Stochastic Geometry Modeling of Cellular Networks: From System-Level Analysis to System-Level Optimization

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Oxford University, September 2017

This Talk: To Analyze Is Good, To Optimize Is Better…

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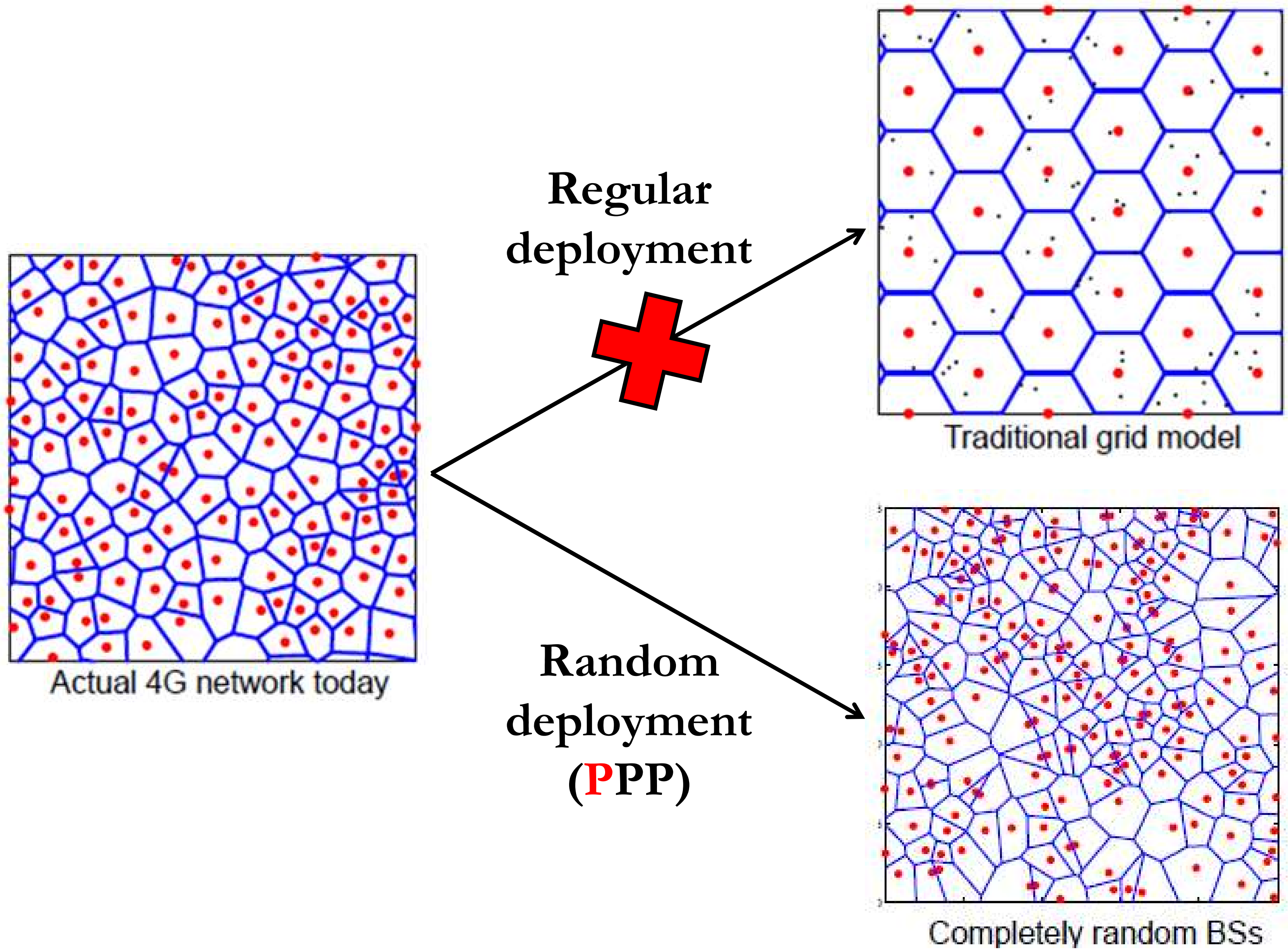
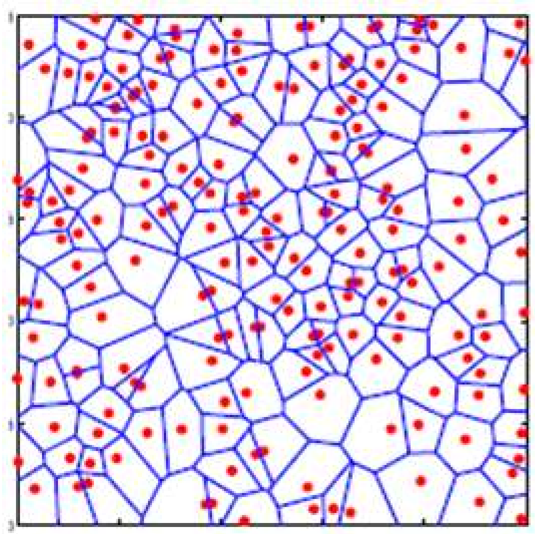
- **Part I: Recap and Motivation Based on Yesterday's Talk**
  - **Stochastic Geometry - A powerful tool to model cellular networks**
  - **The modeling assumptions matter and we need to be careful**
  - **Are current analytical frameworks suitable for optimization ?**

- **Part II: A New Definition of Coverage Probability**
  - **Rationale, motivation, and usefulness**
  - **Application to optimize the energy efficiency of cellular networks**
  - **Application to optimize cellular networks with network slicing**
  - **… and more …**

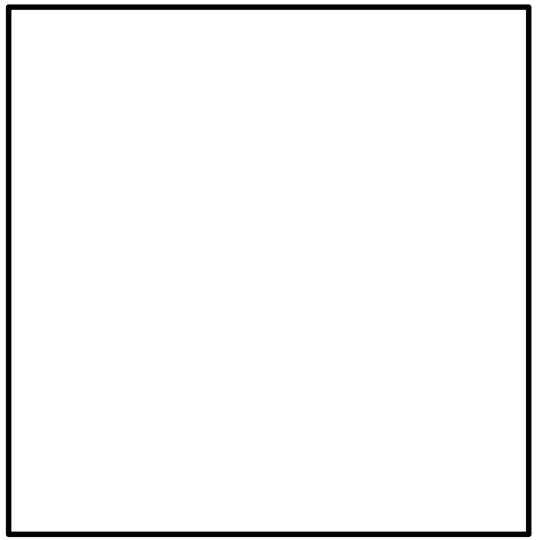
Recap on Yesterday's Talk: A New Model Is Needed



Recap on Yesterday's Talk: SG Is The Only Option



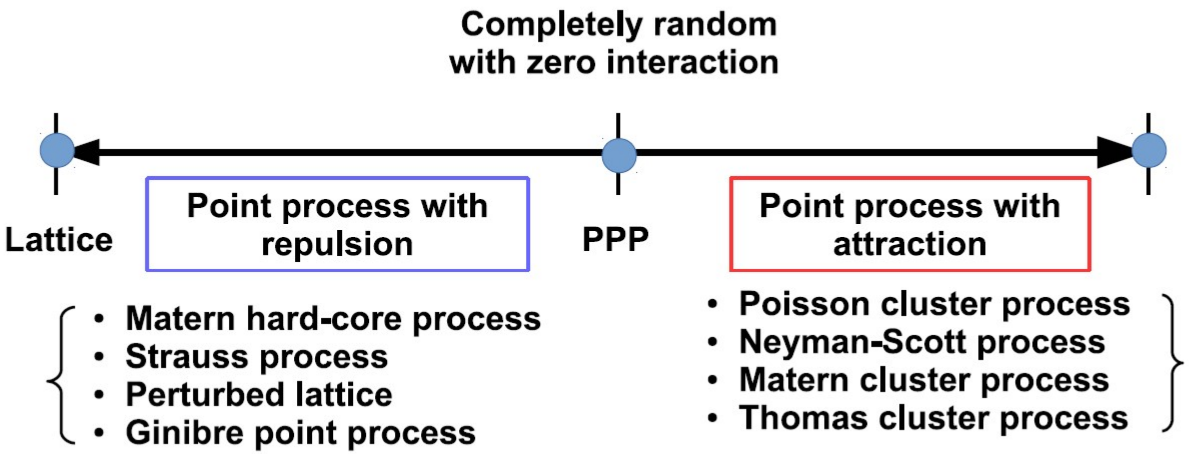
WITH
Stochastic
Geometry

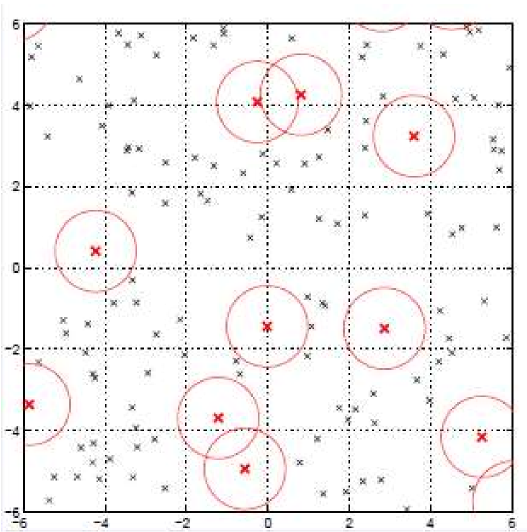


WITHOUT
Stochastic
Geometry

Modeling WITHOUT Stochastic Geometry is just "Point-Less"

Recap on Yesterday's Talk: SG Is A Rich Math. Toolbox



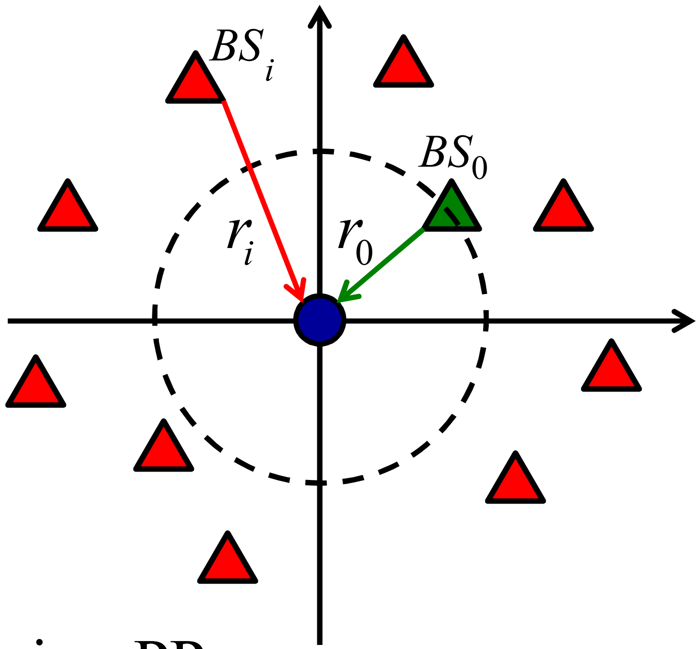


Matern Hard-Core PP

Take a homogeneous PPP and remove any pairs of points that are closer to each other than a predefined minimum distance R

Y. J. Chun, M. O. Hasna, A. Ghrayeb, and M. Di Renzo, "On modeling heterogeneous wireless networks using non-Poisson point processes", IEEE Commun. Mag., submitted. [Online]. Available: http://arxiv.org/pdf/1506.06296.pdf.

Recap on Yesterday's Talk: How SG Works



Φ is a PP

$$\mathrm{P_{cov}} = \Pr\{\mathrm{SINR} > T\}$$

$$\mathrm{SINR} = \frac{P|h_o|^2 r_o^{-\alpha}}{\sigma^2 + I_{agg}(r_0)}$$

$$I_{agg}(r_0) = \sum_{i \in \Phi \setminus BS_0} P|h_i|^2 r_i^{-\alpha}$$

$$\mathrm{P_{cov}} = \Pr\left\{\frac{P|h_o|^2 r_o^{-\alpha}}{\sigma^2 + I_{agg}(r_0)} > T\right\} = \ldots$$

Recap on Yesterday's Talk: How SG Works

$$\mathrm{P_{cov}} = \mathrm{E}_{r_0}\left\{ \exp\left(-T\sigma^2 P^{-1} r_o^{\alpha}\right) \mathrm{MGF}_{I_{agg}(r_0)}\left(P^{-1} T r_o^{\alpha}\right) \right\}$$

$$= \int_0^{+\infty} \exp\left(-T\sigma^2 P^{-1} \xi^{\alpha}\right) \mathrm{MGF}_{I_{agg}(r_0)}\left(P^{-1} T \xi^{\alpha}\right) \mathrm{PDF}_{r_0}\left(\xi\right) d\xi$$

Stochastic Geometry

provides us with the mathematical tools for computing the MGF and the PDF above

- **PDF → Void Probability (contact distance distribution)**
- **MGF → PGFL (reduced Palm distribution)**

Recap on Yesterday's Talk: The JFR's Magic Formula

$$\mathrm{P}_{\mathrm{cov}} = \mathrm{P}_{\mathrm{cov}}\left(T,\alpha\right) = \frac{1}{{}_2F_1\left(1,-2/\alpha,1-2/\alpha,-T\right)}$$

Recap on Yesterday's Talk: The JFR's Magic Formula

$$P_{\mathrm{cov}} = P_{\mathrm{cov}}\left(T,\alpha\right) = \frac{1}{{}_2F_1\left(1,-2/\alpha,1-2/\alpha,-T\right)}$$

The Opinion of the "Typical" Performance Analysis Guy:

AMAZING !!!

Recap on Yesterday's Talk: The JFR's Magic Formula

$$\mathrm{P}_{\mathrm{cov}} = \mathrm{P}_{\mathrm{cov}}\left(T,\alpha\right) = \frac{1}{{}_2F_1\left(1,-2/\alpha,1-2/\alpha,-T\right)}$$

The Opinion of the "Typical" Optimization Guy:

Where is my beloved transmit power ?
Where is my beloved density of base stations ?
Where is my beloved convex function ?

Recap on Yesterday's Talk: JFR's Magic Formula "++"

$$\mathrm{P_{cov}} = \int_0^{+\infty} \exp\left(-T\sigma^2 P^{-1}\xi^{\alpha}\right) \mathrm{MGF}_{I_{agg}(r_0)}\left(P^{-1}T\xi^{\alpha}\right) \mathrm{PDF}_{r_0}(\xi)\, d\xi$$

$$\mathrm{MGF}_{I_{agg}(r_0)}\left(P^{-1}T\xi^{\alpha}\right) = \exp\left(\pi\lambda\xi^2\left(1 - {}_2F_1\left(1, -\frac{2}{\alpha}, 1-\frac{1}{\alpha}, -T\right)\right)\right)$$

$$\mathrm{PDF}_{r_0}(\xi) = 2\pi\lambda\xi \exp\left(-\pi\lambda\xi^2\right)$$

Recap on Yesterday's Talk: JFR's Magic Formula "++"

$$\mathrm{P}_{\mathrm{cov}} = \int_0^{+\infty} \exp\left(-T\sigma^2 P^{-1}\xi^{\alpha}\right) \mathrm{MGF}_{I_{agg}(r_0)}\left(P^{-1}T\xi^{\alpha}\right) \mathrm{PDF}_{r_0}\left(\xi\right) d\xi$$

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$$\mathrm{PDF}_{r_0}\left(\xi\right) = 2\pi\lambda\xi \exp\left(-\pi\lambda\xi^2\right)$$

The Opinion of the "Typical" Optimization Guy:

Where is my beloved <u>closed-form</u> convex function ?

Recap on Yesterday's Talk: With non-Poisson Is Worse

TRANSACTIONS ON WIRELESS COMMUNICATIONS

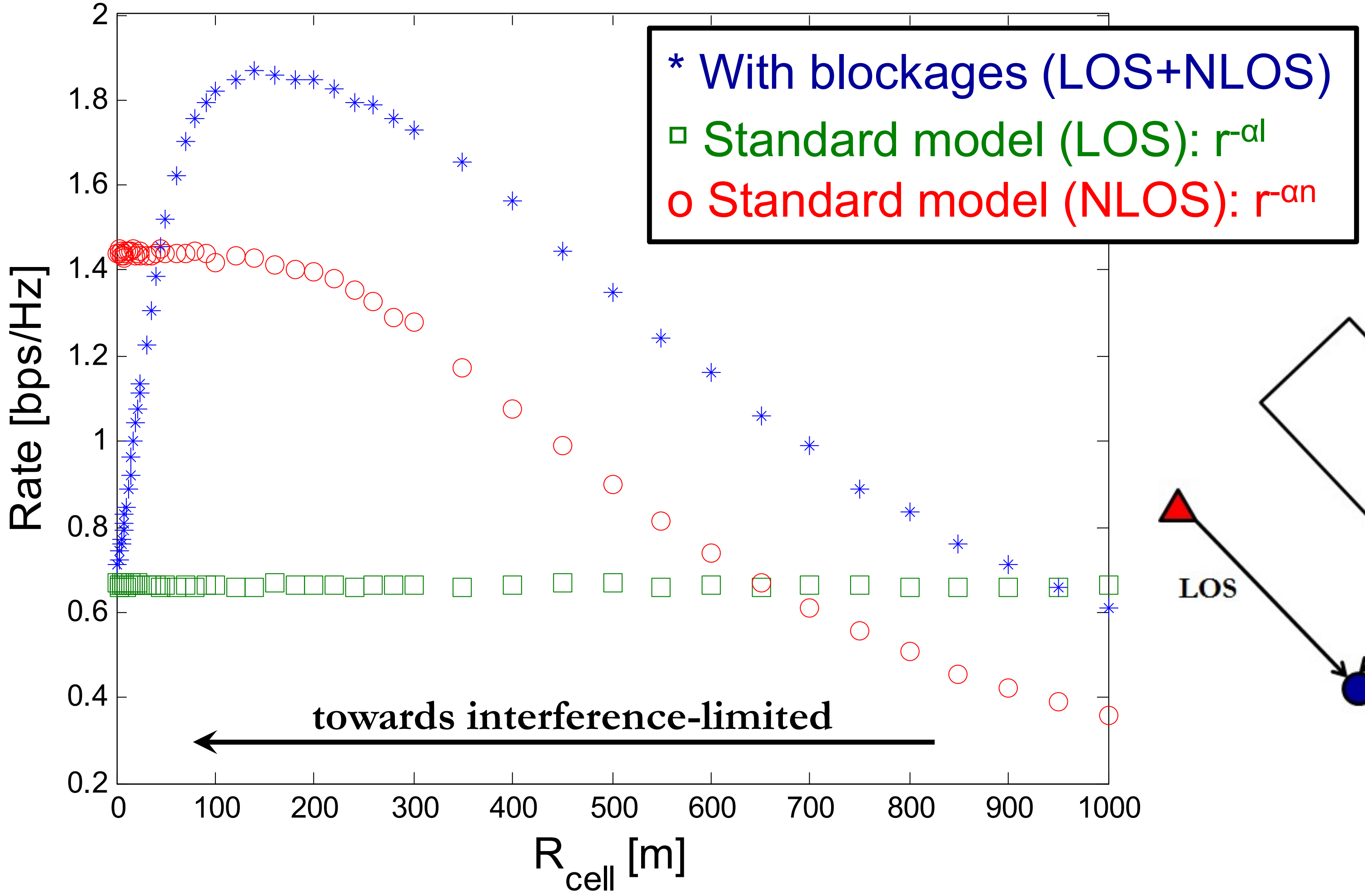
Inhomogeneous Double Thinning – Modeling and Analyzing Cellular Networks Using Inhomogeneous Poisson Point Processes

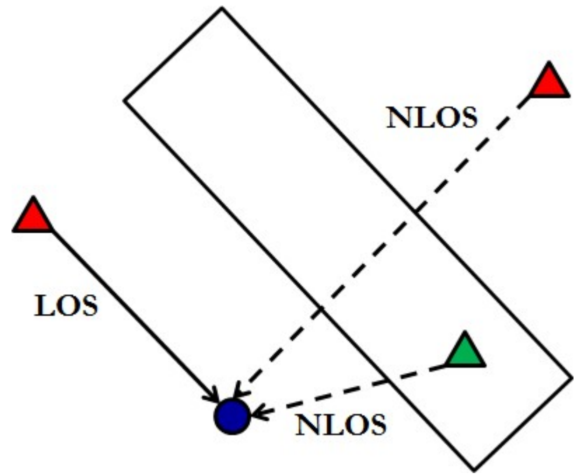
Marco Di Renzo, *Senior Member, IEEE*, and Shanshan Wang, *Student Member, IEEE*

Abstract

In this paper, we introduce a new methodology for modeling and analyzing downlink cellular networks, where the Base Stations (BSs) constitute a stationary Point Process (PP) that exhibits some degree of interaction among the points, i.e., spatial inhibition (repulsiveness) or spatial aggregation (clustering). The proposed approach is based on the theory of Inhomogeneous Poisson Point Processes (I-PPP) and is referred to as Inhomogeneous Double Thinning (IDT) approach. In a stationary PP, the

Recap on Yesterday's Talk: With LOS/NLOS Is Worse





M. Di Renzo, W. Lu, and P. Guan, "The Intensity Matching Approach: A Tractable Stochastic Geometry Approximation to System-Level Analysis of Cellular Networks", IEEE Trans. Wireless Commun., Sep. 2016.

Recap on Yesterday's Talk: With LOS/NLOS Is Worse

$$\mathcal{R} = \mathcal{R}_{\mathrm{LOS,in}} + \mathcal{R}_{\mathrm{LOS,out}} + \mathcal{R}_{\mathrm{NLOS,in}} + \mathcal{R}_{\mathrm{NLOS,out}}$$

$$\mathcal{R}_{s,\mathrm{in}} = \int_0^{\kappa_s \widehat{D}_1^{\alpha_s}} \left(\int_0^{\infty} \exp\left(-zx\frac{\sigma_N^2}{G^{(0)}P_{\mathrm{RB}}}\right) \exp\left(\widehat{\mathcal{T}}_{\mathrm{in}}\left(\frac{z}{G^{(0)}}, x\right)\right) \bar{\mathcal{M}}_{g_s^{(0)}}(z)\, \frac{dz}{z}\right) \times \widehat{\Lambda}^{(1)}_{\Phi_{s,\mathrm{in}}}([0,x)) \exp\left(-\widehat{\Lambda}_{\Phi_{\mathrm{in}}}([0,x))\right) dx$$

$$\mathcal{R}_{s,\mathrm{out}} = \int_{\kappa_s \widehat{D}_1^{\alpha_s}}^{\infty} \left(\int_0^{\infty} \exp\left(-zx\frac{\sigma_N^2}{G^{(0)}P_{\mathrm{RB}}}\right) \exp\left(\widehat{\mathcal{T}}_{\mathrm{out}}\left(\frac{z}{G^{(0)}}, x\right)\right) \bar{\mathcal{M}}_{g_s^{(0)}}(z)\, \frac{dz}{z}\right) \times \widehat{\Lambda}^{(1)}_{\Phi_{s,\mathrm{out}}}([0,x)) \exp\left(-\widehat{\Lambda}_{\Phi_{\mathrm{out}}}([0,x))\right) dx$$

$$\widehat{\mathcal{T}}_{\mathrm{in}}(z,x) = \pi\lambda_{\mathrm{BS}}^{(\mathrm{I})} \sum_{r\in\{\mathrm{LOS,NLOS}\}} \left(\Theta_r \widehat{q}_r^{[0,\widehat{D}_1]}\left(\left(\frac{x}{\kappa_r}\right)^{2/\alpha_r}\mathcal{F}_{r,\mathrm{in}}(z) - \widehat{D}_1^2 \mathcal{F}_{r,\mathrm{in}}\left(\frac{xz}{\kappa_r \widehat{D}_1^{\alpha_r}}\right)\right) + \Theta_r \widehat{q}_r^{[\widehat{D}_1,\infty]} \widehat{D}_1^2 \mathcal{F}_{r,\mathrm{in}}\left(\frac{xz}{\kappa_r \widehat{D}_1^{\alpha_r}}\right)\right)$$

$$\widehat{\mathcal{T}}_{\mathrm{out}}(z,x) = \pi\lambda_{\mathrm{BS}}^{(\mathrm{I})} \sum_{r\in\{\mathrm{LOS,NLOS}\}} \left(\Theta_r \widehat{q}_r^{[\widehat{D}_1,\infty]} (x/\kappa_r)^{2/\alpha_r} \mathcal{F}_{r,\mathrm{out}}(z)\right)$$

$$\widehat{\Lambda}_{\Phi_{\mathrm{in}}}([0,x)) = \pi\lambda_{\mathrm{BS}} \sum_{r\in\{\mathrm{LOS,NLOS}\}} \left(\Theta_r \widehat{q}_r^{[0,\widehat{D}_1]} (x/\kappa_r)^{2/\alpha_r}\right) \overline{\mathcal{H}}\left(x - \kappa_r \widehat{D}_1^{\alpha_r}\right)$$

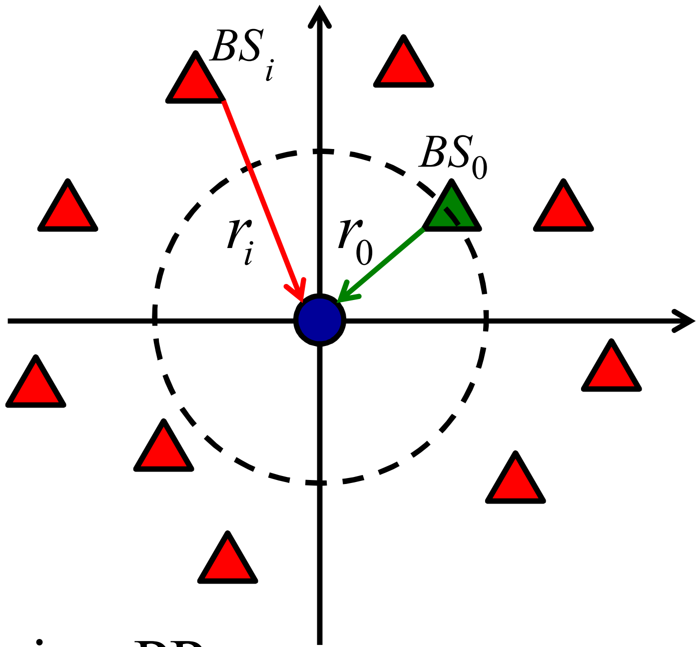
$$\widehat{\Lambda}_{\Phi_{\mathrm{out}}}([0,x)) = \pi\lambda_{\mathrm{BS}} \sum_{r\in\{\mathrm{LOS,NLOS}\}} \left(\Theta_r \widehat{q}_r^{[0,\widehat{D}_1]} \widehat{D}_1^2 + \Theta_r \widehat{q}_r^{[\widehat{D}_1,\infty]} \left((x/\kappa_r)^{2/\alpha_r} - \widehat{D}_1^2\right)\right) \mathcal{H}\left(x - \kappa_r \widehat{D}_1^{\alpha_r}\right)$$

$$\widehat{\Lambda}^{(1)}_{\Phi_{s,\mathrm{in}}}([0,x)) = \pi\lambda_{\mathrm{BS}} \Theta_s \widehat{q}_s^{[0,\widehat{D}_1]} (2/\alpha_s)\, \kappa_s^{-2/\alpha_s} x^{2/\alpha_s - 1}$$

$$\widehat{\Lambda}^{(1)}_{\Phi_{s,\mathrm{out}}}([0,x)) = \pi\lambda_{\mathrm{BS}} \Theta_s \widehat{q}_s^{[\widehat{D}_1,\infty]} (2/\alpha_s)\, \kappa_s^{-2/\alpha_s} x^{2/\alpha_s - 1}$$

Recap on Yesterday's Talk: The Path-Loss Model…

… determines the mathematical complexity …



Φ is a PP

$$\mathrm{P}_{\mathrm{cov}} = \Pr\left\{\mathrm{SINR} > T\right\}$$

$$\mathrm{SINR} = \frac{P\left|h_o\right|^2 l^{-1}\left(r_0\right)}{\sigma^2 + I_{agg}\left(r_0\right)}$$

$$I_{agg}\left(r_0\right) = \sum_{i \in \Phi \setminus BS_0} P\left|h_i\right|^2 l^{-1}\left(r_i\right)$$

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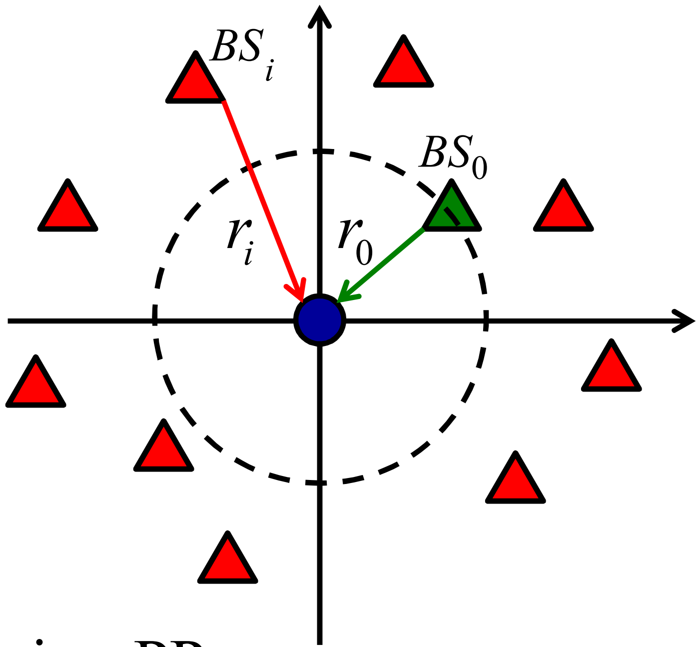
Recap on Yesterday's Talk: Is The Path-Loss The Issue ?

… Power vs. Path-Loss in the interference-limited regime …



Φ is a PP

$$\mathrm{P_{cov}} = \Pr\left\{\mathrm{SIR} > T\right\}$$

$$\mathrm{SIR} = \frac{P\left|h_o\right|^2 l^{-1}\left(r_0\right)}{I_{agg}\left(r_0\right)}$$

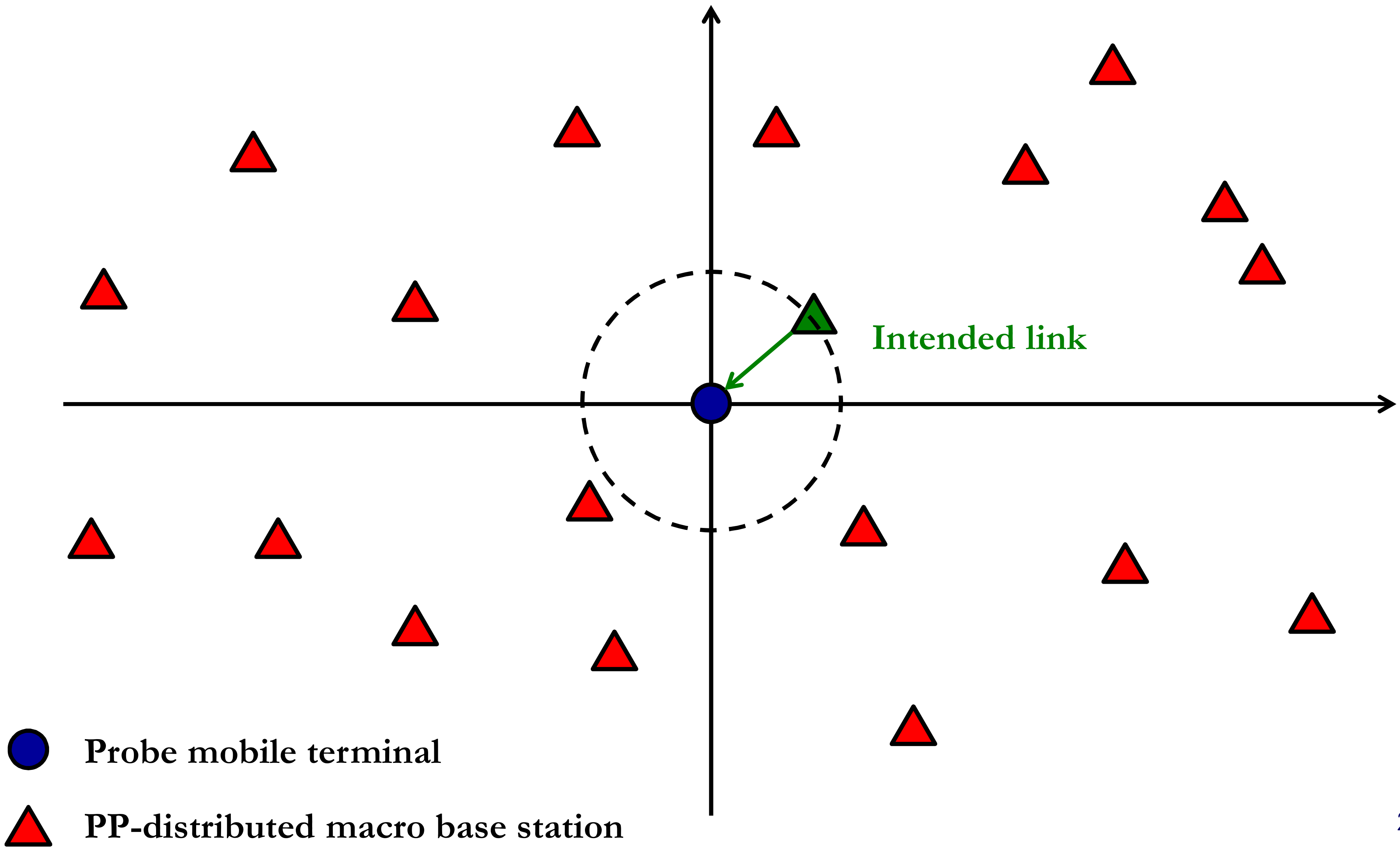
$$I_{agg}\left(r_0\right) = \sum_{i\in\Phi\backslash BS_0} P\left|h_i\right|^2 l^{-1}\left(r_i\right)$$

$$\mathrm{P_{cov}} = \Pr\left\{\frac{P\left|h_o\right|^2 l^{-1}\left(r_0\right)}{I_{agg}\left(r_0\right)} > T\right\} = \Pr\left\{\frac{\left|h_o\right|^2 l^{-1}\left(r_0\right)}{\sum_{i\in\Phi\backslash BS_0}\left|h_i\right|^2 l^{-1}\left(r_i\right)} > T\right\} = \ldots$$

Today's Main Question…

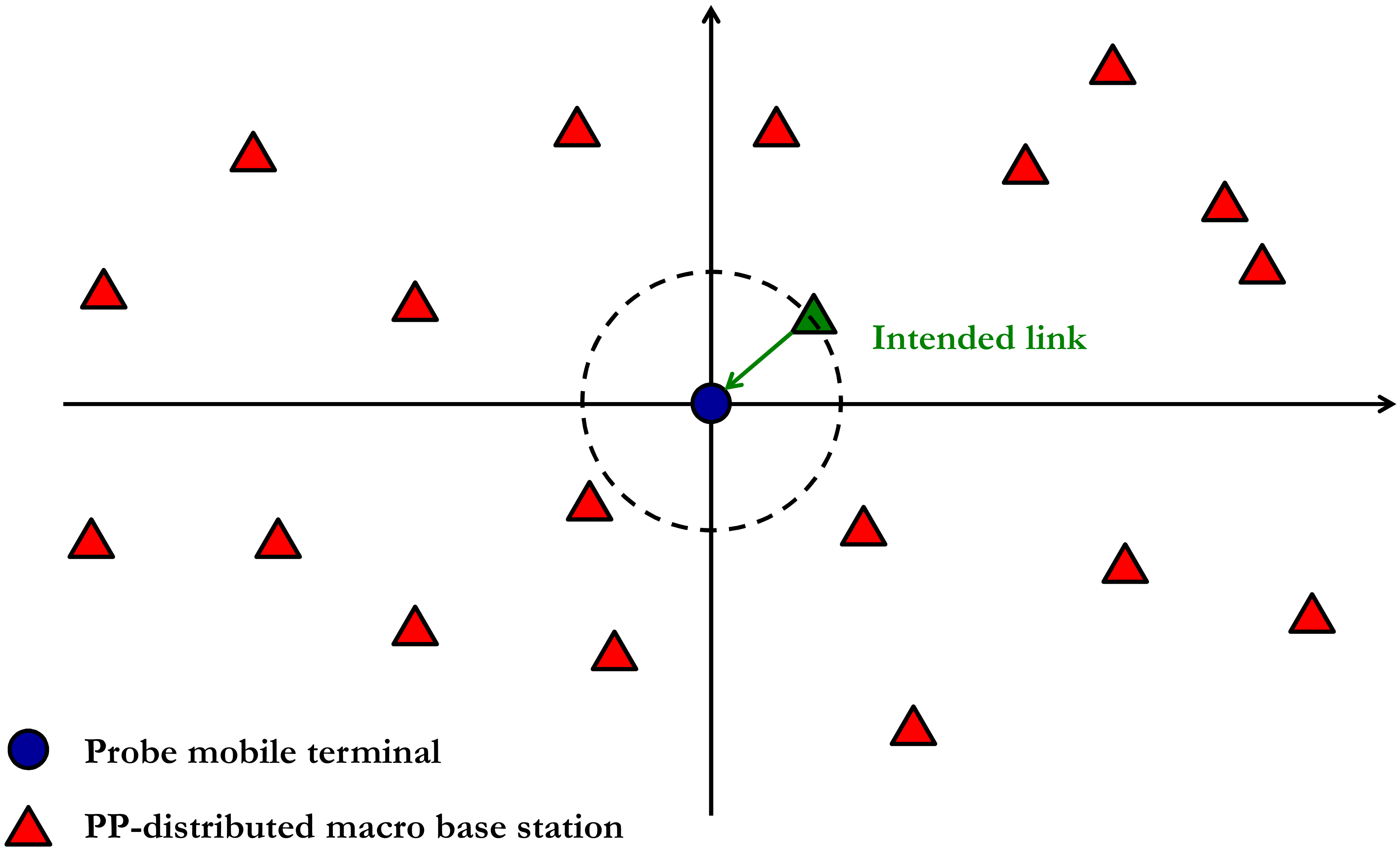
…Can We Develop a *Meaningful* Analytical Framework that is *Accurate & Tractable* for System-Level Optimization ?

Cellular Networks: First Detecting, Then Transmitting



Cellular Networks: First Detecting, Then Transmitting

… and if I cannot detect any base stations … ?



Cellular Networks: First Detecting, Then Transmitting

C. Cell Association Criterion

A cell association criterion based on the highest average received power is assumed. Let $\mathrm{BS}_n \in \Psi_{\mathrm{BS}}$ denote a generic BS of the network. The serving BS, BS_0, is obtained as follows:

$$\mathrm{BS}_0 = \arg\max_{\mathrm{BS}_n \in \Psi_{\mathrm{BS}}} \left\{ 1/l\left(r_n\right) \right\} = \arg\max_{\mathrm{BS}_n \in \Psi_{\mathrm{BS}}} \left\{ 1/L_n \right\} \tag{2}$$

where the short-hand notation $L_n = l\left(r_n\right)$ is used. As for the intended link, $L_0 = \min_{r_n \in \Psi_{\mathrm{BS}}} \left\{ L_n \right\}$ holds. It is worth noting that, for the considered system, i.e., a single-tier cellular network with BSs having the same transmit power, (2) is equivalent to the shortest distance cell association.

Cellular Networks: First Detecting, Then Transmitting

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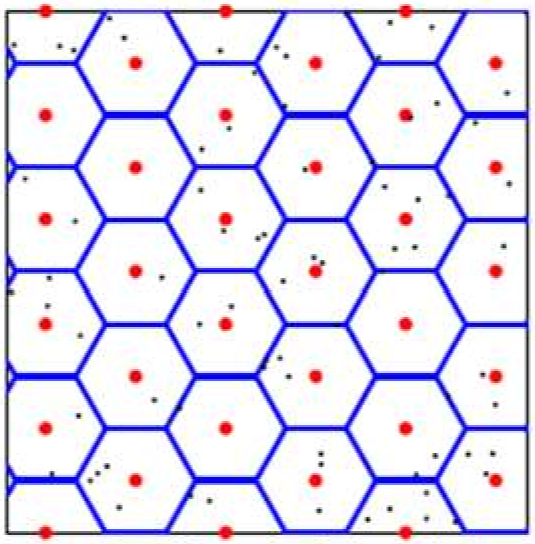
What does it happen if $1/L_0$ is too weak to be detected ?

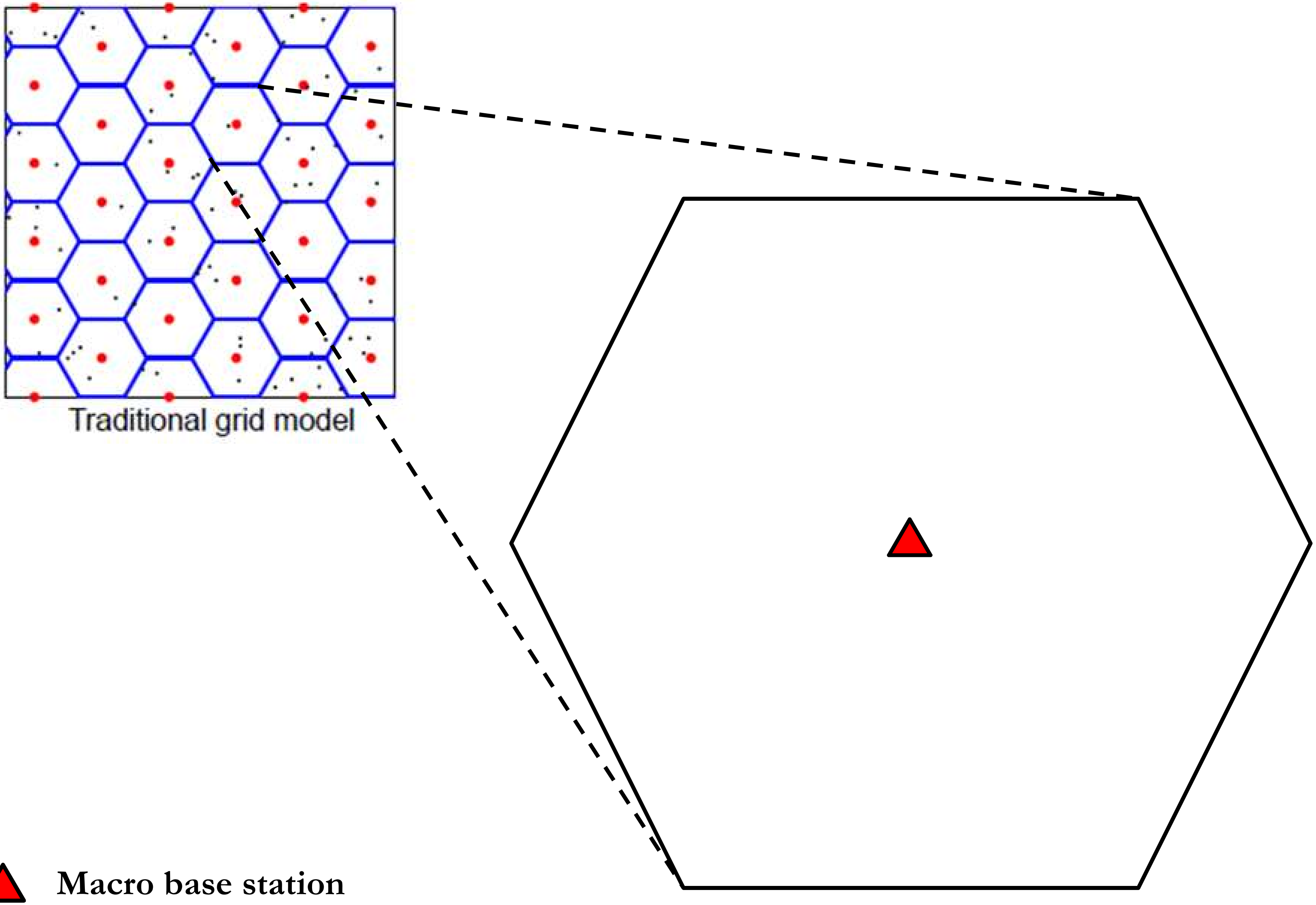
The Tight Interplay Between Tx Power & BSs Density



Traditional grid model

Macro base station

The Tight Interplay Between Tx Power & BSs Density

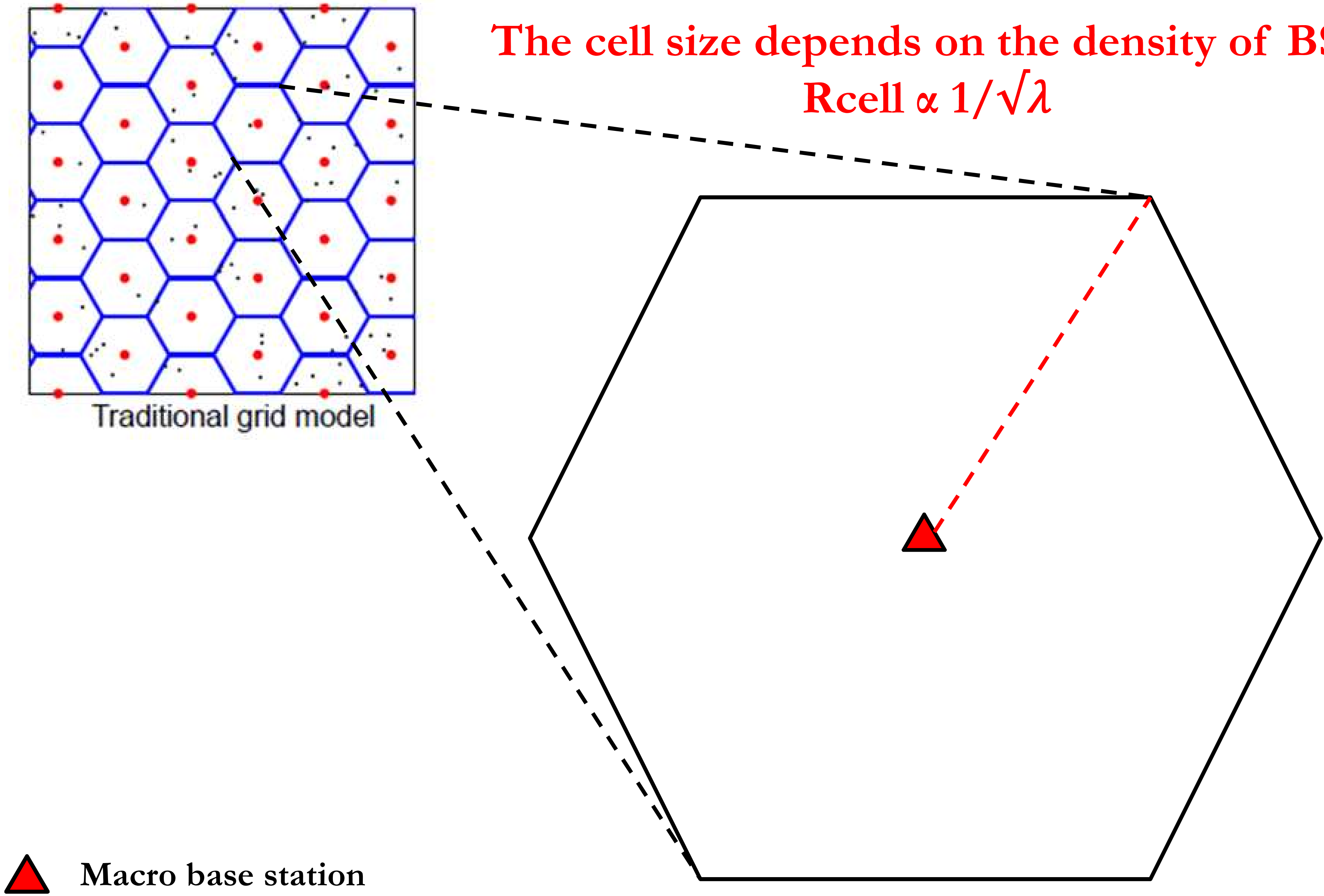


▲ **Macro base station**

The Tight Interplay Between Tx Power & BSs Density

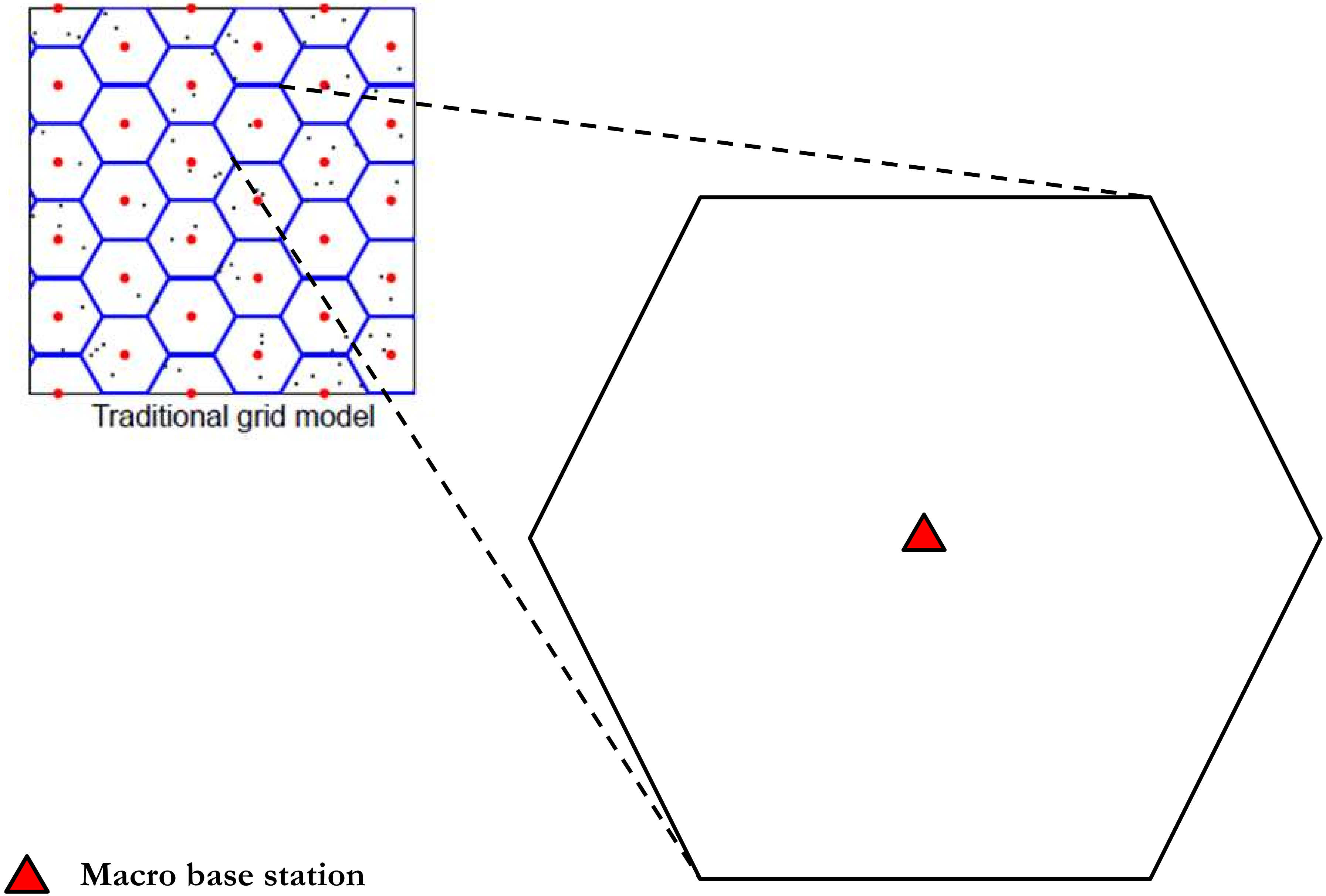
The cell size depends on the density of BSs

Rcell ∝ 1/√λ



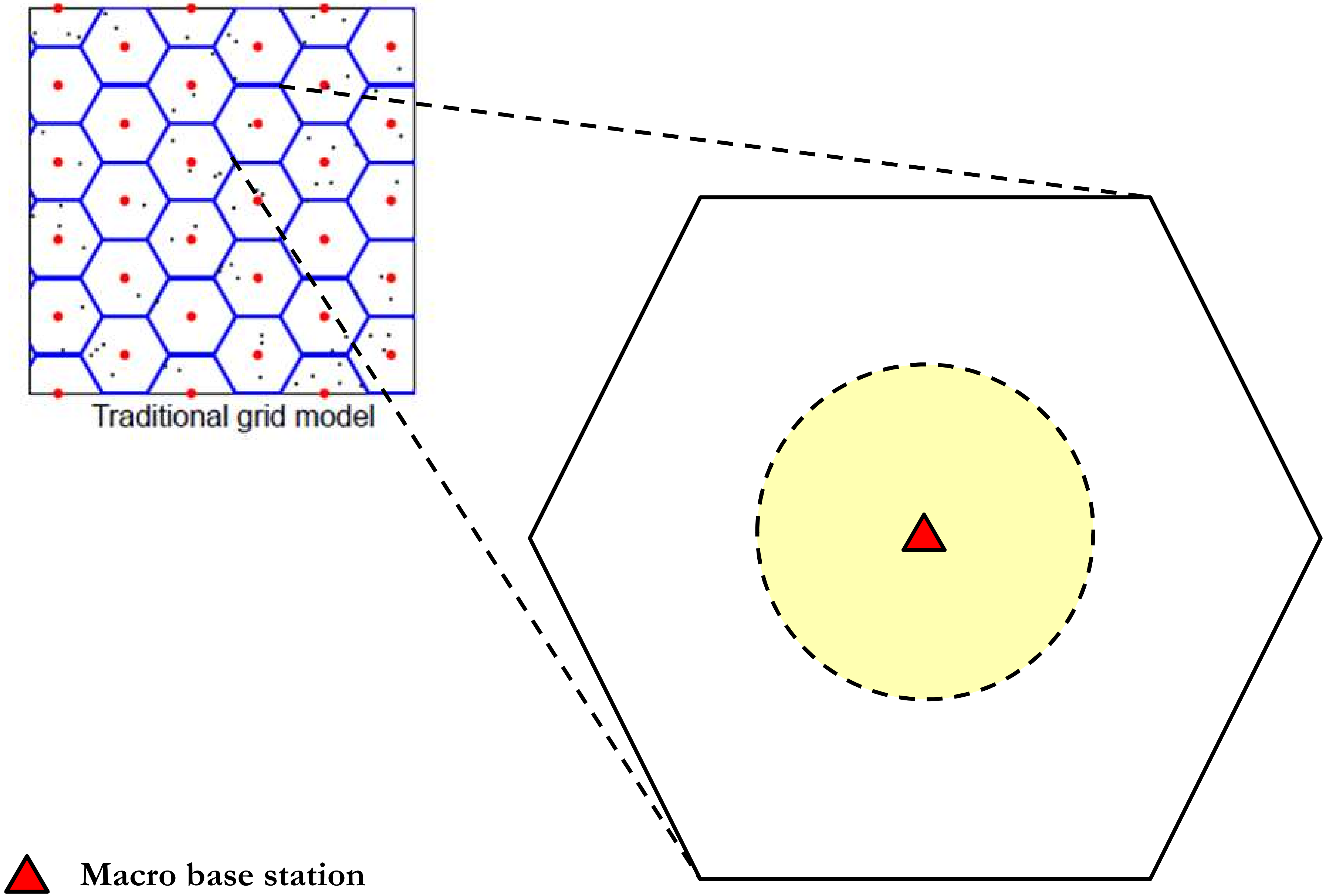
Macro base station

The Tight Interplay Between Tx Power & BSs Density



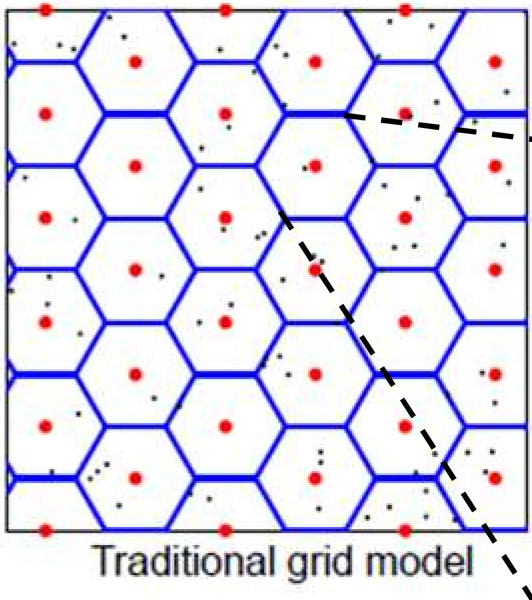
Macro base station

The Tight Interplay Between Tx Power & BSs Density



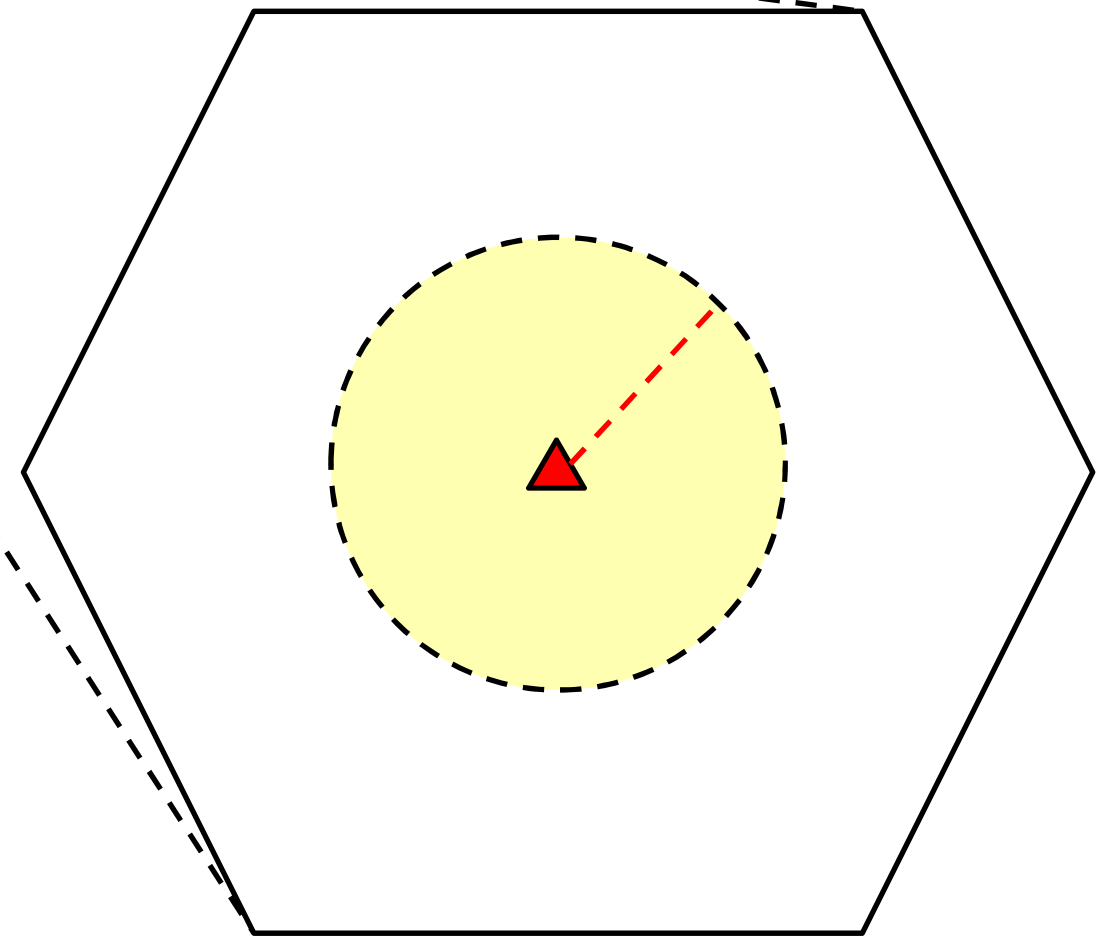
Macro base station

The Tight Interplay Between Tx Power & BSs Density



The cell coverage depends on the Tx power

$P_{rx} \propto P_{tx}/l(r) > \gamma_{min}$

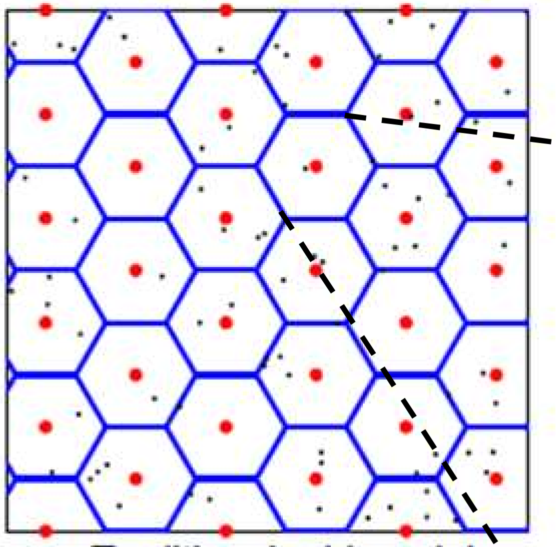


▲ **Macro base station**

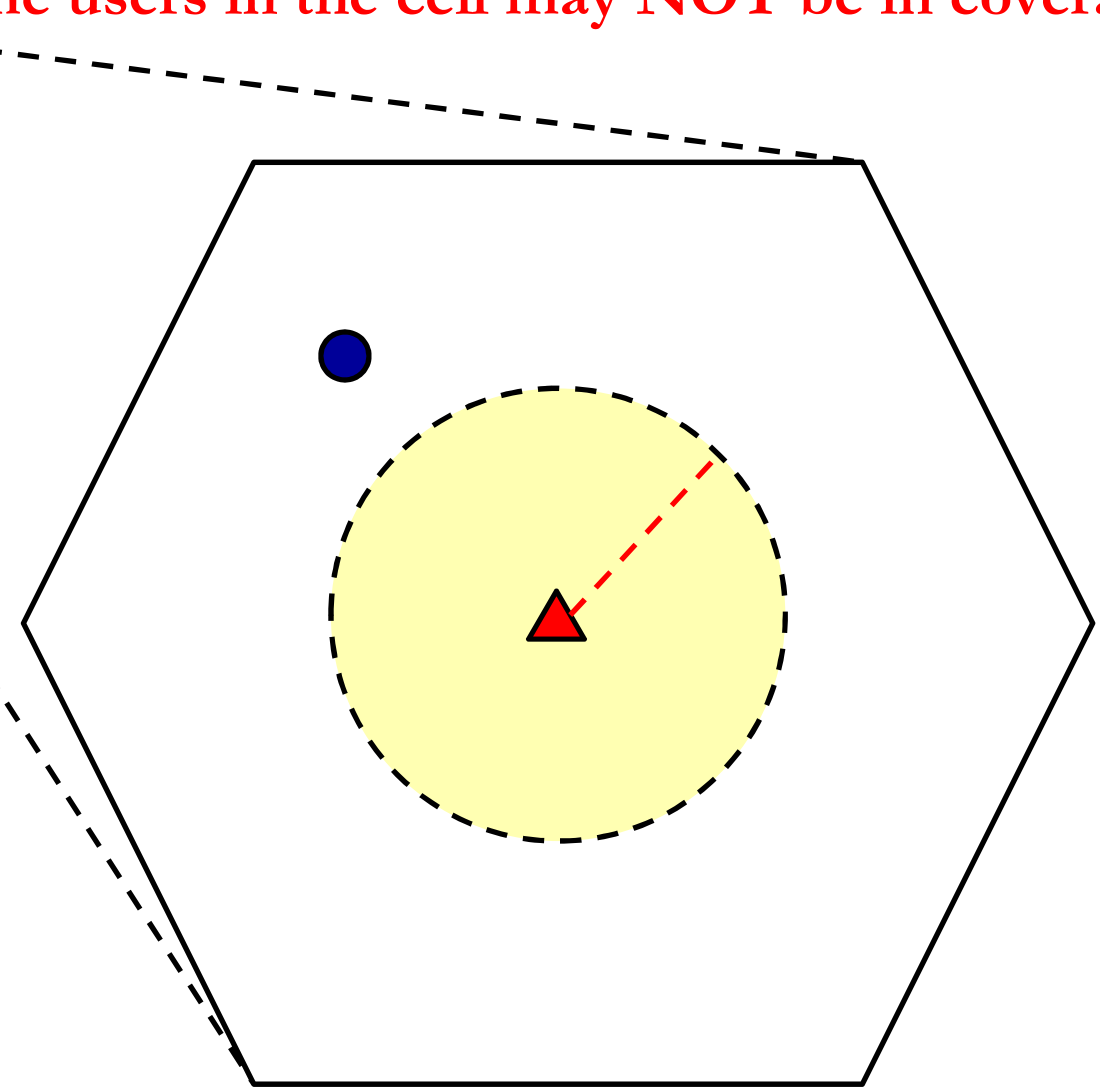
The Tight Interplay Between Tx Power & BSs Density

Ptx vs. λ :

Some users in the cell may NOT be in coverage



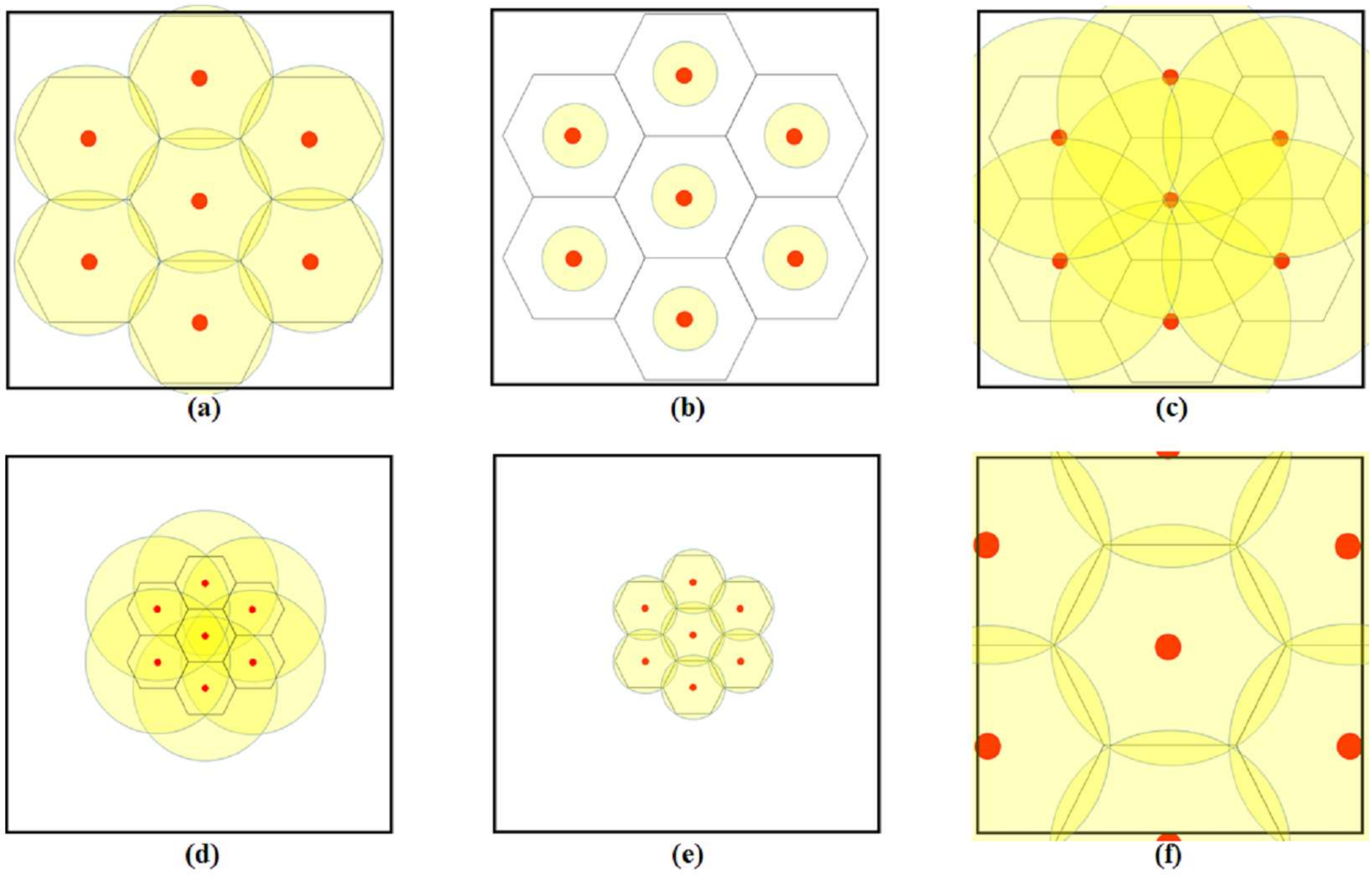
Traditional grid model



Probe mobile terminal

Macro base station

The Tight Interplay Between Tx Power & BSs Density


(a) (b) (c) (d) (e) (f)

A New Definition of Coverage Probability

III. A New Mathematical Formulation of the PSE

In this section, we introduce and motivate a new definition of coverage probability, $\mathrm{P_{cov}}$, and PSE, which overcomes the limitations of currently available mathematical frameworks and is suitable for system-level optimization (see Section I-A). All symbols are defined in Table I.

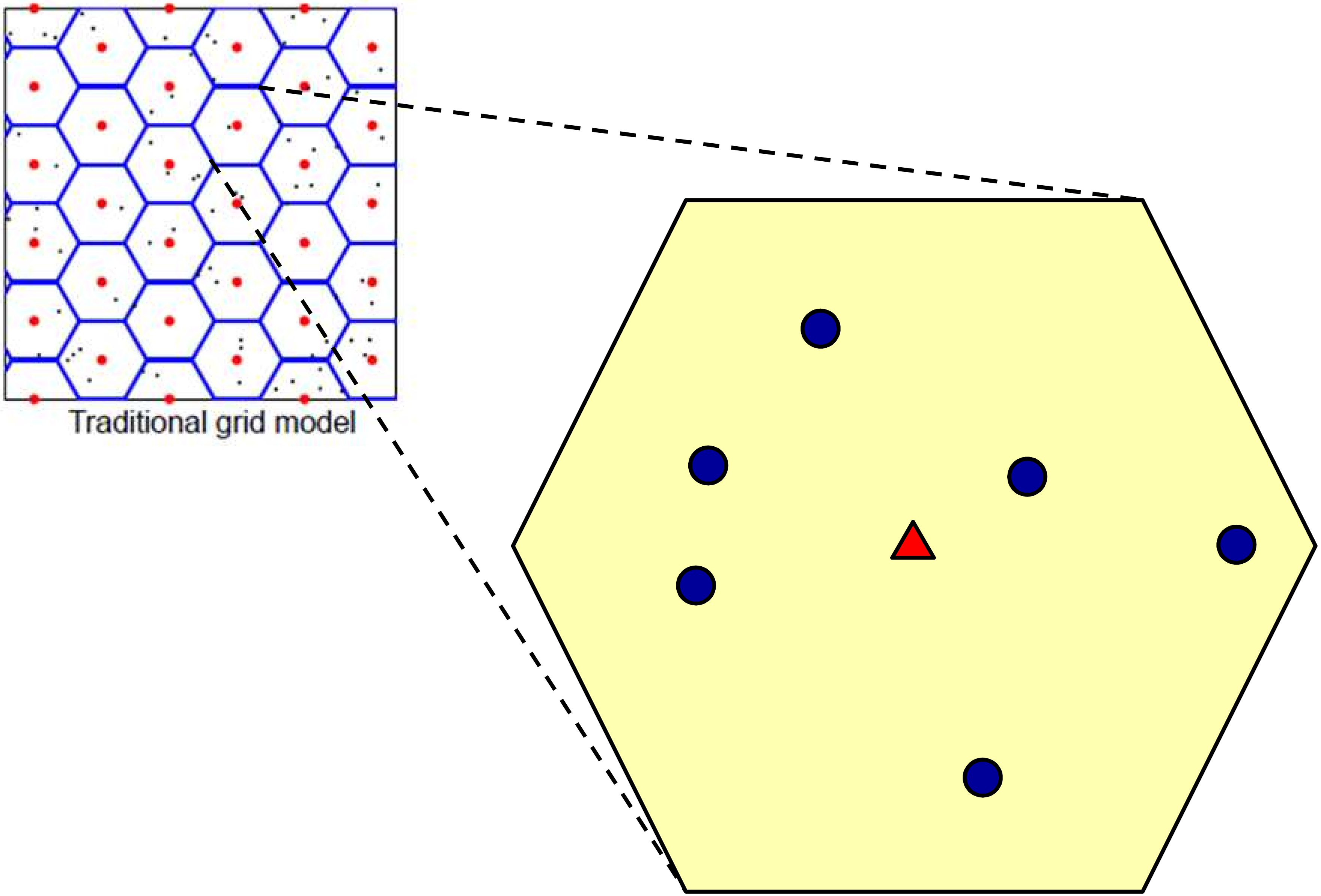
Definition 1: Let γ_{D} and γ_{A} be the reliability thresholds for successfully decoding the information data and for successfully detecting the nearest (serving) BS, BS_0, respectively. The coverage probability, $\mathrm{P_{cov}}$, of the typical MT, MT_0, is defined as follows:

$$\mathrm{P_{cov}}\left(\gamma_{\mathrm{D}}, \gamma_{\mathrm{A}}\right) = \begin{cases} \Pr\left\{\mathrm{SIR} \ge \gamma_{\mathrm{D}}, \overline{\mathrm{SNR}} \ge \gamma_{\mathrm{A}}\right\} & \text{if} \quad \mathrm{MT}_0 \text{ is selected} \\ 0 & \text{if} \quad \mathrm{MT}_0 \text{ is not selected} \end{cases} \tag{3}$$

where the Signal-to-Interference-Ratio (SIR) and the average Signal-to-Noise-Ratio ($\overline{\mathrm{SNR}}$) can be formulated, for the network model under analysis, as follows:

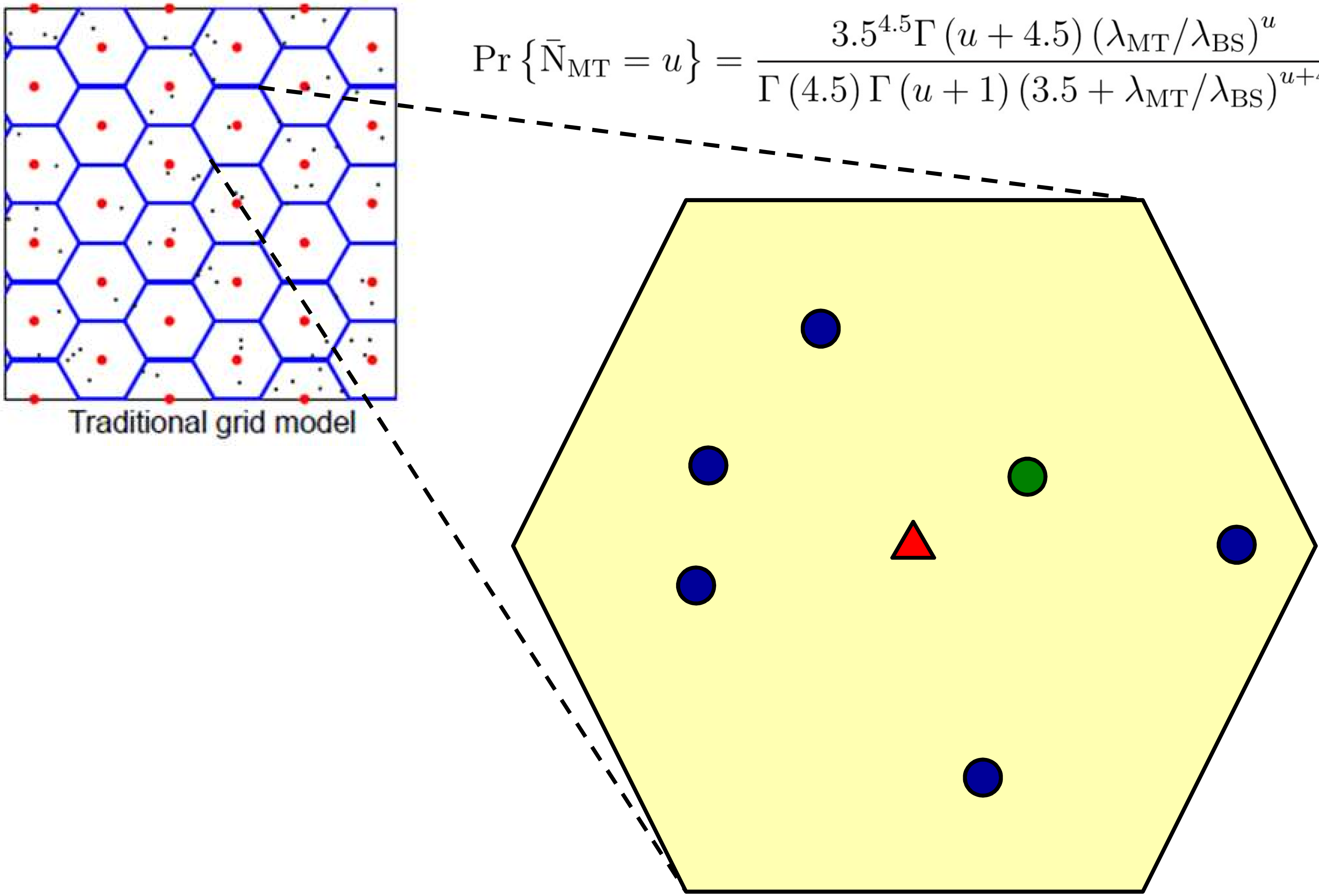
$$\mathrm{SIR} = \frac{\mathrm{P_{tx}} g_0 / L_0}{\sum_{\mathrm{BS}_i \in \Psi_{\mathrm{BS}}^{(\mathrm{I})}} \mathrm{P_{tx}} g_i / L_i \mathbb{1}\left(L_i > L_0\right)} \qquad \overline{\mathrm{SNR}} = \frac{\mathrm{P_{tx}} / L_0}{\sigma_{\mathrm{N}}^2} \tag{4}$$

On User Selection…

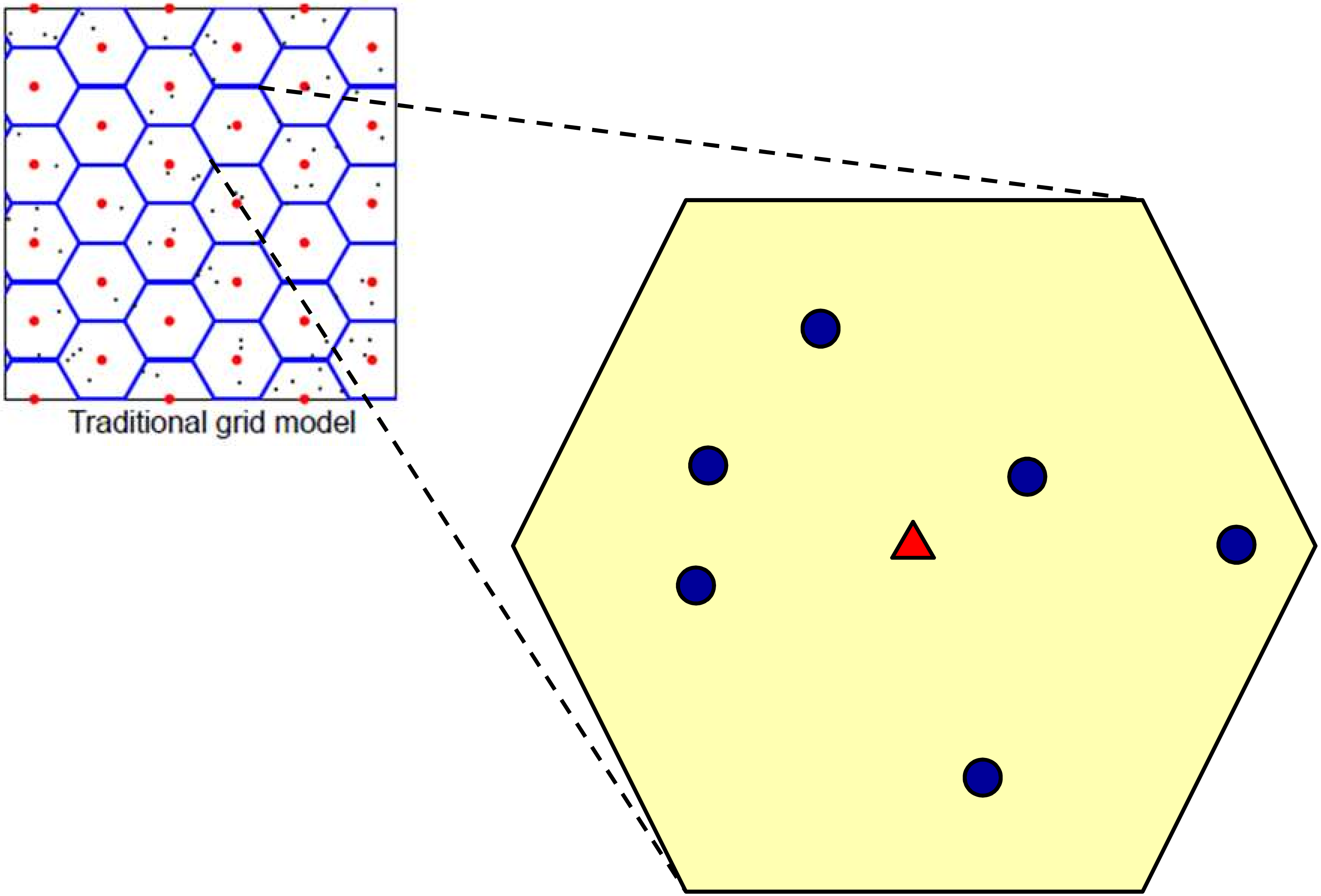


How Many Users In A Cell ?

$$\Pr\left\{\bar{\mathrm{N}}_{\mathrm{MT}} = u\right\} = \frac{3.5^{4.5}\Gamma\left(u+4.5\right)\left(\lambda_{\mathrm{MT}}/\lambda_{\mathrm{BS}}\right)^{u}}{\Gamma\left(4.5\right)\Gamma\left(u+1\right)\left(3.5+\lambda_{\mathrm{MT}}/\lambda_{\mathrm{BS}}\right)^{u+4.5}}$$

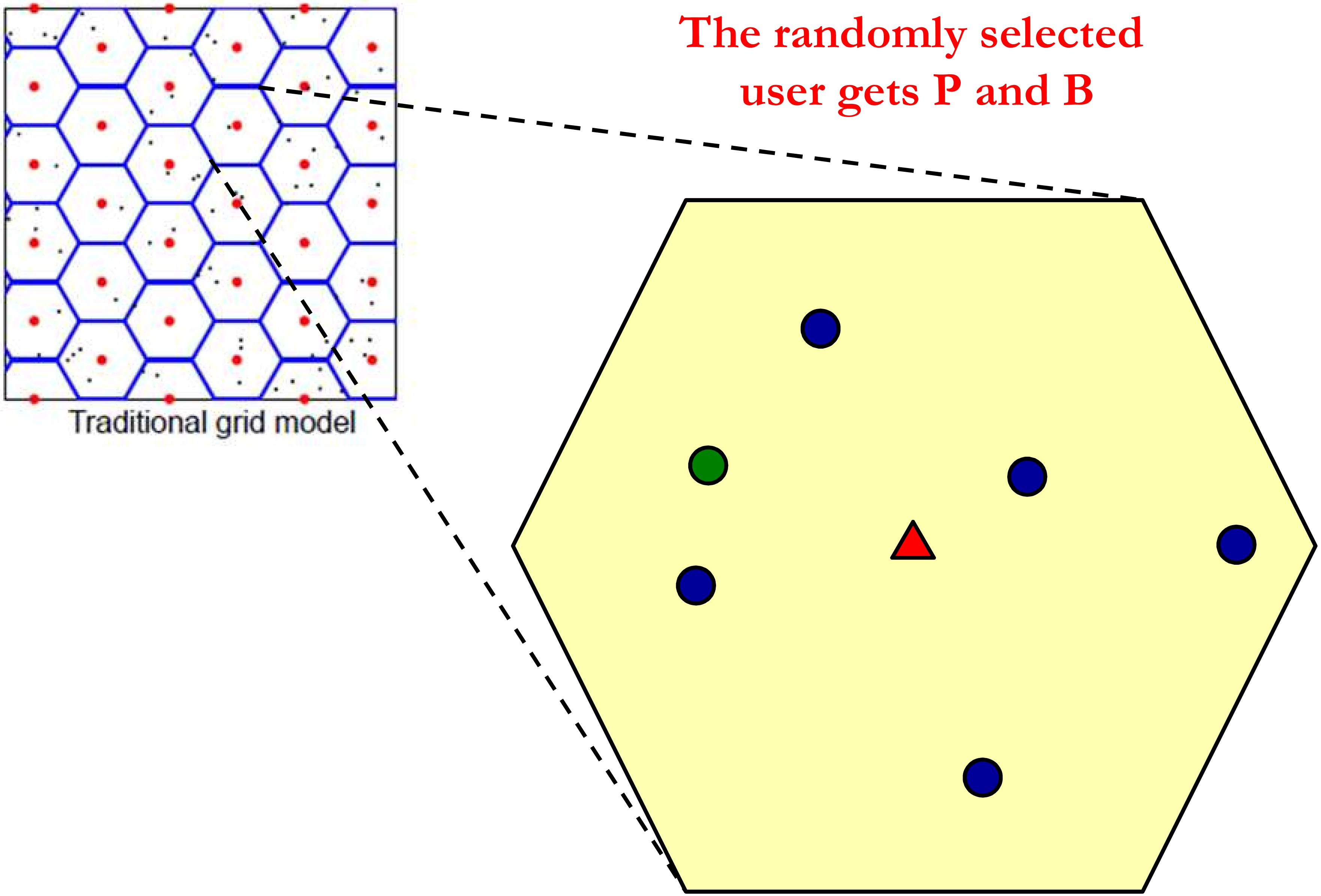


How To Schedule Users And Allocate P & B ?



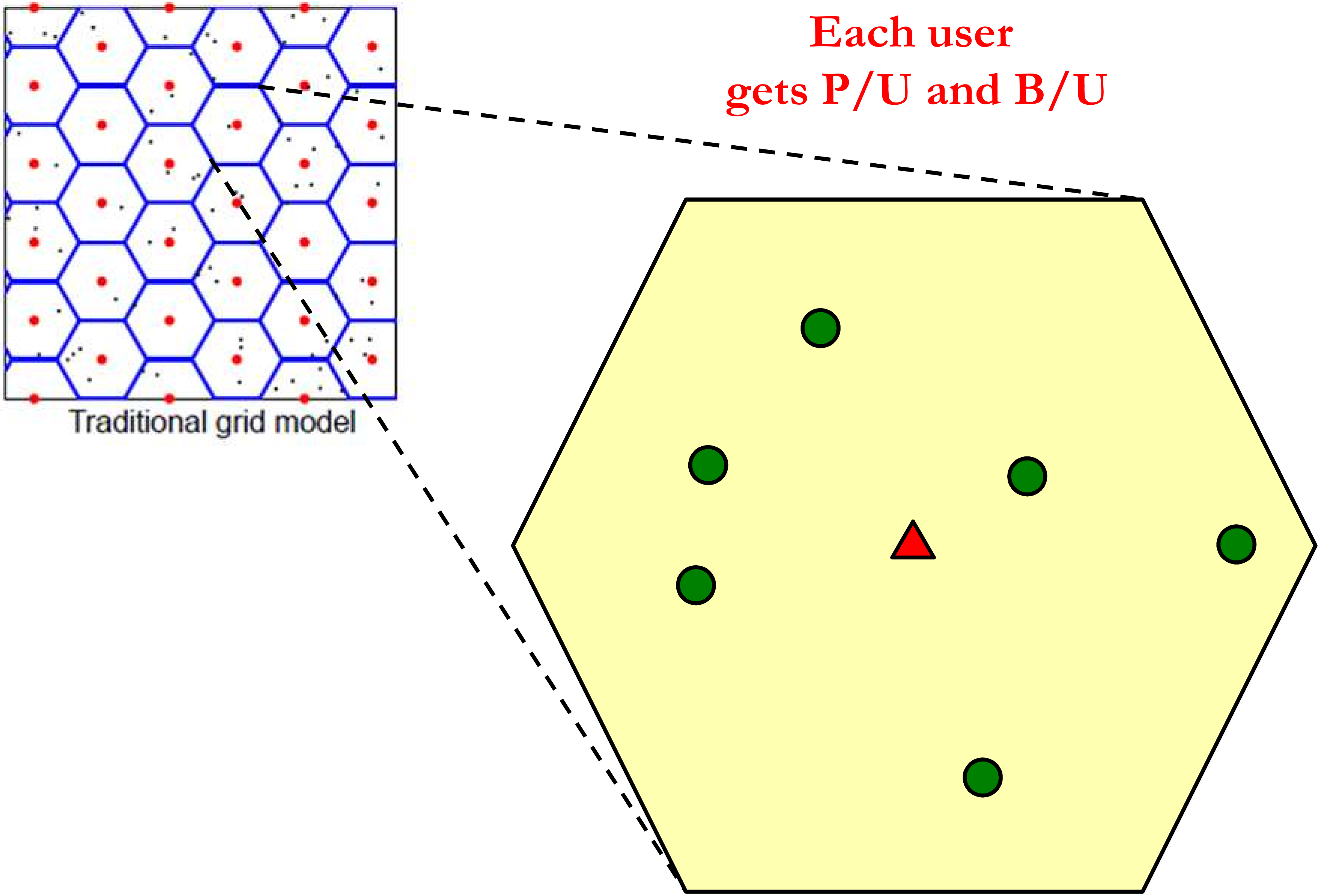
How To Schedule Users And Allocate P & B ?

The randomly selected user gets P and B



How To Schedule Users And Allocate P & B ?

Each user
gets P/U and B/U



Definition of Potential Spectral Efficiency (PSE)

Lemma 2: Let *Load Model 2* be assumed. The PSE (bit/sec/m^2) can be formulated as follows:

$$\begin{aligned} \mathrm{PSE}\left(\gamma_{\mathrm{D}}, \gamma_{\mathrm{A}}\right) &= \mathbb{E}_{\bar{\mathrm{N}}_{\mathrm{MT}}}\left\{\mathrm{PSE}\left(\gamma_{\mathrm{D}}, \gamma_{\mathrm{A}} \middle| \bar{\mathrm{N}}_{\mathrm{MT}}\right)\right\} \\ &\overset{(b)}{=} \sum_{u=0}^{+\infty} \lambda_{\mathrm{MT}} \frac{\mathrm{B_W}}{u+1} \log_2\left(1+\gamma_{\mathrm{D}}\right) \Pr\left\{\mathrm{SIR} \geq \gamma_{\mathrm{D}}, \overline{\mathrm{SNR}} \geq \gamma_{\mathrm{A}}\right\} \Pr\left\{\bar{\mathrm{N}}_{\mathrm{MT}} = u\right\} \\ &= \lambda_{\mathrm{MT}} \mathrm{B_W} \log_2\left(1+\gamma_{\mathrm{D}}\right) \Pr\left\{\mathrm{SIR} \geq \gamma_{\mathrm{D}}, \overline{\mathrm{SNR}} \geq \gamma_{\mathrm{A}}\right\} \sum_{u=0}^{+\infty} \frac{\Pr\left\{\bar{\mathrm{N}}_{\mathrm{MT}} = u\right\}}{u+1} \end{aligned} \quad (7)$$

New Closed-Form Mathematical Expression of PSE

Proposition 1: Consider either *Load Model 1* or *Load Model 2*. Assume notation and functions given in Tables I and II. The PSE (bit/sec/m^2) can be formulated, in closed-form, as follows:

$$\mathrm{PSE}\left(\gamma_{\mathrm{D}}, \gamma_{\mathrm{A}}\right) = \mathrm{B_W}\log_2\left(1+\gamma_{\mathrm{D}}\right)\frac{\lambda_{\mathrm{BS}}\mathcal{L}\left(\lambda_{\mathrm{MT}}/\lambda_{\mathrm{BS}}\right)}{1+\Upsilon\mathcal{L}\left(\lambda_{\mathrm{MT}}/\lambda_{\mathrm{BS}}\right)}\mathcal{Q}\left(\lambda_{\mathrm{BS}}, \mathrm{P_{tx}}, \lambda_{\mathrm{MT}}/\lambda_{\mathrm{BS}}\right) \quad (9)$$

Proof: See Appendix A. □

$$\mathcal{L}\left(x\right) = 1-\left(1+x/\alpha\right)^{-\alpha},\ \alpha = 3.5$$

$$\mathcal{Q}\left(x,y,z\right) = 1-\exp\left(-\pi x(y/\eta)^{2/\beta}\left(1+\Upsilon\mathcal{L}\left(z\right)\right)\right)$$

$$\Upsilon = {}_2F_1\left(-2/\beta, 1, 1-2/\beta, -\gamma_{\mathrm{D}}\right)-1 \geq 0$$

New Closed-Form Mathematical Expression of PSE

Proposition 1: Consider either *Load Model 1* or *Load Model 2*. Assume notation and functions given in Tables I and II. The PSE (bit/sec/m^2) can be formulated, in closed-form, as follows:

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Proof: See Appendix A. □

Corollary 1: If $\gamma_{\mathrm{A}} = 0$, i.e., the conventional definition of $\mathrm{P_{cov}}$ is used, the PSE in (9) simplifies as follows:

$$\mathrm{PSE}\left(\gamma_{\mathrm{D}}, \gamma_{\mathrm{A}}=0\right) = \mathrm{B_W} \log_2\left(1+\gamma_{\mathrm{D}}\right) \frac{\lambda_{\mathrm{BS}} \mathcal{L}\left(\lambda_{\mathrm{MT}}/\lambda_{\mathrm{BS}}\right)}{1+\Upsilon \mathcal{L}\left(\lambda_{\mathrm{MT}}/\lambda_{\mathrm{BS}}\right)} \tag{10}$$

$$\mathcal{L}\left(x\right) = 1-\left(1+x/\alpha\right)^{-\alpha},\ \alpha = 3.5$$

$$\mathcal{Q}\left(x,y,z\right) = 1-\exp\left(-\pi x (y/\eta)^{2/\beta}\left(1+\Upsilon\mathcal{L}\left(z\right)\right)\right)$$

$$\Upsilon = {}_2F_1\left(-2/\beta, 1, 1-2/\beta, -\gamma_{\mathrm{D}}\right) - 1 \geq 0$$

New Closed-Form Mathematical Expression of PSE

Proposition 1: Consider either *Load Model 1* or *Load Model 2*. Assume notation and functions given in Tables I and II. The PSE (bit/sec/m^2) can be formulated, in closed-form, as follows:

$$\mathrm{PSE}(\gamma_{\mathrm{D}}, \gamma_{\mathrm{A}}) = \mathrm{B_W} \log_2 (1 + \gamma_{\mathrm{D}}) \frac{\lambda_{\mathrm{BS}} \mathcal{L}(\lambda_{\mathrm{MT}}/\lambda_{\mathrm{BS}})}{1 + \Upsilon \mathcal{L}(\lambda_{\mathrm{MT}}/\lambda_{\mathrm{BS}})} \mathcal{Q}(\lambda_{\mathrm{BS}}, \mathrm{P_{tx}}, \lambda_{\mathrm{MT}}/\lambda_{\mathrm{BS}}) \tag{9}$$

Proof: See Appendix A. □

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$$\mathrm{PSE}(\gamma_{\mathrm{D}}, \gamma_{\mathrm{A}} = 0) = \mathrm{B_W} \log_2 (1 + \gamma_{\mathrm{D}}) \frac{\lambda_{\mathrm{BS}} \mathcal{L}(\lambda_{\mathrm{MT}}/\lambda_{\mathrm{BS}})}{1 + \Upsilon \mathcal{L}(\lambda_{\mathrm{MT}}/\lambda_{\mathrm{BS}})} \tag{10}$$

$$\mathcal{L}(x) = 1 - (1 + x/\alpha)^{-\alpha}, \ \alpha = 3.5$$

$$\mathcal{Q}(x, y, z) = 1 - \exp\left(-\pi x (y/\eta)^{2/\beta} (1 + \Upsilon \mathcal{L}(z))\right)$$

$$\Upsilon = {}_2F_1(-2/\beta, 1, 1 - 2/\beta, -\gamma_{\mathrm{D}}) - 1 \geq 0$$

Example 1

System-Level Energy Efficiency Optimization

System-Level Energy Efficiency Optimization

IV. System-Level EE Optimization: Formulation and Solution

A unified formulation of the EE (bit/Joule) of the cellular network under analysis is as follows:

$$
\begin{aligned}
\mathrm{EE}\left(\mathrm{P_{tx}}, \lambda_{\mathrm{BS}}\right) &= \frac{\mathrm{PSE}}{\mathrm{P_{grid}}} \\
&= \frac{\mathrm{B_W} \log_2\left(1+\gamma_{\mathrm{D}}\right) \mathcal{L}\left(\lambda_{\mathrm{MT}}/\lambda_{\mathrm{BS}}\right) \mathcal{Q}\left(\lambda_{\mathrm{BS}}, \mathrm{P_{tx}}, \lambda_{\mathrm{MT}}/\lambda_{\mathrm{BS}}\right)}{\left[1+\Upsilon \mathcal{L}\left(\lambda_{\mathrm{MT}}/\lambda_{\mathrm{BS}}\right)\right]\left[\mathcal{L}\left(\lambda_{\mathrm{MT}}/\lambda_{\mathrm{BS}}\right)\left(\mathrm{P_{tx}}+\mathrm{P_{circ}}-\mathrm{P_{idle}}\right)+\mathrm{P_{idle}}+\mathcal{M}\left(\lambda_{\mathrm{MT}}/\lambda_{\mathrm{BS}}\right)\mathrm{P_{circ}}\right]}
\end{aligned}
\qquad (14)
$$

Optimal Transmit Power Given The BSs' Density

B. Optimal Transmit Power Given the Density of the BSs

In this section, we analyze whether there exists an optimal and unique transmit power, $P_{tx}^{(opt)}$, that maximizes the EE formulated in (14), while all the other parameters, including λ_{BS}, are fixed and given. In mathematical terms, the optimization problem can be formulated as follows:

$$\max_{P_{tx}} EE\left(P_{tx}, \lambda_{BS}\right) \quad \text{subject to } P_{tx} \in \left[P_{tx}^{(min)}, P_{tx}^{(max)}\right] \tag{15}$$

where $P_{tx}^{(min)} \geq 0$ and $P_{tx}^{(max)} \geq 0$ are the minimum and maximum power budget of the BSs, respectively. One may assume, without loss of generality, $P_{tx}^{(min)} \to 0$ and $P_{tx}^{(max)} \to \infty$.

The following theorem completely characterizes the solution of (15).

Theorem 1: Let $\mathcal{S}_{\mathcal{P}}\left(\cdot\right)$ be the function defined in Table II. The EE in (14) is a unimodal and strictly pseudo-concave function in P_{tx}. The optimization problem in (15) has a unique solution given by $P_{tx}^{(opt)} = \max\left\{P_{tx}^{(min)}, \min\left\{P_{tx}^{*}, P_{tx}^{(max)}\right\}\right\}$, where P_{tx}^{*} is the only stationary point of the EE in (14) that is obtained as the unique solution of the following equation:

$$\dot{EE}_{P_{tx}}\left(P_{tx}^{*}, \lambda_{BS}\right) = P_{idle} - \mathcal{S}_{\mathcal{P}}\left(P_{tx}^{*}\right) = 0 \Leftrightarrow \mathcal{S}_{\mathcal{P}}\left(P_{tx}^{*}\right) = P_{idle} \tag{16}$$

Optimal BSs' Density Given The Transmit Power

C. Optimal Density Given the Transmit Power of the BSs

In this section, we analyze whether there exists an optimal and unique density of BSs, $\lambda_{\rm BS}^{\rm (opt)}$, that maximizes the EE formulated in (14), while all the other parameters, including $\rm P_{tx}$, are fixed and given. In mathematical terms, the optimization problem can be formulated as follows:

$$\max\nolimits_{\lambda_{\rm BS}} {\rm EE}\left({\rm P_{tx}}, \lambda_{\rm BS}\right) \quad \text{subject to } \lambda_{\rm BS} \in \left[\lambda_{\rm BS}^{\rm (min)}, \lambda_{\rm BS}^{\rm (max)}\right] \tag{17}$$

where $\lambda_{\rm BS}^{\rm (min)} \geq 0$ and $\lambda_{\rm BS}^{\rm (max)} \geq 0$ are the minimum and maximum allowed density of the BSs, respectively. One may assume, without loss of generality, $\lambda_{\rm BS}^{\rm (min)} \to 0$ and $\lambda_{\rm BS}^{\rm (min)} \to \infty$.

The following theorem completely characterizes the solution of (17).

Theorem 2: Let $\mathcal{S}_{\mathcal{D}}\left(\cdot\right)$ be the function defined in Table II. The EE in (14) is a unimodal and strictly pseudo-concave function in $\lambda_{\rm BS}$. The optimization problem in (17) has a unique solution given by $\lambda_{\rm BS}^{\rm (opt)} = \max\left\{\lambda_{\rm BS}^{\rm (min)}, \min\left\{\lambda_{\rm BS}^{*}, \lambda_{\rm BS}^{\rm (max)}\right\}\right\}$, where $\lambda_{\rm BS}^{*}$ is the only stationary point of the EE in (14) that is obtained as the unique solution of the following equation:

$$\dot{\rm EE}_{\lambda_{\rm BS}}\left({\rm P_{tx}}, \lambda_{\rm BS}^{*}\right) = \mathcal{S}_{\mathcal{D}}\left(\lambda_{\rm BS}^{*}\right) - {\rm P_{idle}} = 0 \Leftrightarrow \mathcal{S}_{\mathcal{D}}\left(\lambda_{\rm BS}^{*}\right) = {\rm P_{idle}} \tag{18}$$

Joint Optimal Tx Power and BSs' Density

… alternating optimization …

Algorithm

`Let` $\mathrm{P_{tx}} \in \left[\mathrm{P_{tx}^{(min)}}, \mathrm{P_{tx}^{(max)}}\right]$; $\lambda_{\mathrm{BS}} \in \left[\lambda_{\mathrm{BS}}^{(\mathrm{min})}, \lambda_{\mathrm{BS}}^{(\mathrm{max})}\right]$;

`Set` $\lambda_{\mathrm{BS}} = \overline{\lambda}_{\mathrm{BS}}^{(\mathrm{opt})} \in \left[\lambda_{\mathrm{BS}}^{(\mathrm{min})}, \lambda_{\mathrm{BS}}^{(\mathrm{max})}\right]$ (initial guess); $V = 0$; $\epsilon > 0$;

Repeat

$V_0 = V$;

$\overline{\mathrm{P}}_{\mathrm{tx}}^{*} \leftarrow \dot{\mathrm{E}}\mathrm{E}_{\mathrm{P_{tx}}}\left(\mathrm{P_{tx}}, \overline{\lambda}_{\mathrm{BS}}^{(\mathrm{opt})}\right) = 0; \quad \overline{\mathrm{P}}_{\mathrm{tx}}^{(\mathrm{opt})} = \max\left\{\mathrm{P_{tx}^{(min)}}, \min\left\{\overline{\mathrm{P}}_{\mathrm{tx}}^{*}, \mathrm{P_{tx}^{(max)}}\right\}\right\};$ (16)

$\overline{\lambda}_{\mathrm{BS}}^{*} \leftarrow \dot{\mathrm{E}}\mathrm{E}_{\lambda_{\mathrm{BS}}}\left(\overline{\mathrm{P}}_{\mathrm{tx}}^{(\mathrm{opt})}, \lambda_{\mathrm{BS}}\right) = 0; \quad \overline{\lambda}_{\mathrm{BS}}^{(\mathrm{opt})} = \max\left\{\lambda_{\mathrm{BS}}^{(\mathrm{min})}, \min\left\{\overline{\lambda}_{\mathrm{BS}}^{*}, \lambda_{\mathrm{BS}}^{(\mathrm{max})}\right\}\right\};$ (18)

$V = \mathrm{EE}\left(\overline{\mathrm{P}}_{\mathrm{tx}}^{(\mathrm{opt})}, \overline{\lambda}_{\mathrm{BS}}^{(\mathrm{opt})}\right);$ (14)

Until $|V - V_0| / V \le \epsilon$;

`Return` $\mathrm{P_{tx}^{(opt)}} = \overline{\mathrm{P}}_{\mathrm{tx}}^{(\mathrm{opt})}$; $\lambda_{\mathrm{BS}}^{(\mathrm{opt})} = \overline{\lambda}_{\mathrm{BS}}^{(\mathrm{opt})}$.

The EE is Unimodal and Pseudo-Concave in P_{tx}

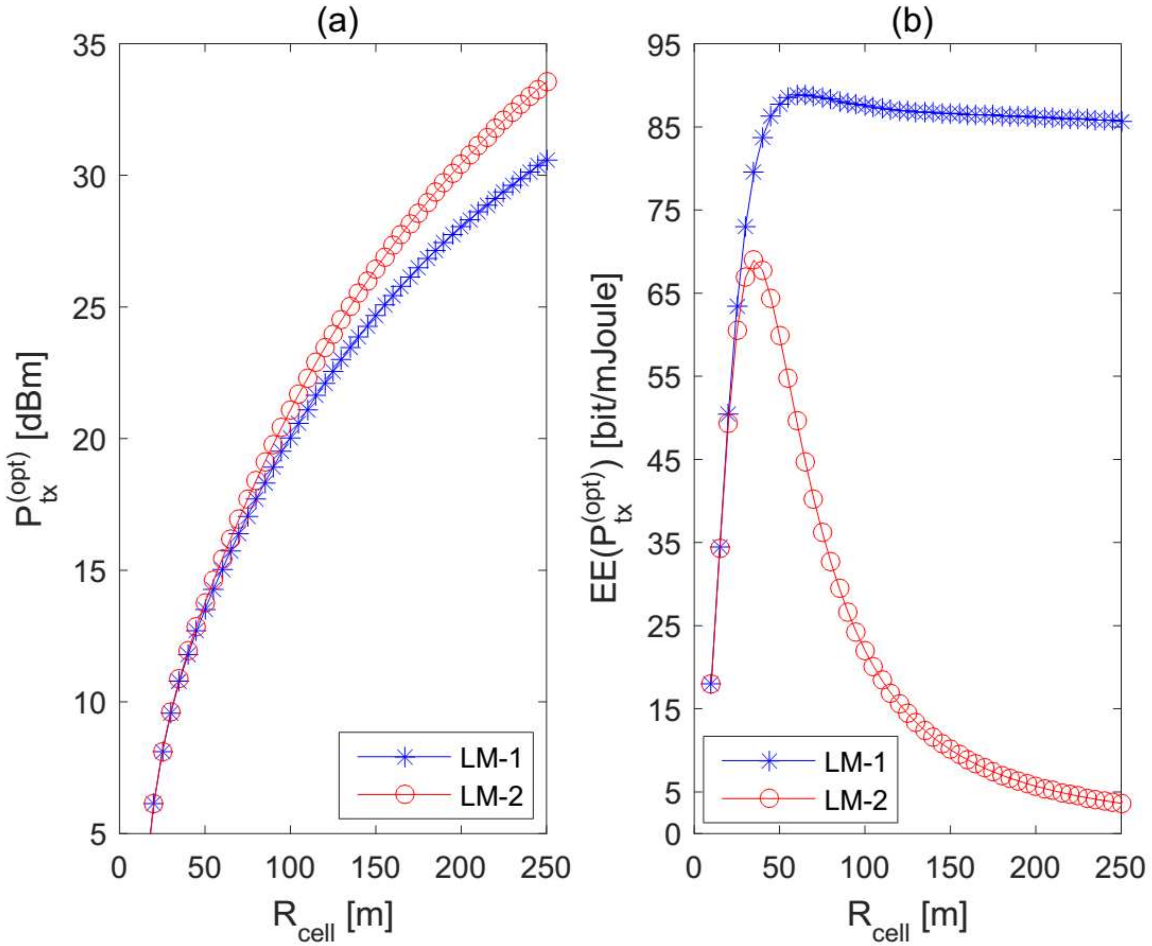


Fig. 5: Optimal transmit power (a) and energy efficiency (b) versus $\mathrm{R_{cell}}$. Solid lines: Optimum from *Theorem 1*. Markers: Optimum from a brute-force search of (15). LM-1: Load Model 1 and LM-2: Load Model 2.

The EE is Unimodal and Pseudo-Concave in P_{tx} (???!!!)

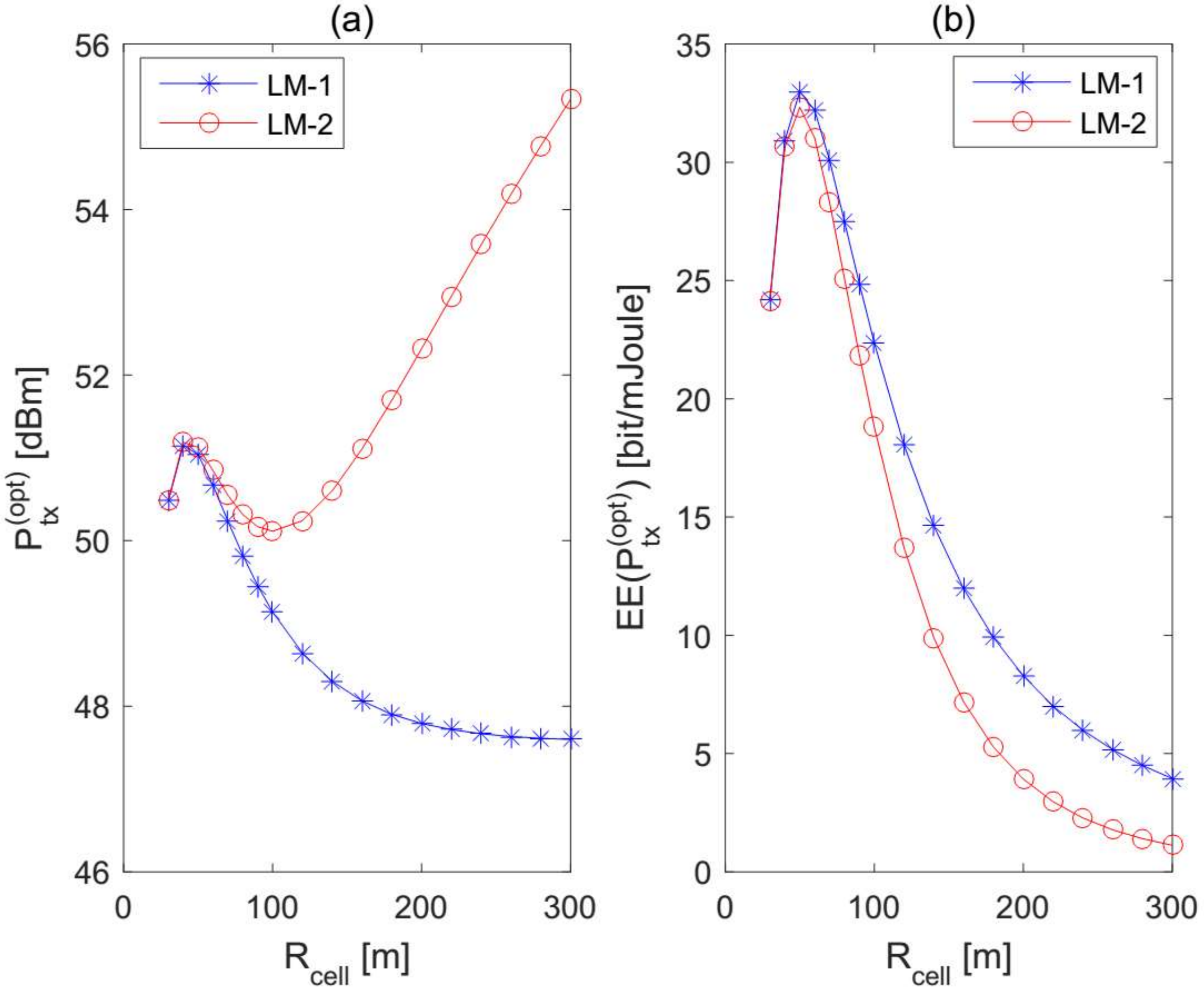


Fig. 2: Optimal transmit power (a) and energy efficiency (b) versus R_{cell}. Solid lines: Optimum from *Theorem 1*. Markers: Optimum from a brute-force search of (15).

The EE is Unimodal and Pseudo-Concave in λ_{BS}

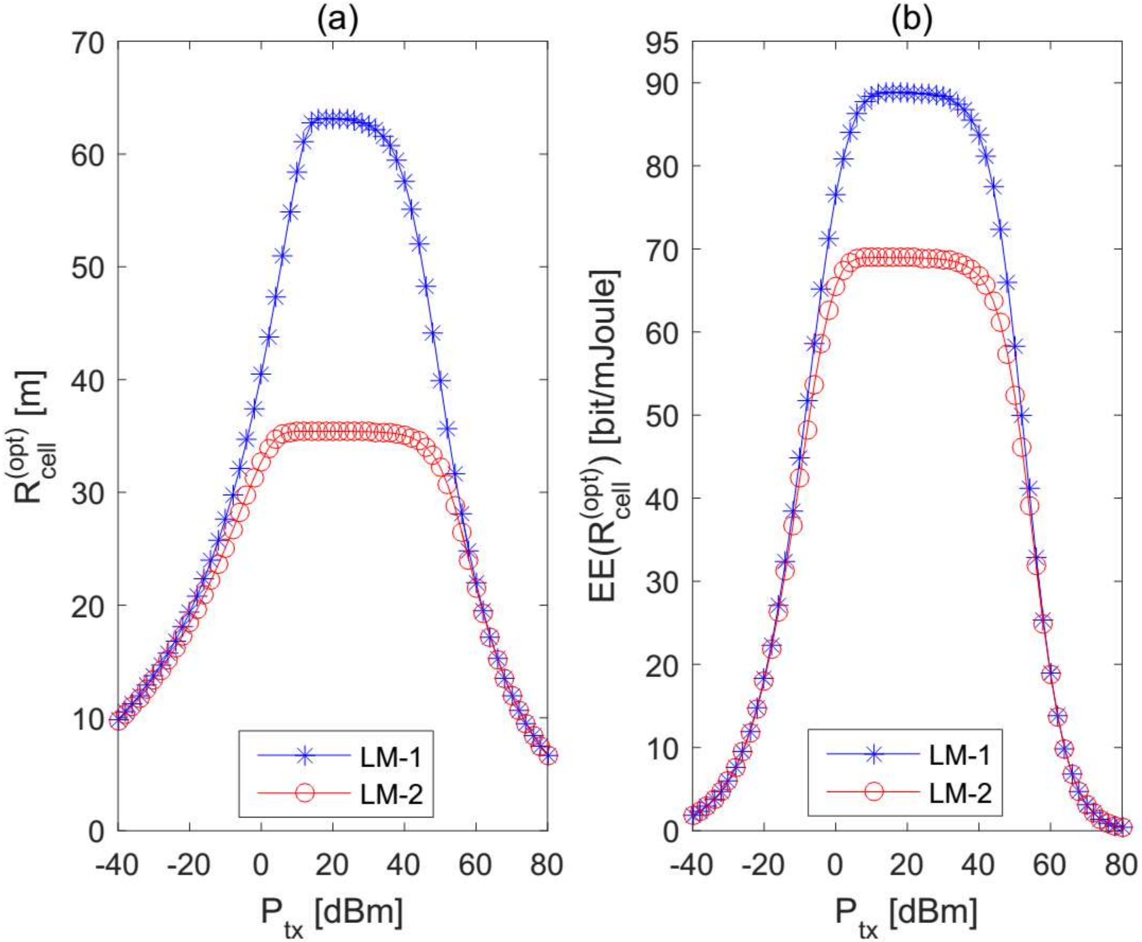


Fig. 6: Optimal density of BSs (R_{cell}) (a) and energy efficiency (b) versus the transmit power. Solid lines: Optimum from *Theorem 2*. Markers: Optimum from a brute-force search of (17). LM-1:Load Model 1, LM-2:Load Model 2.

Intrigued Enough ? – Our Submitted Paper to TWC

TRANSACTIONS ON WIRELESS COMMUNICATIONS

System-Level Modeling and Optimization of the Energy Efficiency in Cellular Networks – A Stochastic Geometry Framework

Marco Di Renzo, *Senior Member, IEEE*, Alessio Zappone, *Senior Member, IEEE*,
Thanh Tu Lam, *Student Member, IEEE*, and Mérouane Debbah, *Fellow, IEEE*

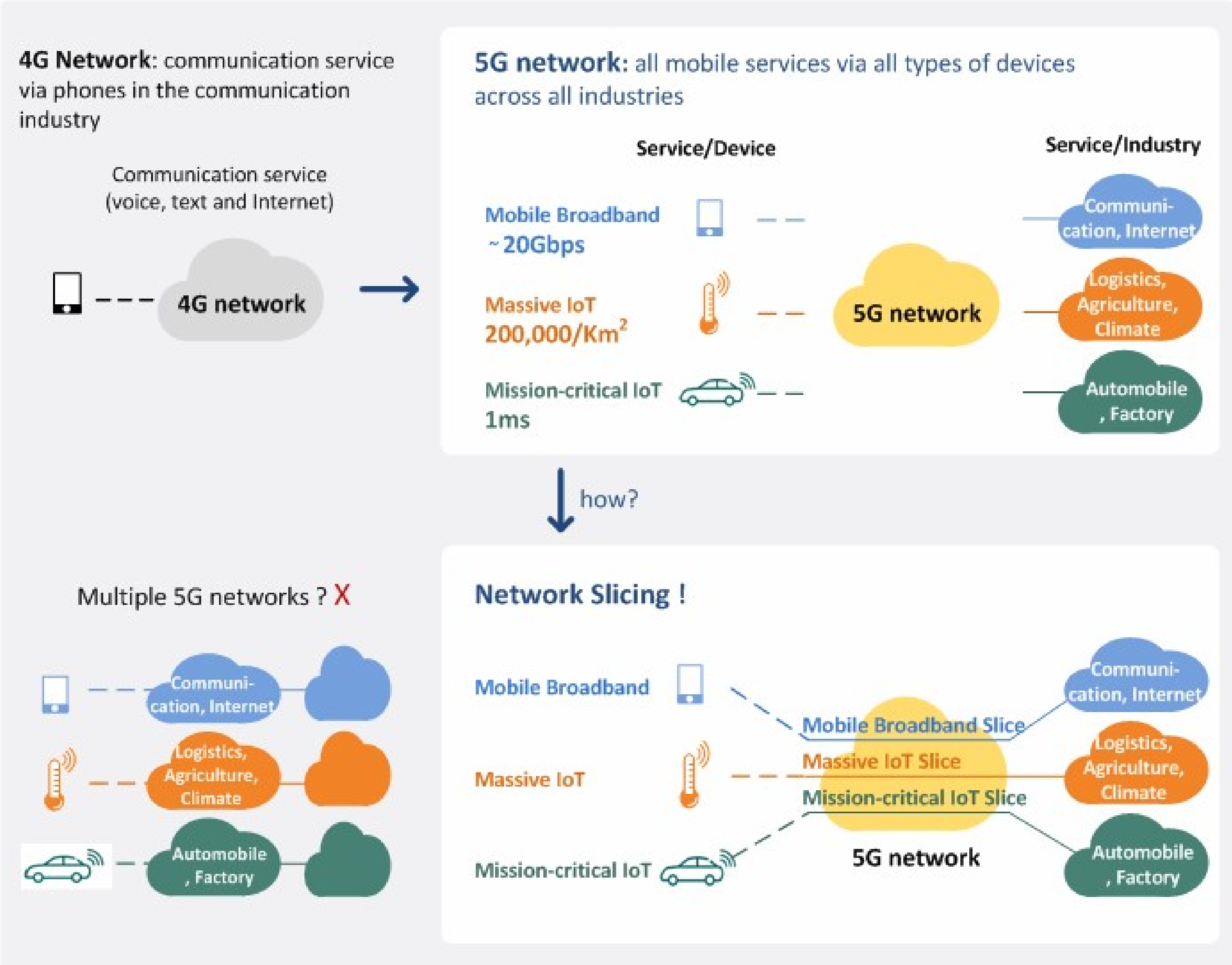
Abstract

In this paper, we analyze and optimize the energy efficiency of downlink cellular networks. With the aid of tools from stochastic geometry, we introduce a new closed-form mathematical expression of the potential spectral efficiency (bit/sec/m^2) in the interference-limited regime. Unlike currently available mathematical frameworks, the proposed analytical formulation explicitly depends on the transmit power and density of the base stations. This is obtained by generalizing the definition of coverage probability

Example 2

System-Level Network Slicing Optimization

On Slicing the Radio Access Network



On Slicing the End-To-End Network

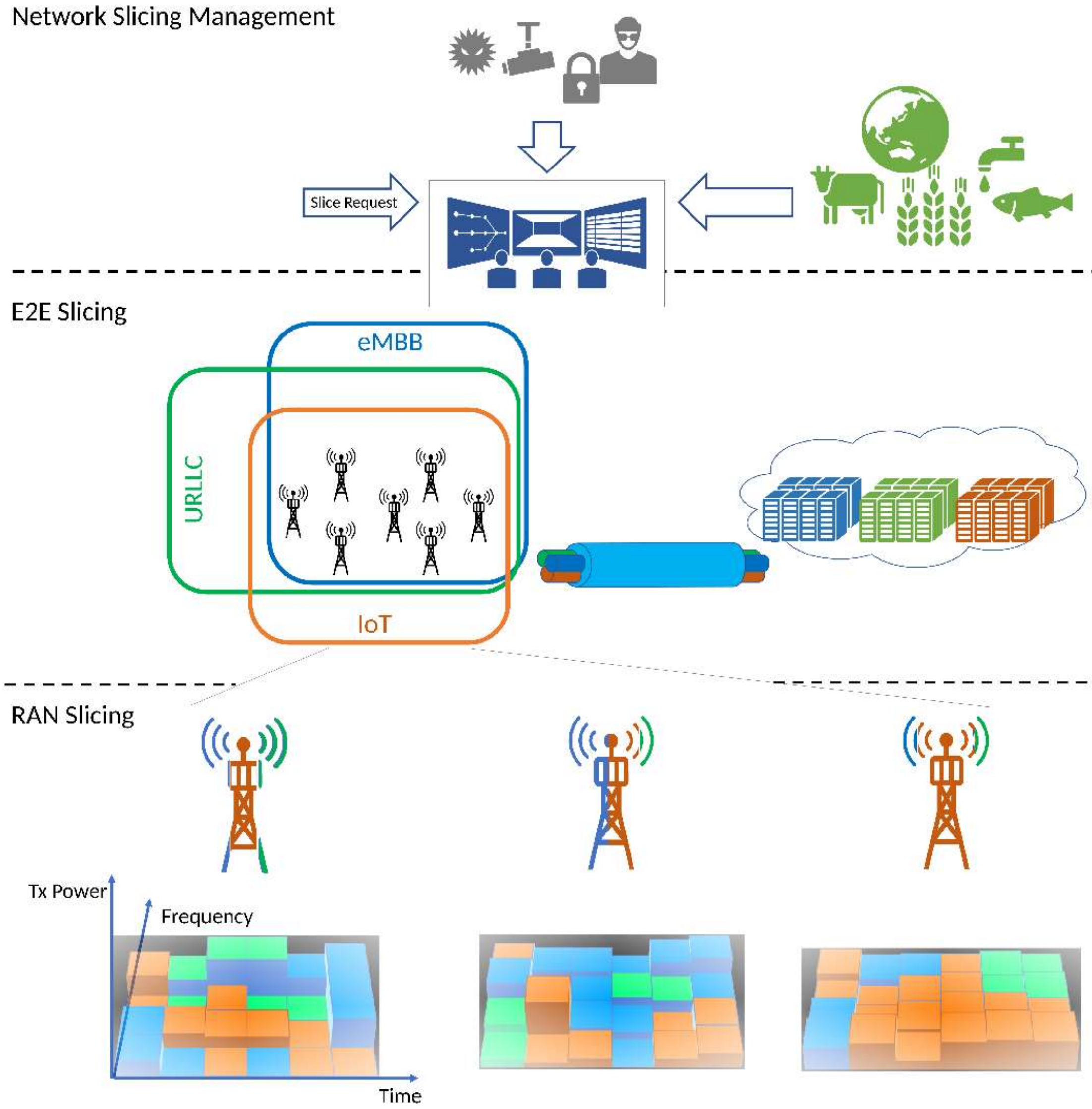


Fig. 1: Illustration of the Network Slicing Concept

Two Network Slicing Questions…

… How a Telecom Operator Should Optimally Allocate its Spectrum and Power to Different Slices ?

… Is Network Slicing Just a Business Need or Does it Lead to a Better Network Design ?

A New Definition of PSE

Proposition 1. *The exact mathematical expression of the PSE is given in Eq.* (6) *below:*

$$\mathrm{PSE}\left(P,B,\lambda_{\mathrm{T}}\right)=B\log_2\left(1+\gamma_I\right)\frac{\lambda_{\mathrm{BS}}L\left(\frac{\lambda_{\mathrm{T}}}{\lambda_{\mathrm{BS}}}\right)}{1+L\left(\frac{\lambda_{\mathrm{T}}}{\lambda_{\mathrm{BS}}}\right)\Upsilon\left(\gamma_I,\beta\right)}$$
$$\times\left[1-\exp\left(-\pi\lambda_{\mathrm{BS}}\left(\tau_A\frac{P}{B}\right)^{2/\beta}\left(1+L\left(\frac{\lambda_{\mathrm{T}}}{\lambda_{\mathrm{BS}}}\right)\Upsilon\left(\gamma_I,\beta\right)\right)\right)\right] \tag{6}$$

where $\tau_A=\left(\kappa\gamma_A N_0\right)^{-1}$ and:

$$L\left(\frac{\lambda_{\mathrm{T}}}{\lambda_{\mathrm{BS}}}\right)=1-\left(1+\frac{1}{3.5}\frac{\lambda_{\mathrm{T}}}{\lambda_{\mathrm{BS}}}\right)^{-3.5}\geq 0 \tag{7}$$

$$\Upsilon\left(\gamma_I,\beta\right)={}_2F_1\left(-\frac{2}{\beta},1,1-\frac{2}{\beta},-\gamma_I\right)-1\geq 0 \tag{8}$$

with ${}_2F_1\left(\cdot\right)$ denoting the Gauss hypergeometric function.

Network Slicing System-Level Optimization

Problem `Bi-Sharing`:

$$\begin{aligned}
\text{minimize} \quad & \mathbb{1} \\
\text{subject to} \quad & k_1^{(\mathrm{T1})} B_{\mathrm{T1}} \left(1 - e^{-(\frac{P_{\mathrm{T1}}}{B_{\mathrm{T1}}})^{2/\beta} k_2^{(\mathrm{T1})}} \right) \geq \alpha_{\mathrm{T1}} \mathrm{PSE}_{\mathrm{NoSlicing}}; \\
& k_1^{(\mathrm{T2})} B_{\mathrm{T2}} \left(1 - e^{-(\frac{P_{\mathrm{T2}}}{B_{\mathrm{T2}}})^{2/\beta} k_2^{(\mathrm{T2})}} \right) \geq \alpha_{\mathrm{T2}} \mathrm{PSE}_{\mathrm{NoSlicing}}; \\
& B_{\mathrm{T1}} + B_{\mathrm{T2}} \leq B_{\mathrm{tot}}; \\
& P_{\mathrm{T1}} + P_{\mathrm{T2}} \leq P_{\mathrm{tot}}; \\
& B_{\mathrm{T1}}, B_{\mathrm{T2}}, P_{\mathrm{T1}}, P_{\mathrm{T2}} \in \mathbb{R}_+;
\end{aligned}$$

STORNS: Proposed Stochastic RAN Slicer

Algorithm 1 Stochastic RAN Slicer (STORNS)

1) Initialise set $\boldsymbol{\mu}$ and γ.
2) Initialise sets $\boldsymbol{b} \leftarrow 0$, $\boldsymbol{p} \leftarrow 0$ and value $k \leftarrow 0$.
3) Solve Problem `MultiTenant-Optimizer (DUAL)` (with INPUT $\boldsymbol{b}^{(k)}$ and $\boldsymbol{p}^{(k)}$) and get $\boldsymbol{\mu}^{(k)}$.
4) Calculate $\boldsymbol{\mu}^{(k+1)}$ based on Eq. (17).
5) Update $\boldsymbol{\zeta}^{(k+1)}$ based on Eq. (15).
6) Solve Problem `MultiTenant-Optimizer (UNCONSTRAINED)` (with INPUT $\boldsymbol{\mu}^{(k+1)}$) and get $\boldsymbol{b}^{(k+1)}$ and $\boldsymbol{p}^{(k+1)}$.
7) If $(\boldsymbol{\mu}^{(k+1)} \neq \boldsymbol{\mu}^{(k)})$, then increase $k = k + 1$ and Go to step (3).
8) Mark $\boldsymbol{\mu}^{(*)} = \boldsymbol{\mu}^{(k+1)}$ as the optimal Lagrange multipliers.
9) Solve Problem `MultiTenant-Optimizer (UNCONSTRAINED)` (with INPUT $\boldsymbol{\mu}^{(*)}$) and get the optimal solution of $\boldsymbol{b}^{(*)}$ and $\boldsymbol{p}^{(*)}$.

STORNS: Performance Close to the Optimum

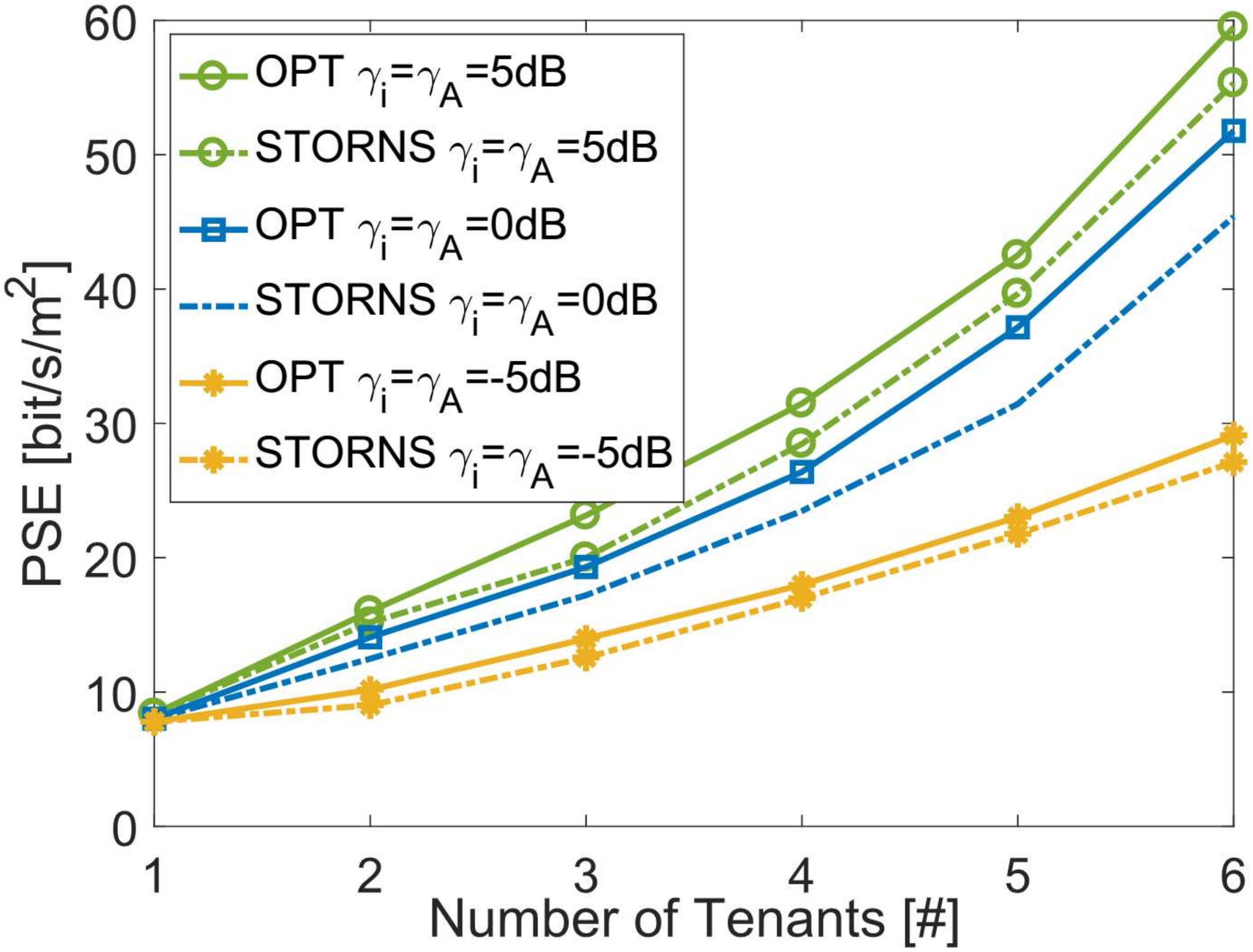


Fig. 4: Optimal vs STORNS

STORNS: Lower Computational Complexity than OPT

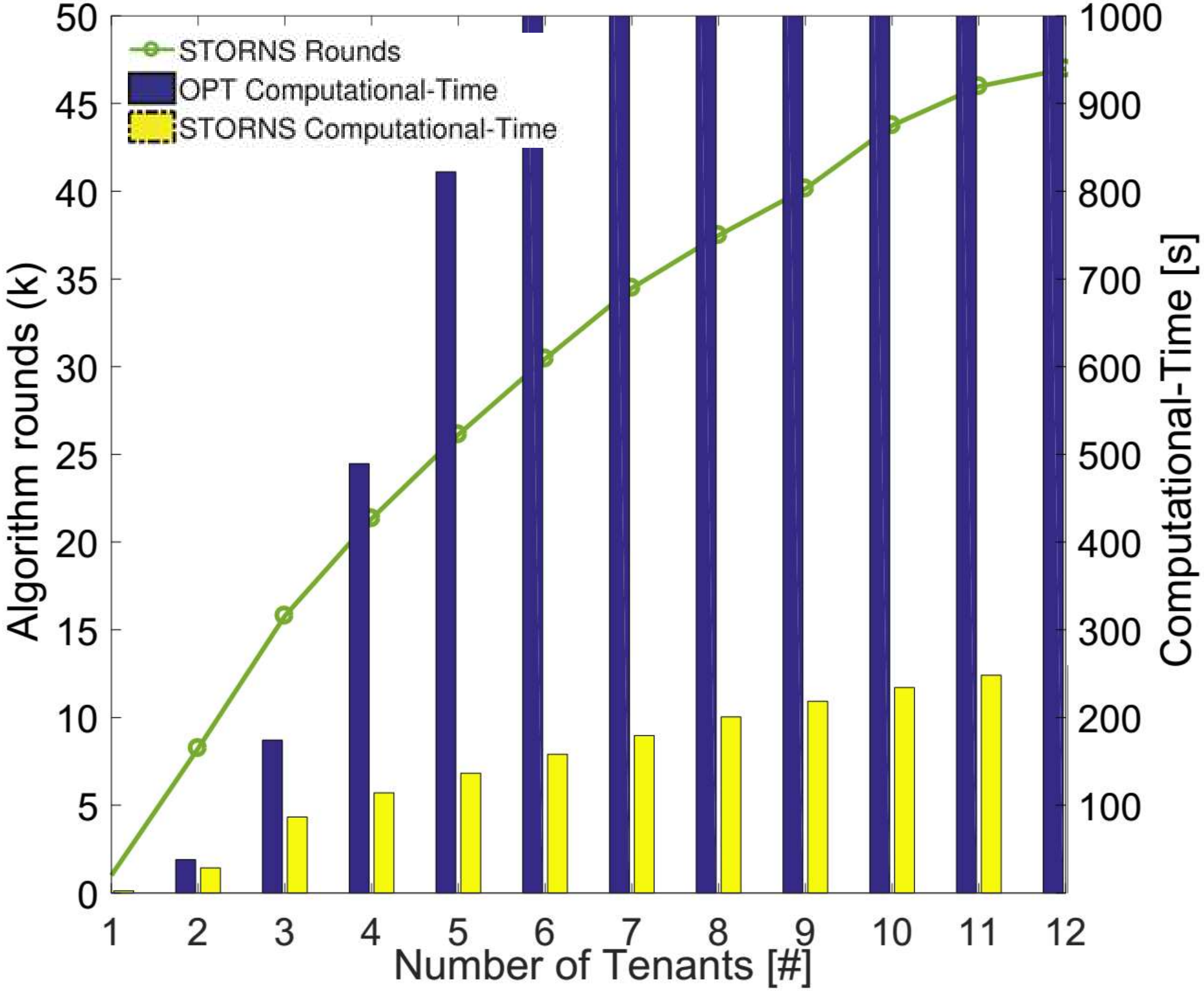


Fig. 6: Computational Analysis

STORNS: Slicing is Better than Not-to-Slicing

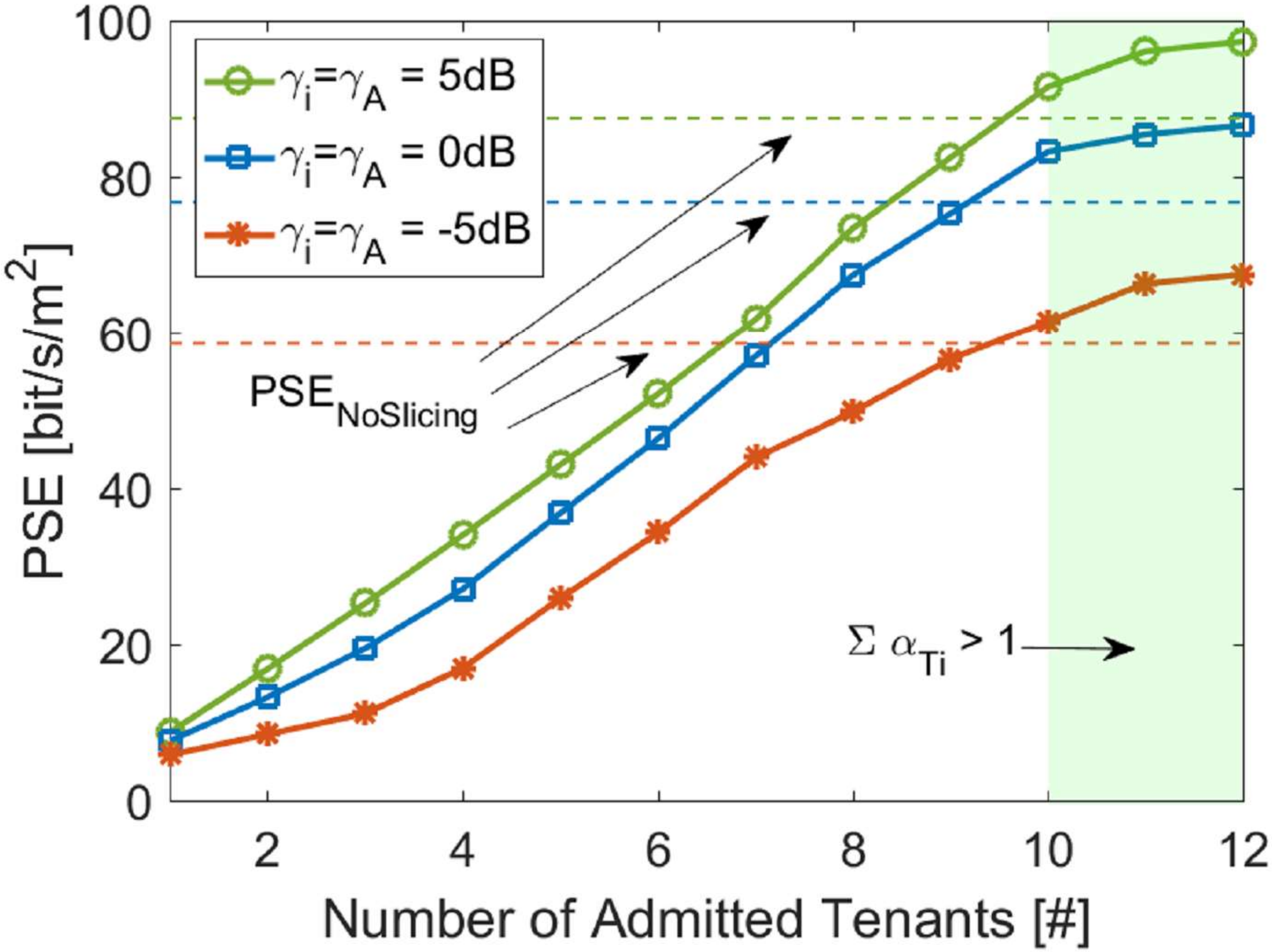


Fig. 5: Potential benefits of network slicing

Intrigued Enough ? Our Submitted Paper to INFOCOM

STORNS: Stochastic Radio Access Network Slicing

Vincenzo Sciancalepore, Marco Di Renzo, Xavier Costa-Perez
NEC Europe Labs (Germany) & CNRS / Paris-Saclay University (France)

Abstract*—Network virtualization and softwarization are key enablers of the novel *network slicing* concept. Network slicing introduces new business models such as allowing telecom providers to lease virtualized slices of their infrastructure to *tenants*, e.g., industry verticals (automotive, e-health, factories, etc.). However, this new paradigm poses a major challenge when applied to radio access networks (RAN): *How to achieve revenue maximization while meeting the diverse service level agreements (SLAs) requested by the tenants?

In this paper, we propose a new analytical framework, based on stochastic geometry theory, to model realistic RANs that leverage the business opportunities offered by network slicing. Moreover, we mathematically prove the benefits of network as compared to un-sliced RANs. Based on this finding, we design a new admission control functional block, STORNS, which takes decisions considering per slice SLA guaranteed average experienced throughput. A radio resource allocation strategy is introduced to optimally allocate transmit power and bandwidth (i.e., a slice of radio access resources) to the users of each tenant of the cellular network. Numerical results are illustrated to validate our proposed solution in terms of potential spectral efficiency and to compare it against un-sliced RANs.

I. Introduction

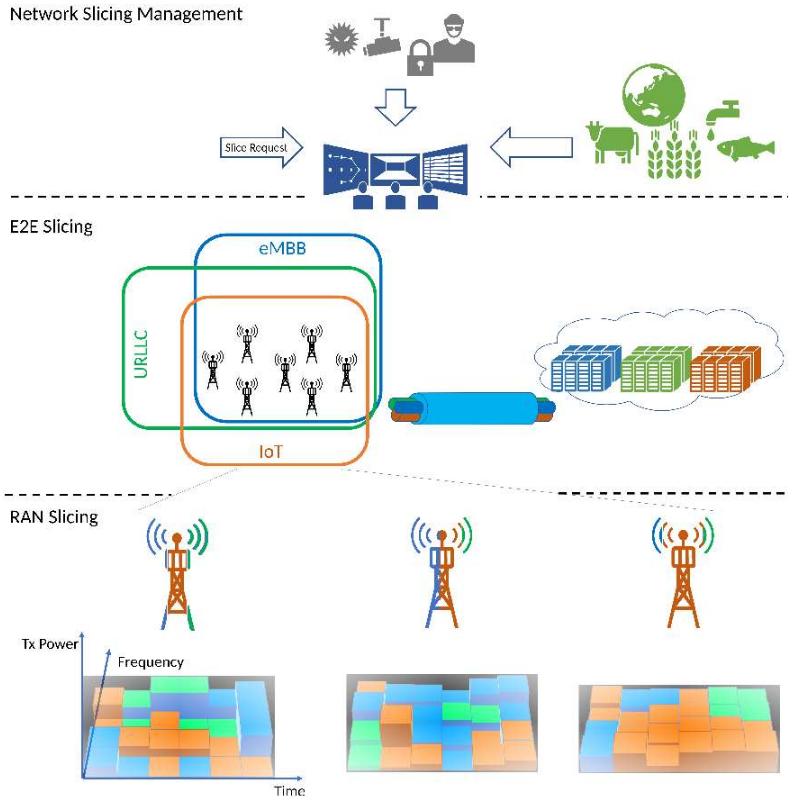
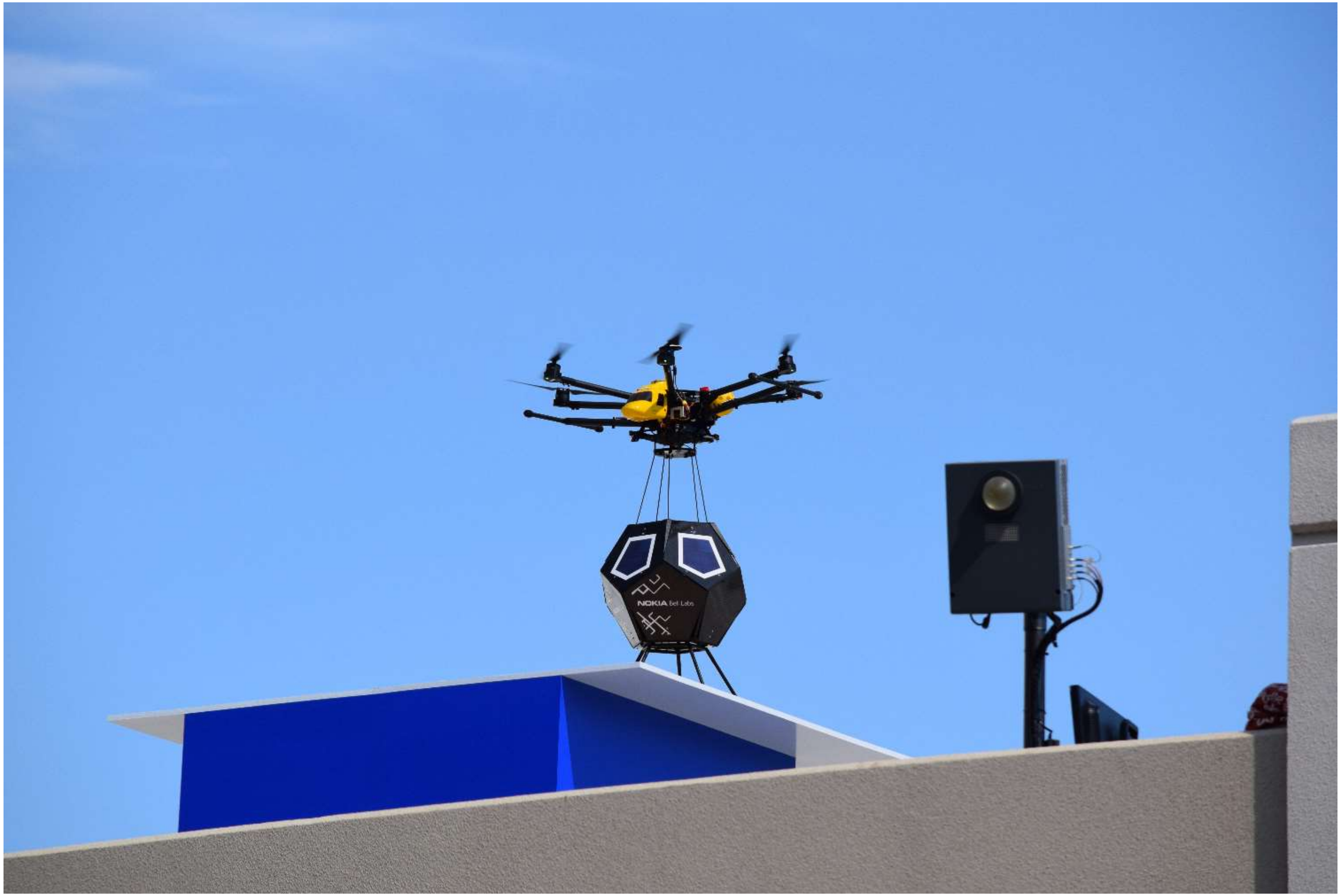


Fig. 1: Illustration of the Network Slicing Concept

Example 4

Energy-Neutral Cellular Networks

What Is The Density of BSs Powered By Renewables ?



Example 3

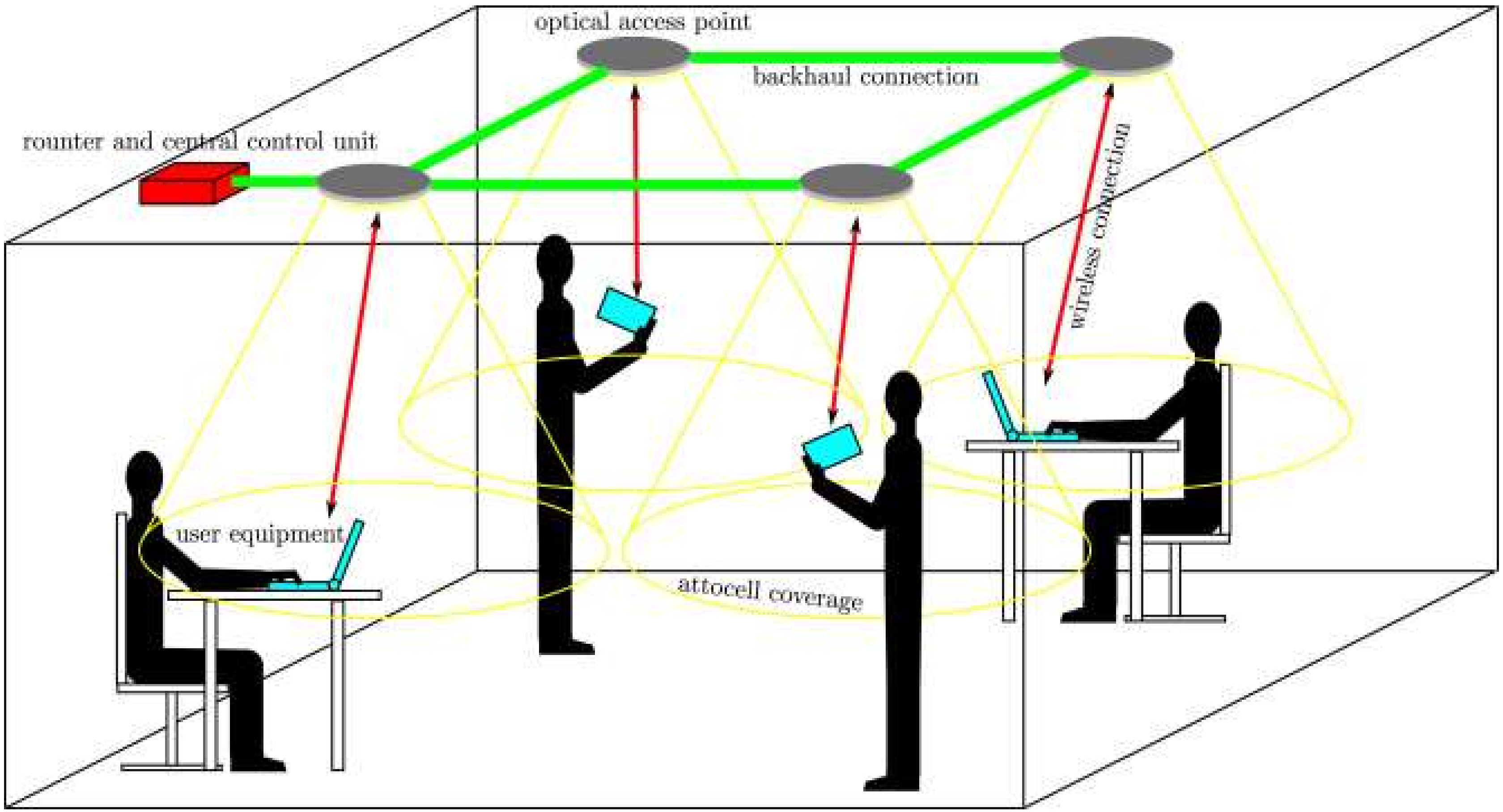
From 2D to 3D Cellular Networks

What Is The Optimal Elevation of the Base Stations ?

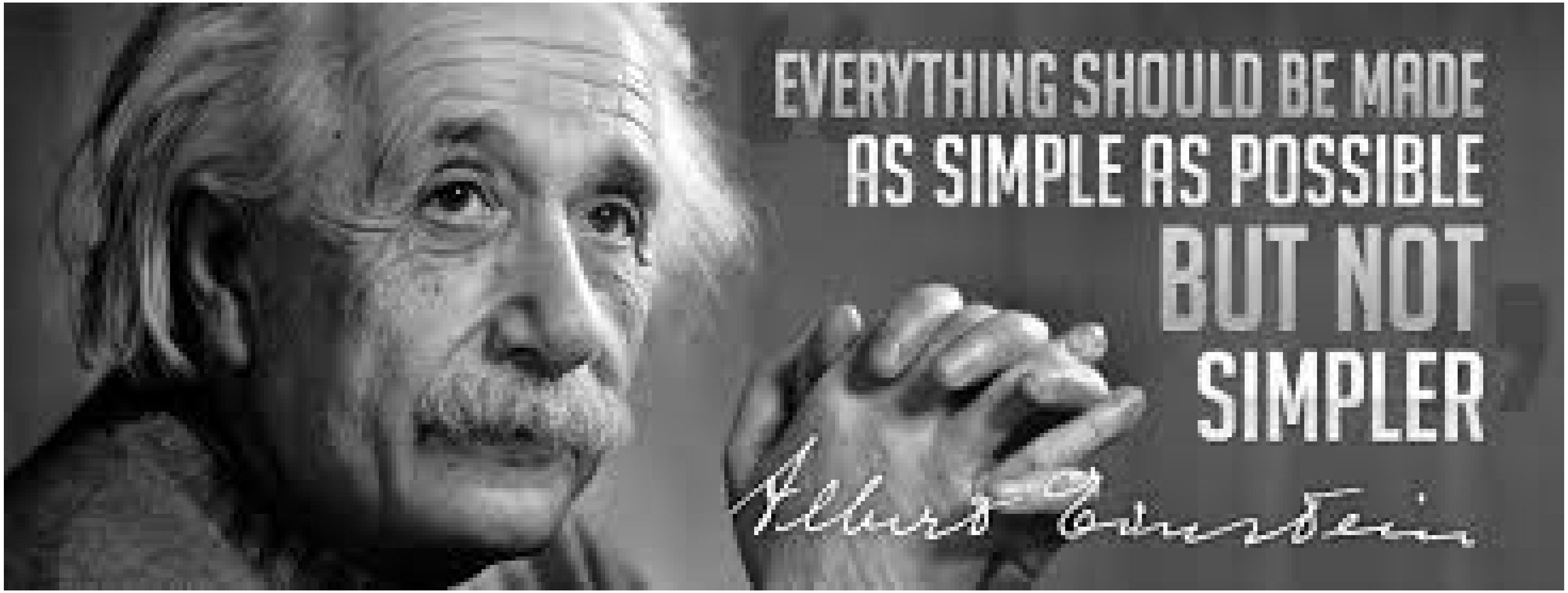
Example 4

From WiFi to LiFi: That Light Be !

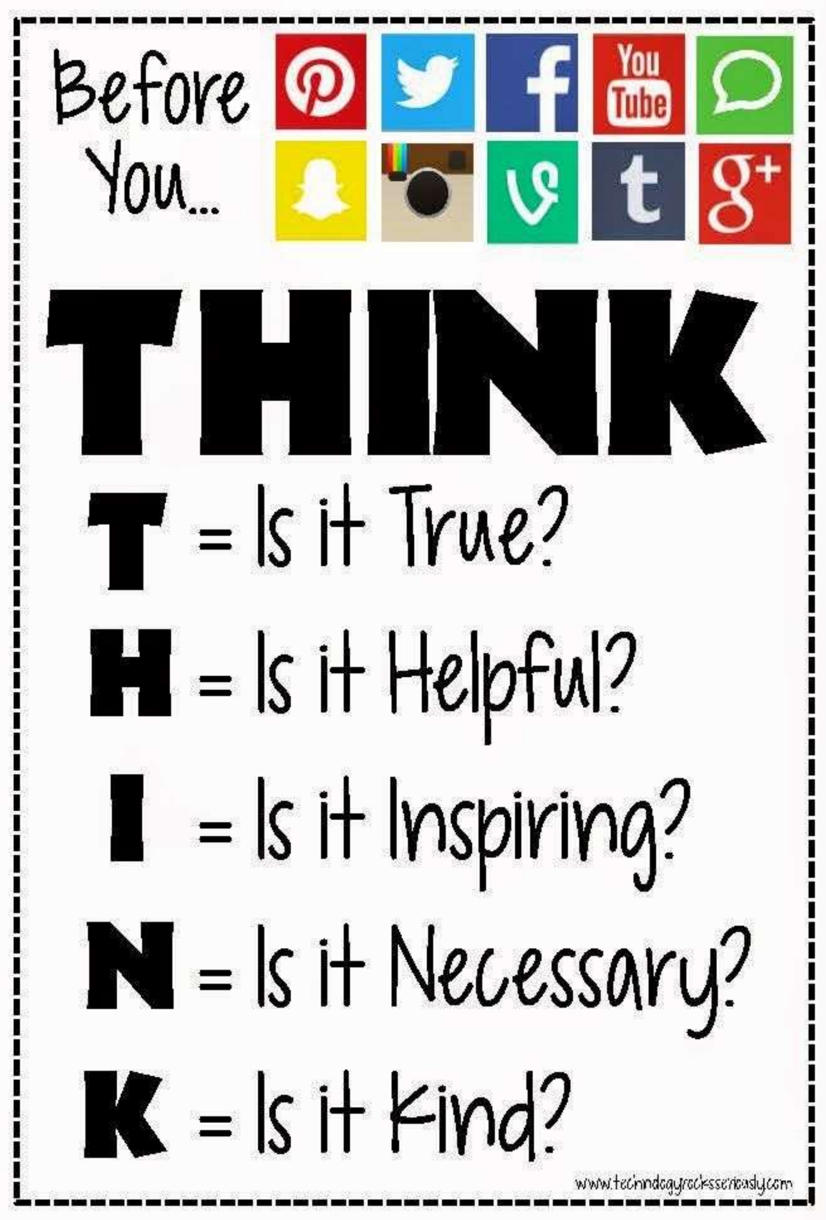
What Is The Optimal Elevation and Density of LEDs ?



The Bottom Line of SG Modeling According to "ME"



The Bottom Line of SG Modeling According to "ME"



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  "Modeling mmWave cellular communications via hyper dense small cell deployments"

- **This research project is concerned with assessing fundamental trade-offs between spectrum and infrastructure sharing, by combining SDN and NFV principles and to quantify the resulting network performance gains**

- **Ph.D. Scholarship CNRS-2:**

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- **This research project is concerned with rethinking the cell-centric cellular architecture by developing user-centric concepts that maximizes throughput and minimizes power consumption, to enable multiple per-user connections with several access points for infinite capacity and zero latency perception**

… best way to apply …
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Many Thanks to Our 5Gwireless & 5Gaura Students !!!

Thank You for Your Attention

- **ETN-5Gwireless (H2020-MCSA, grant 641985)**
- **ETN-5Gaura (H2020-MCSA, grant 675806)**

Two European Training Networks on 5G Wireless Networks

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