Markov Marked Point Processes for Vertex creation in Multi-layer Spatially Embedded Random Networks ?

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HAMMERSLEY-CLIFFORD-RIPLEY-KELLY THEOREM

INTERACTION FUNCTION

$$f(\bar{x}) \propto h(\bar{x}) = \prod_{z \subseteq \bar{x}} \phi(z)$$

CONDITIONAL INTENSITY

$$\lambda(u, \bar{x}) = \frac{f(\bar{x} \cup \{u\})}{f(\bar{x})} = \frac{h(\bar{x} \cup \{u\})}{h(\bar{x})} \quad u \notin \bar{x}$$

PAIRWISE INTERACTION PROCESS

$$f(\bar{x}) \propto h(\bar{x}) = \prod_{x_i \in \bar{x}} \phi(x_i) \prod_{x_i x_j \in x} \phi(x_i, x_j)$$

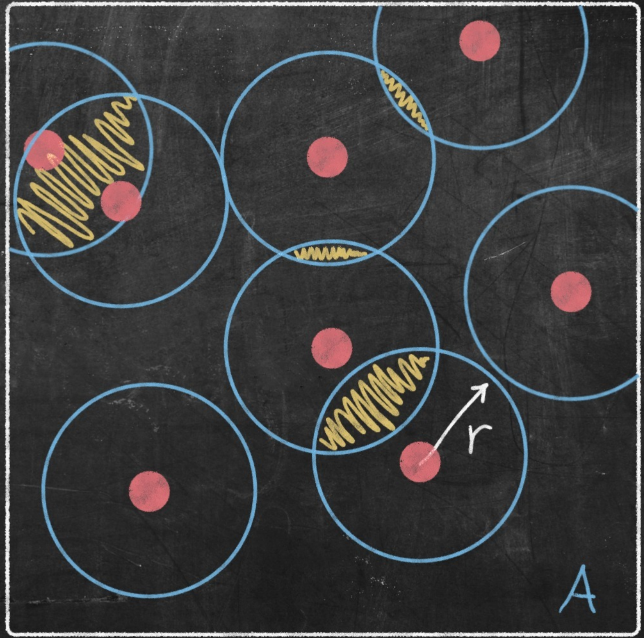
STRAUSS PROCESS

$$\phi(x_i) = \beta \qquad \phi(x_i, x_j) = \begin{cases} \gamma & \text{if } \|x_i - x_j\| < r \\ 1 & \text{otherwise} \end{cases}$$

$\gamma = 1$ POISSON PROCESS

$0 < \gamma < 1$ $\qquad \gamma > 1$?

$\gamma = 0$ HARD CORE PROCESS



$$\bar{x} = \{x_1, \dots, x_n\} \quad x_i \in A \quad n \geq 0$$

$$\lambda(u, \bar{x}) = \beta \gamma^{t(u, \bar{x})}$$

Hammersley-Clifford-Ripley-Kelly Theorem

INTERACTION FUNCTION

$$f(\bar{y}) \propto h(\bar{y}) = \prod_{z \subseteq \bar{y}} \phi(z)$$

CONDITIONAL INTENSITY

$$\lambda((u,a),\bar{y}) = \frac{f(\bar{y} \cup \{(u,a)\})}{f(\bar{y})} = \frac{h(\bar{y} \cup \{(u,a)\})}{h(\bar{y})}$$

PAIRWISE INTERACTION PROCESS

$$f(\bar{y}) \propto h(\bar{y}) = \prod_{(u,a) \in \bar{y}} \phi((u,a)) \prod_{(u,a),(v,b) \in \bar{y}} \phi((u,a),(v,b))$$

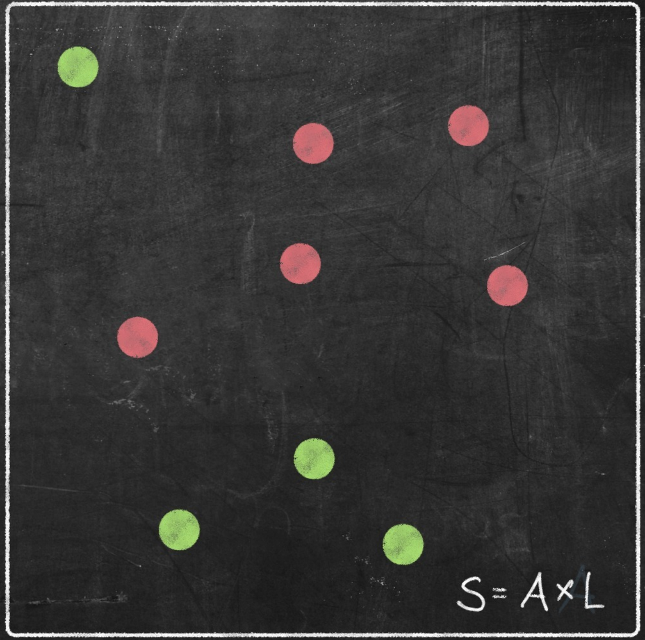
STRAUSS PROCESS

$$\phi(u,a) = \beta_a \qquad \phi((u,a),(v,b)) = \begin{cases} \gamma_{ab} & \text{if } dist(u,v) < r_{ab} \\ 1 & \text{otherwise} \end{cases}$$

$\bar{Y}(A,L)$ INDEPENDENT

$\bar{Y}(L|A)$ CONDITIONAL ON LOCATION

$\bar{Y}(A|L)$ CONDITIONAL ON MARKS



$\bar{y} = \{(u,a), \ldots, (v,b)\}$ $\quad u,v \in A$, $\; a,b \in L$

$$\lambda((u,a),\bar{y}) = \beta_a \prod_{b \in L} \gamma_{ab}^{\,t_{ab}((u,a),\bar{y})}$$

REAL-WORLD NETWORK

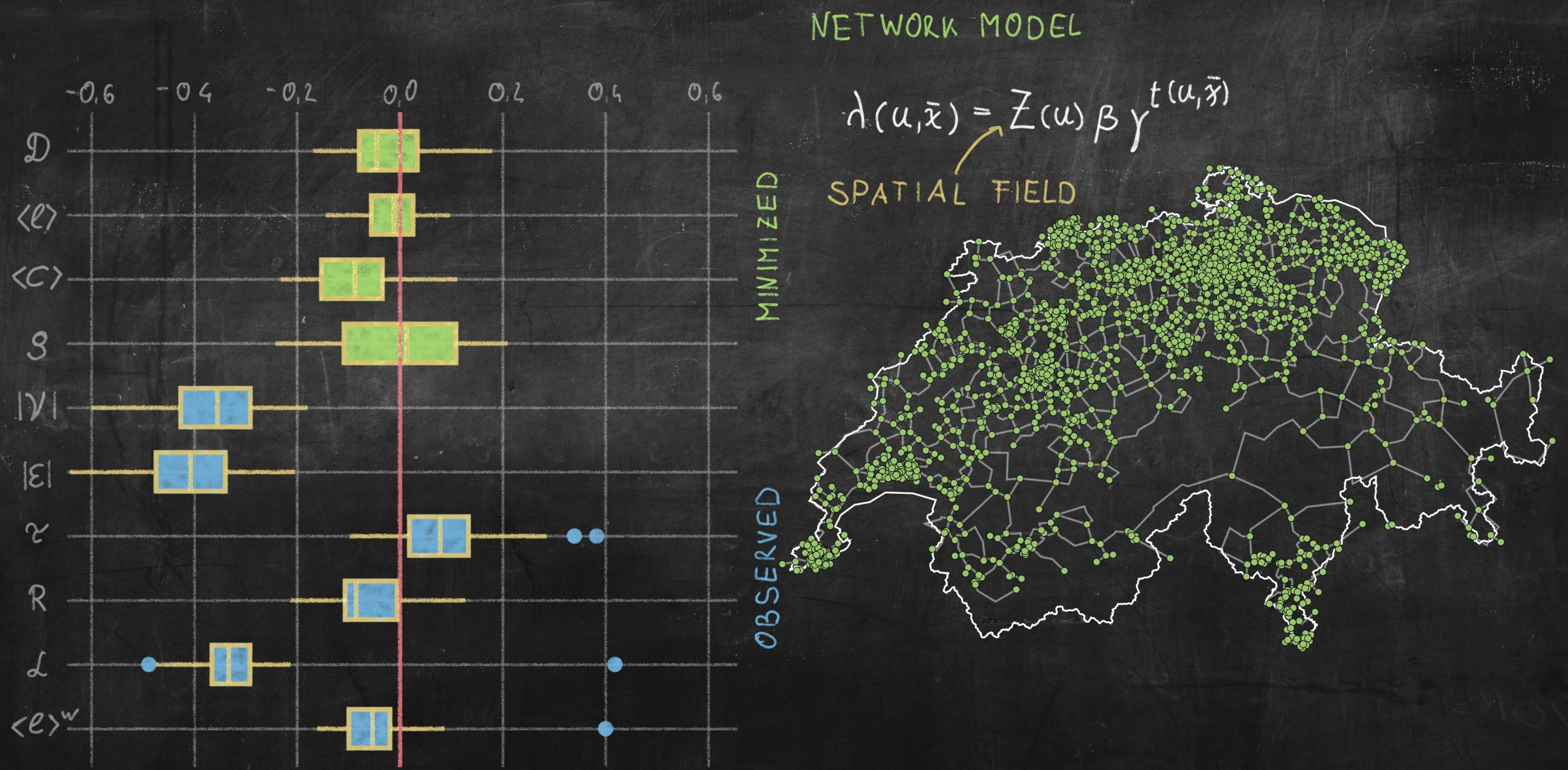
NETWORK MODEL

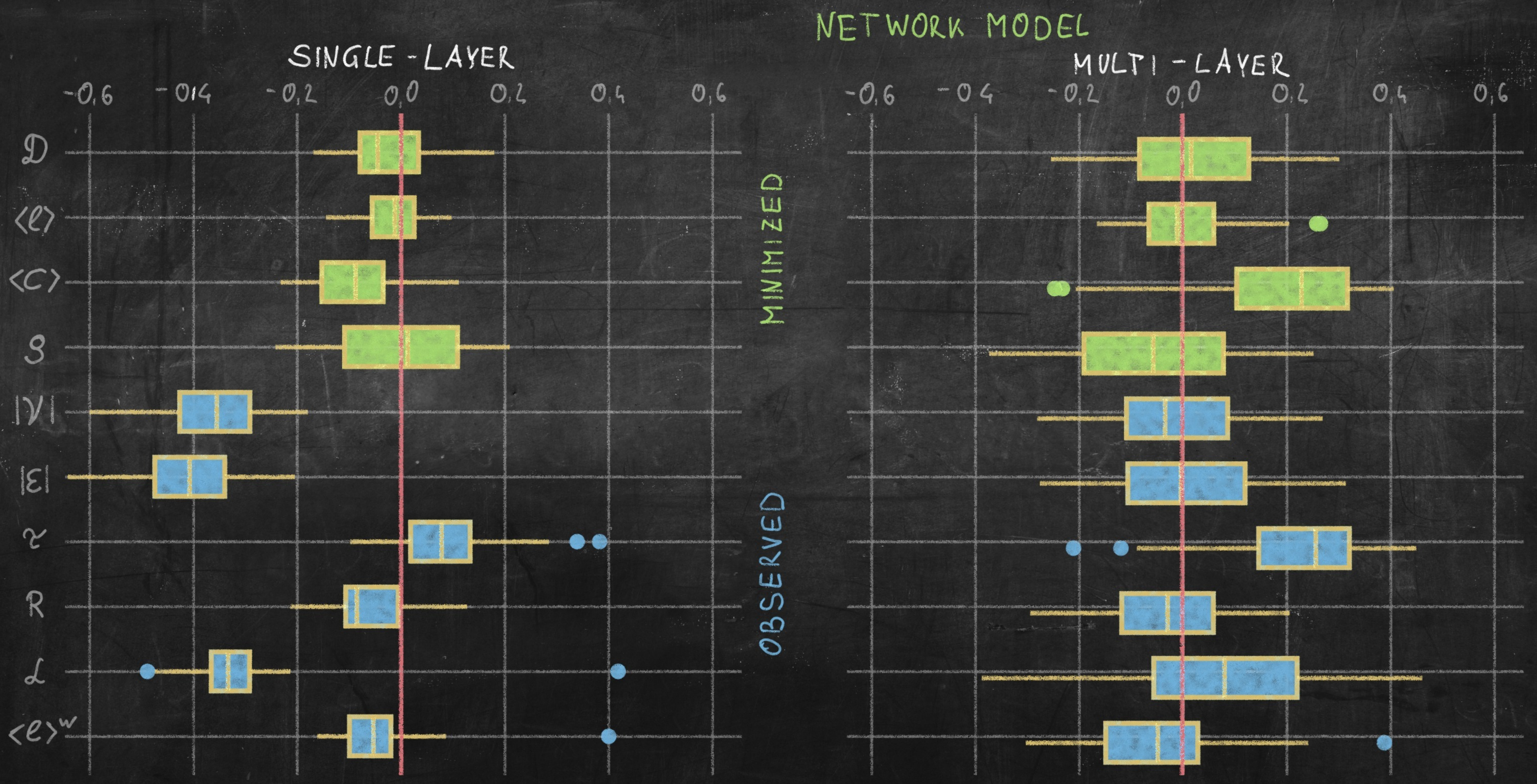
$$\lambda(u,\bar{x}) = Z(u)\,\beta\,\gamma^{t(u,\bar{x})}$$

SPATIAL FIELD

$|\mathcal{V}| = 5585$ $\quad \mathcal{D} = 101$ $\quad \langle \ell \rangle = 42{,}621$

$|\mathcal{E}| = 8294$ $\quad \langle C \rangle = 7{,}497\mathrm{e}{-2}$ $\quad \mathcal{S} = 2{,}954\,\mathrm{e}{-3}$


NETWORK MODEL
$\lambda(u,\bar{x}) = Z(u)\,\beta\,\gamma^{t(u,\bar{x})}$
SPATIAL FIELD
-0,6
-0,4
-0,2
0,0
0,2
0,4
0,6
$\mathcal{D}$
$\langle \ell \rangle$
$\langle C \rangle$
$\mathcal{S}$
$|\mathcal{V}|$
$|\mathcal{E}|$
R
$\mathcal{L}$
$\langle e \rangle^w$
MINIMIZED
OBSERVED


NETWORK MODEL
SINGLE-LAYER
MULTI-LAYER
-0,6 -0,4 -0,2 0,0 0,2 0,4 0,6
D
⟨ℓ⟩
⟨C⟩
S
|V|
|E|
r
R
L
⟨e⟩w
MINIMIZED
OBSERVED

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