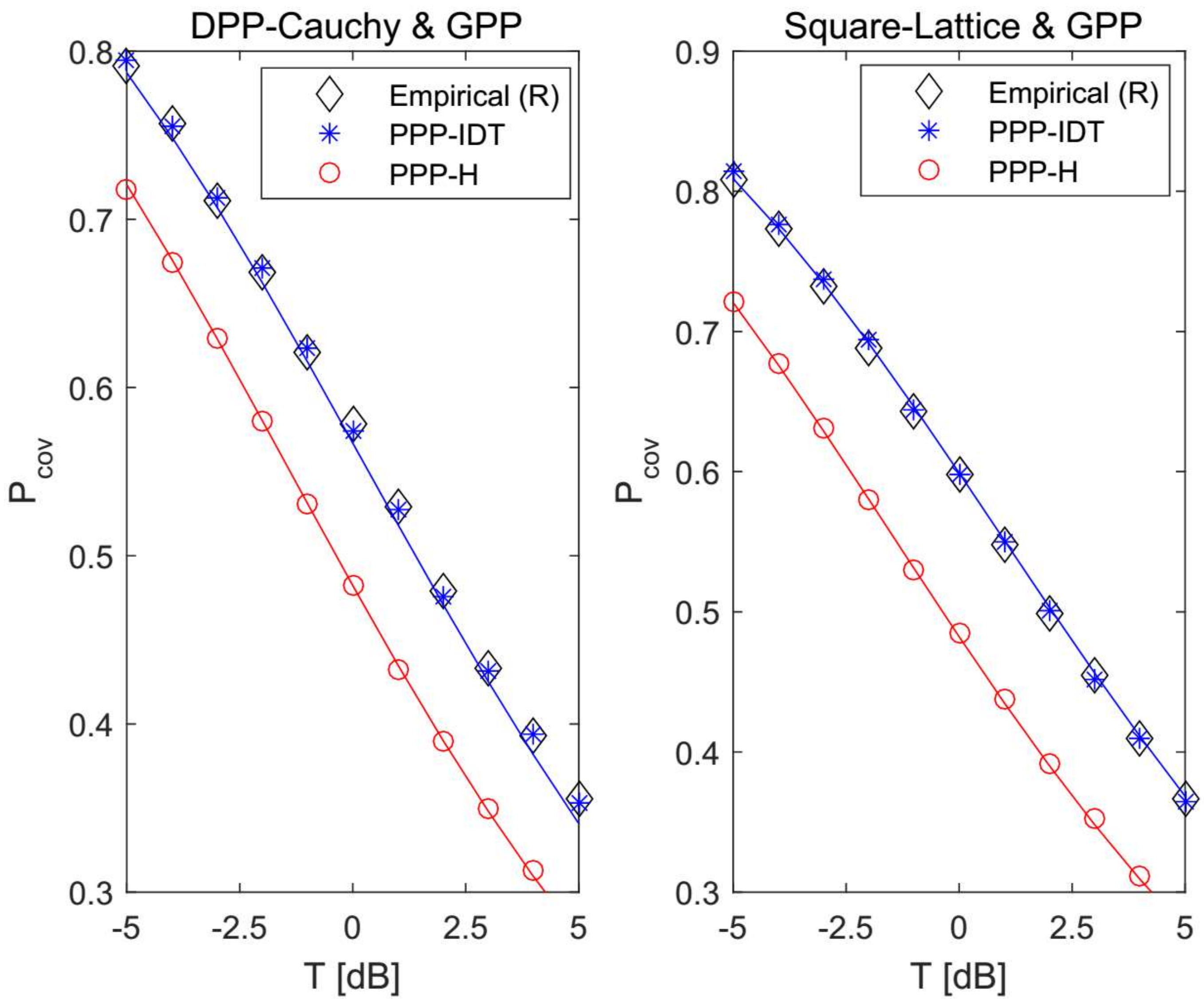
How Good Is The IDT Approach ? – Lattice



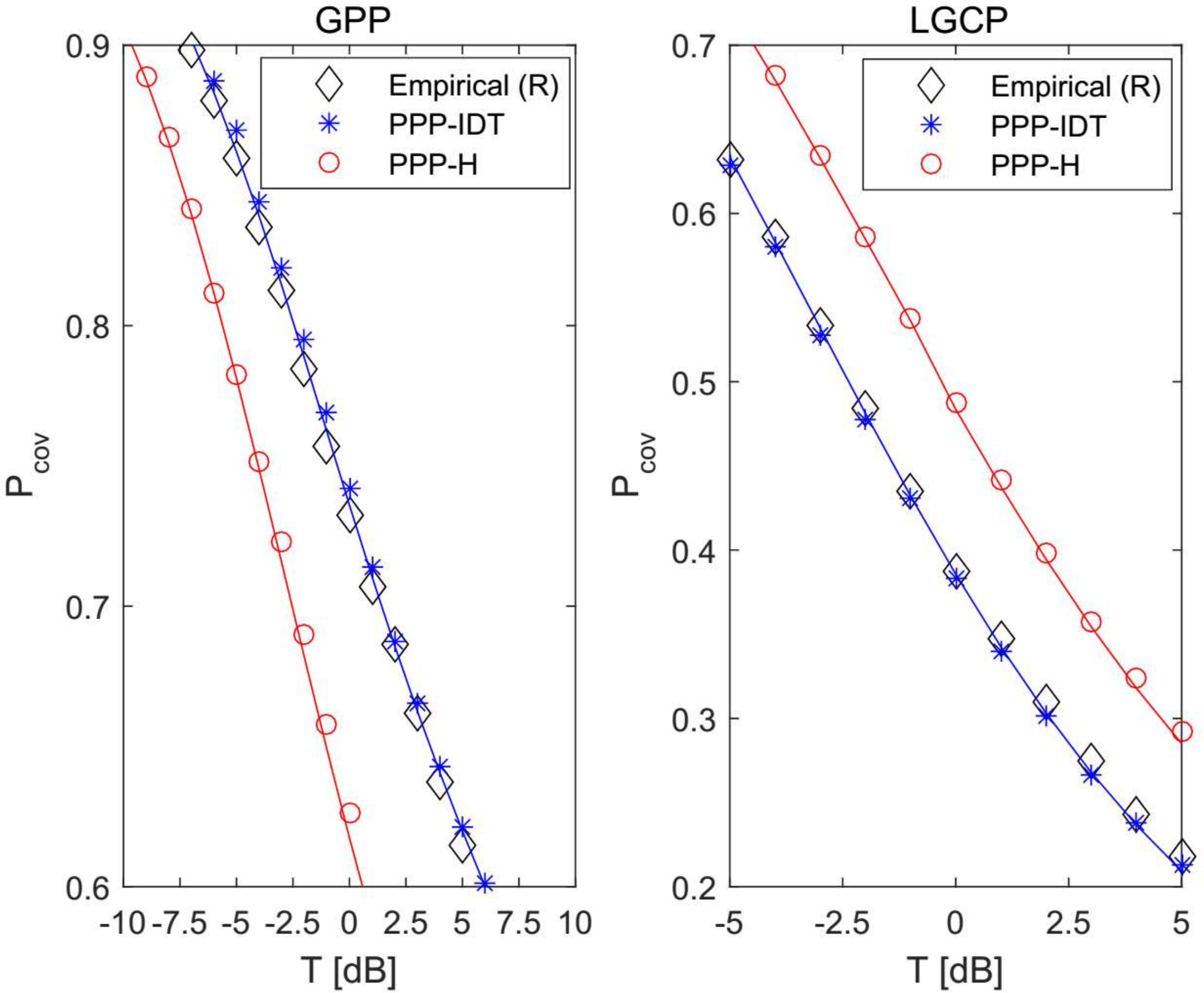
How Good Is The IDT Approach ? – LGCP



How Good Is The IDT Approach ? – Two-Tier



How Good Is The IDT Approach ? – LOS & NLOS



How General Is The IDT Approach ?

… We Tested it for a Variety of Scenarios …

- Single-tier repulsive and attractive PPs
- Multi-tier repulsive and attractive PPs
- Repulsive and attractive PPs with LOS/NLOS links
- Repulsive and attractive PPs with bounded path-loss models
- Repulsive and attractive PPs with elevated base stations
- and combinations of the above…

How To Use The IDT Approach ?

… It is NOT Just an Approximation …

- **Given a PP, you can use it as an approximation …**

- **Given some empirical data, you can estimate the F-Function, the K-Function, and "match" them to two I-PPPs …**

- **You can use it as a new parametric approach for modeling cellular networks, analyzing the trade-offs of different radio access technologies, optimizing their performance, etc. …**

- **It provides, in addition, a simple and easy-to-compute approach for estimating the "deployment gain" and for applying the ASAPPP approach (repulsive PPs) …**

Intrigued Enough ? … Here Is The Paper…

TRANSACTIONS ON WIRELESS COMMUNICATIONS

Inhomogeneous Double Thinning – Modeling and Analyzing Cellular Networks Using Inhomogeneous Poisson Point Processes

Marco Di Renzo, *Senior Member, IEEE*, and Shanshan Wang, *Student Member, IEEE*

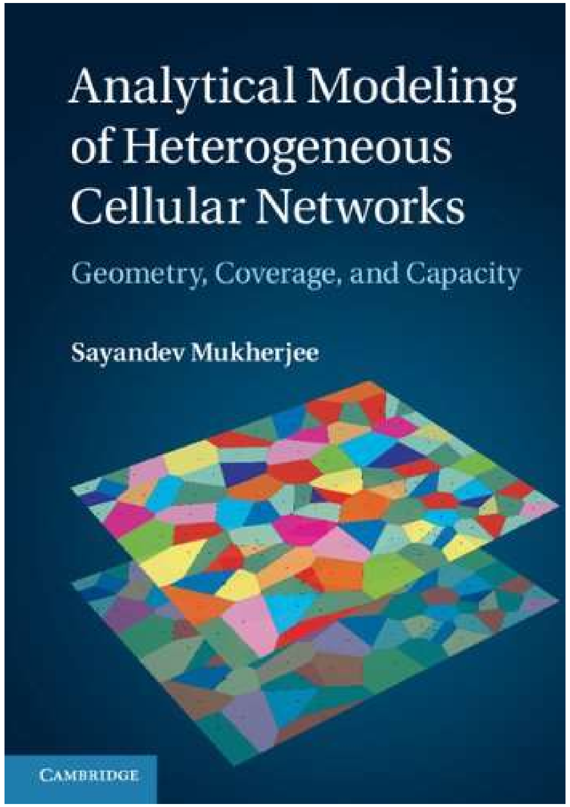
Abstract

In this paper, we introduce a new methodology for modeling and analyzing downlink cellular networks, where the Base Stations (BSs) constitute a stationary Point Process (PP) that exhibits some degree of interaction among the points, i.e., spatial inhibition (repulsiveness) or spatial aggregation (clustering). The proposed approach is based on the theory of Inhomogeneous Poisson Point Processes (I-PPP) and is referred to as Inhomogeneous Double Thinning (IDT) approach. In a stationary PP, the

… final thoughts and takes …

On The Role Of Stochastic Geometry

1.4 The role of analytic modeling



The analytic-modeling-based investigation of deployment scenarios has two phases. In the first phase, we use probabilistic models for the locations of the BSs to determine analytic expressions for the CCDF of the SINR in the deployment region. In other words, the use of a stochastic model (Poisson point process, or PPP) for the locations of the BSs allows us to write an analytic expression for the expectation of (1.4) with respect to either the joint distribution of $(R_0, R_1, \ldots, R_M)$ or the conditional joint distribution of $(R_1, \ldots, R_M)$ given $R_0 = r_0$. Further, these results can be extended to arbitrary fading distributions and arbitrary numbers of tiers of BSs.

As we shall see, this has the benefit of providing insights into the *combinations* of deployment parameters that affect the CCDF of the SINR, and therefore the different sets of deployment parameters that are *equivalent* in that they yield the same CCDF of the SINR. This analytic phase allows us to sift through the large space of combinations of deployment parameters to settle quickly on certain equivalence classes of deployment parameters, each class corresponding to some desired CCDF of the SINR. The service provider may then choose a set of deployment parameters from one of these equivalence classes based on its economic utility function.

In the next phase of the network design, the shortlist of deployment scenarios (as defined by the deployment parameters) chosen in the first phase may be investigated in depth via simulation. This effectively uses the power of detailed simulation, incorporating all relevant aspects whose behavior and impact on performance is to be investigated, for a few selected deployment scenarios.

S. Mukherjee: "Analytical Modeling of Heterogeneous Cellular Networks", Cambridge University Press, January 2014.

… IEEE TCOM Nov. 2011 – now …

THE IMPACT OF COMMUNICATION THEORY ON TECHNOLOGY DEVELOPMENT:

IS THE BEST BEHIND US, OR AHEAD?"

aka "Is Communication Theory Dead"?

PLENARY PANEL: "The Impact of Communication Theory on Technology Development: Is the Best Behind us or Ahead?", IEEE Communications Theory Workshop, May 2010.

Stochastic Geometry For Commun. – Bottom Line

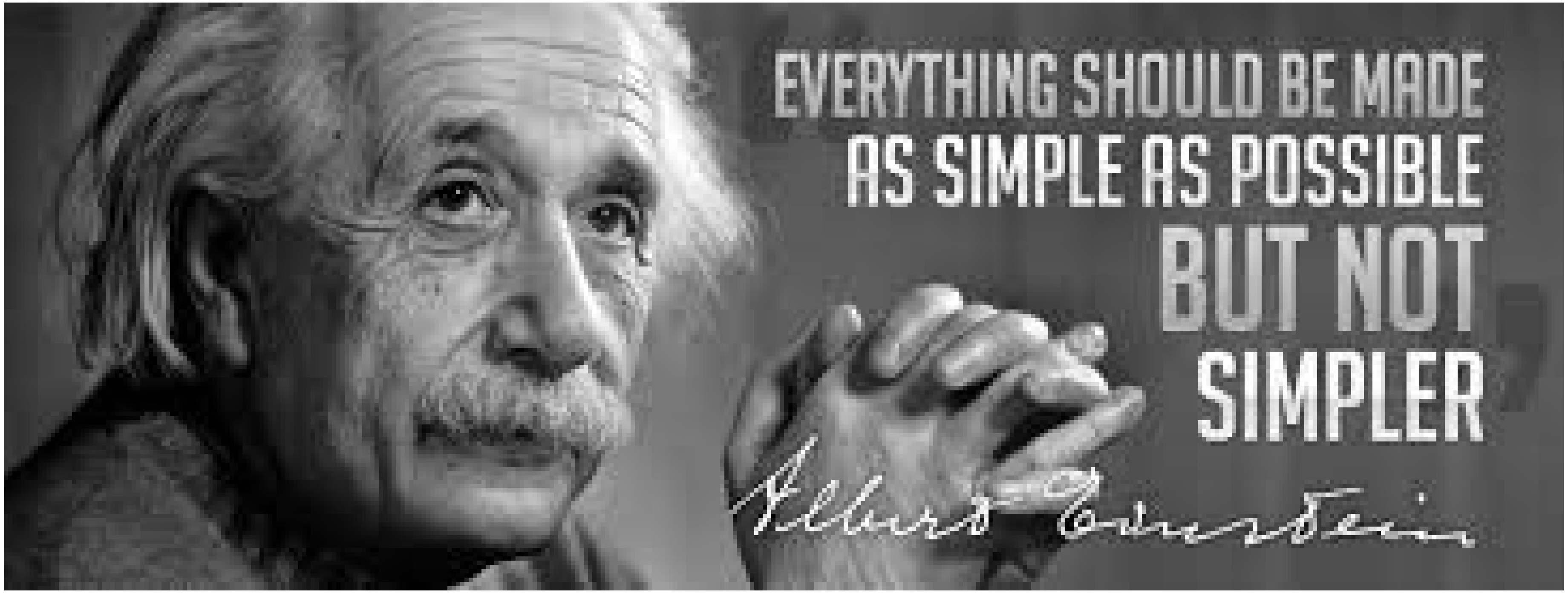
- Stochastic geometry provides us with suitable mathematical models and appropriate statistical methods for analyzing and optimizing heterogeneous (future deployments) cellular networks

- It is instrumental for identifying subsets of candidate (feasible, relevant) solutions based on which finer-grained simulations can be conducted, thus significantly reducing the time and cost of optimizing complex communication networks

- Its application to cellular network designs, however, necessitates to abandon conventional and comfortable assumptions
  - Poisson (complete spatially random) models
  - Simplistic path-loss models
  - Simplistic transmission schemes
  - …

- Relying upon adequate approximations to avoid oversimplifying the system model is not an option. CAUTION is, however, mandatory

Stochastic Geometry For Commun. – Bottom Line



Stochastic Geometry For Cellular Nets – YouTube Video

https://youtu.be/MB8IvOYYvB0

… System-Level Optimization …

(latest results – just submitted)

System-Level Energy Efficiency Optimization

System-Level Modeling and Optimization of the Energy Efficiency in Cellular Networks – A Stochastic Geometry Framework

Marco Di Renzo, *Senior Member, IEEE*, Alessio Zappone, *Senior Member, IEEE*, Thanh Tu Lam, *Student Member, IEEE*, and Mérouane Debbah, *Fellow, IEEE*

Abstract

In this paper, we analyze and optimize the energy efficiency of downlink cellular networks. With the aid of tools from stochastic geometry, we introduce a new closed-form mathematical expression of the potential spectral efficiency (bit/sec/m^2) in the interference-limited regime. Unlike currently available mathematical frameworks, the proposed analytical formulation explicitly depends on the transmit power and density of the base stations. This is obtained by generalizing the definition of coverage probability

STORNS: Stochastic Radio Access Network Slicing

Vincenzo Sciancalepore, Marco Di Renzo, Xavier Costa-Perez
NEC Europe Labs (Germany) & CNRS / Paris-Saclay University (France)

Abstract*—Network virtualization and softwarization are key enablers of the novel *network slicing* concept. Network slicing introduces new business models such as allowing telecom providers to lease virtualized slices of their infrastructure to *tenants*, e.g., industry verticals (automotive, e-health, factories, etc.). However, this new paradigm poses a major challenge when applied to radio access networks (RAN): *How to achieve revenue maximization while meeting the diverse service level agreements (SLAs) requested by the tenants?

In this paper, we propose a new analytical framework, based on stochastic geometry theory, to model realistic RANs that leverage the business opportunities offered by network slicing. Moreover, we mathematically prove the benefits of network as compared to un-sliced RANs. Based on this finding, we design a new admission control functional block, STORNS, which takes decisions considering per slice SLA guaranteed average experienced throughput. A radio resource allocation strategy is introduced to optimally allocate transmit power and bandwidth (i.e., a slice of radio access resources) to the users of each tenant of the cellular network. Numerical results are illustrated to validate our proposed solution in terms of potential spectral efficiency and to compare it against un-sliced RANs.

I. Introduction

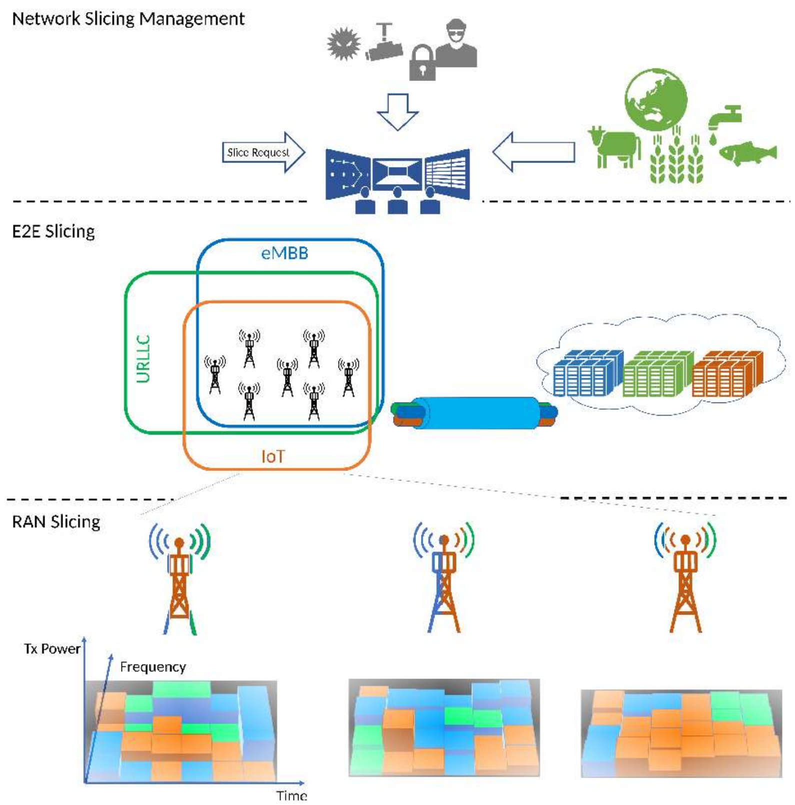


Fig. 1: Illustration of the Network Slicing Concept

… back tomorrow about this …

2 EU-Funded Ph.D. Scholarships Available

- **Ph.D. Scholarship CNRS-1:**

  "Modeling mmWave cellular communications via hyper dense small cell deployments"

- **This research project is concerned with assessing fundamental trade-offs between spectrum and infrastructure sharing, by combining SDN and NFV principles and to quantify the resulting network performance gains**

- **Ph.D. Scholarship CNRS-2:**

  "Device- or user-centric wireless access and multi-connectivity design at mmWave frequencies"

- **This research project is concerned with rethinking the cell-centric cellular architecture by developing user-centric concepts that maximizes throughput and minimizes power consumption, to enable multiple per-user connections with several access points for infinite capacity and zero latency perception**

… best way to apply …
email me: marco.di.renzo@gmail.com

Thank You for Your Attention

- **ETN-5Gwireless (H2020-MCSA, grant 641985)**
- **ETN-5Gaura (H2020-MCSA, grant 675806)**

Two European Training Networks on 5G Wireless Networks

Marco Di Renzo, Ph.D., H.D.R.

Chargé de Recherche CNRS (Associate Professor)

Associate EiC, IEEE Communications Letters

Editor, IEEE Transactions on Communications

Editor, IEEE Transactions on Wireless Communications

Distinguished Lecturer, IEEE Communications Society

Distinguished Lecturer, IEEE Vehicular Technol. Society

Paris-Saclay University

Laboratory of Signals and Systems (L2S) – UMR-8506

CNRS – CentraleSupelec – University Paris-Sud

3 rue Joliot-Curie, 91192 Gif-sur-Yvette (Paris), France

E-Mail: marco.direnzo@l2s.centralesupelec.fr