Contagion dynamics for topological data analysis

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UNIVERSITY OF OXFORD
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MRC Medical Research Council
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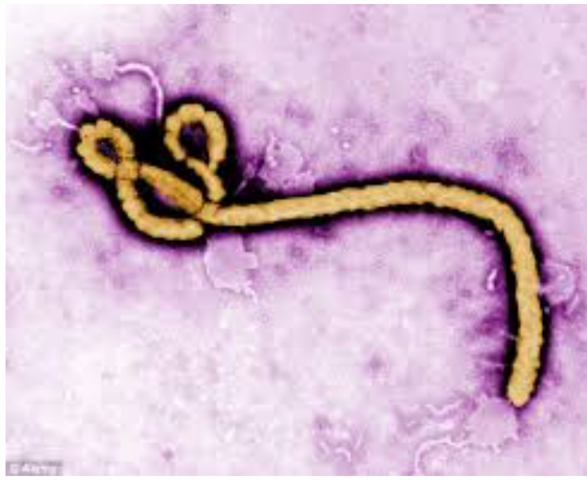
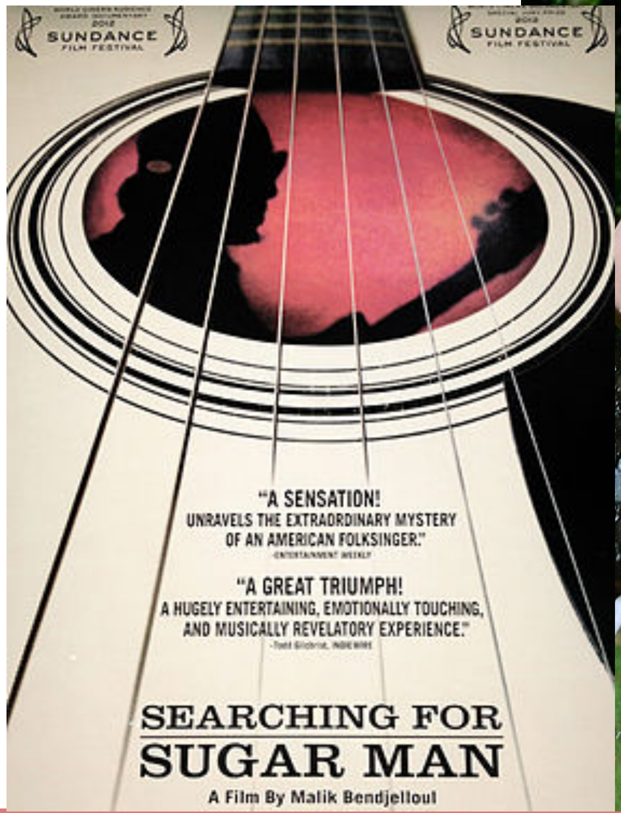
Complex Contagions

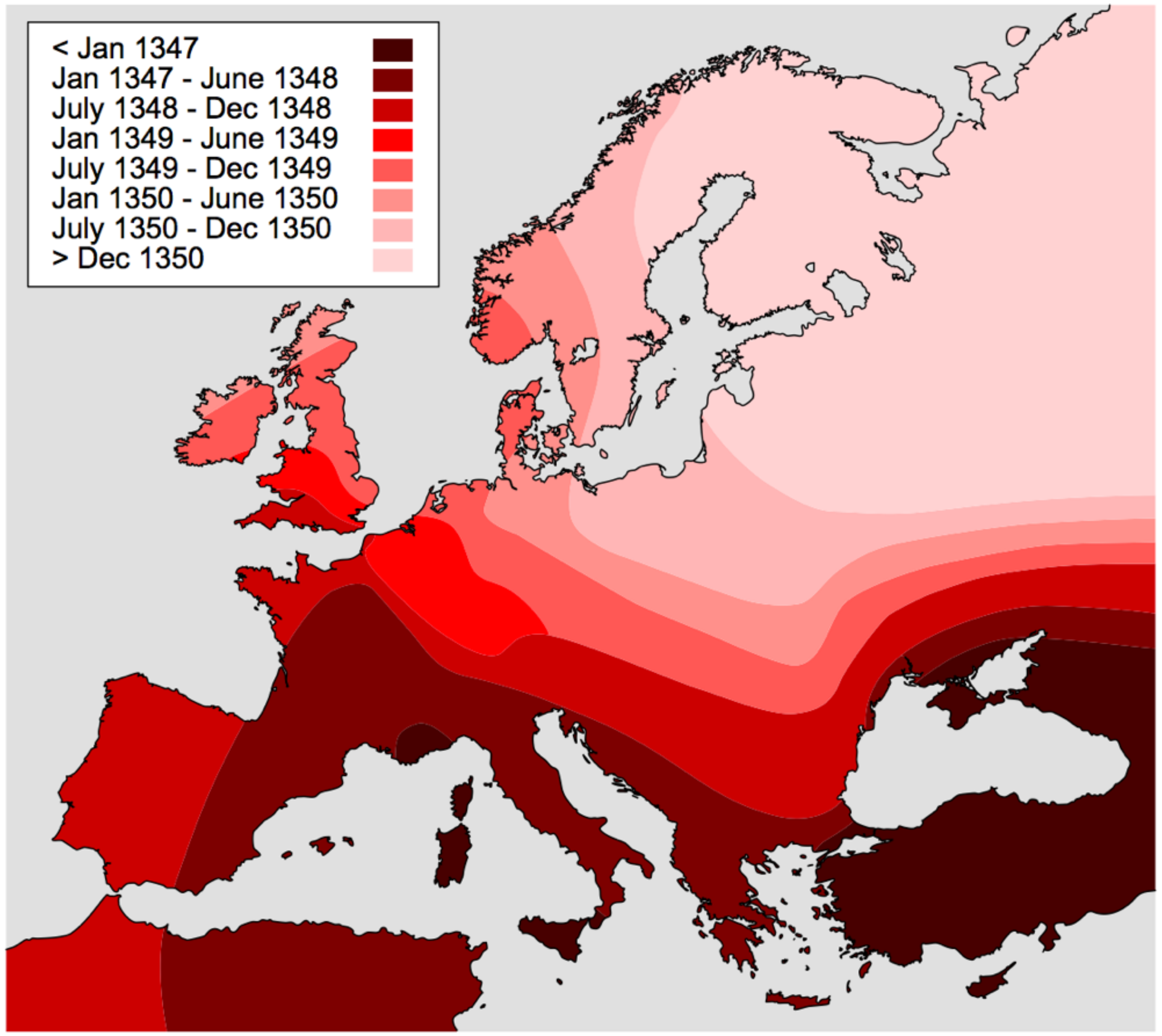


- **Epidemics**
  - Influenza, measles, etc. across social networks
  - Computer viruses across technological networks
- **Social contagion**
  - Viral marketing, viral memes
  - Music preference, voting, trends



Epidemics: Then and Now

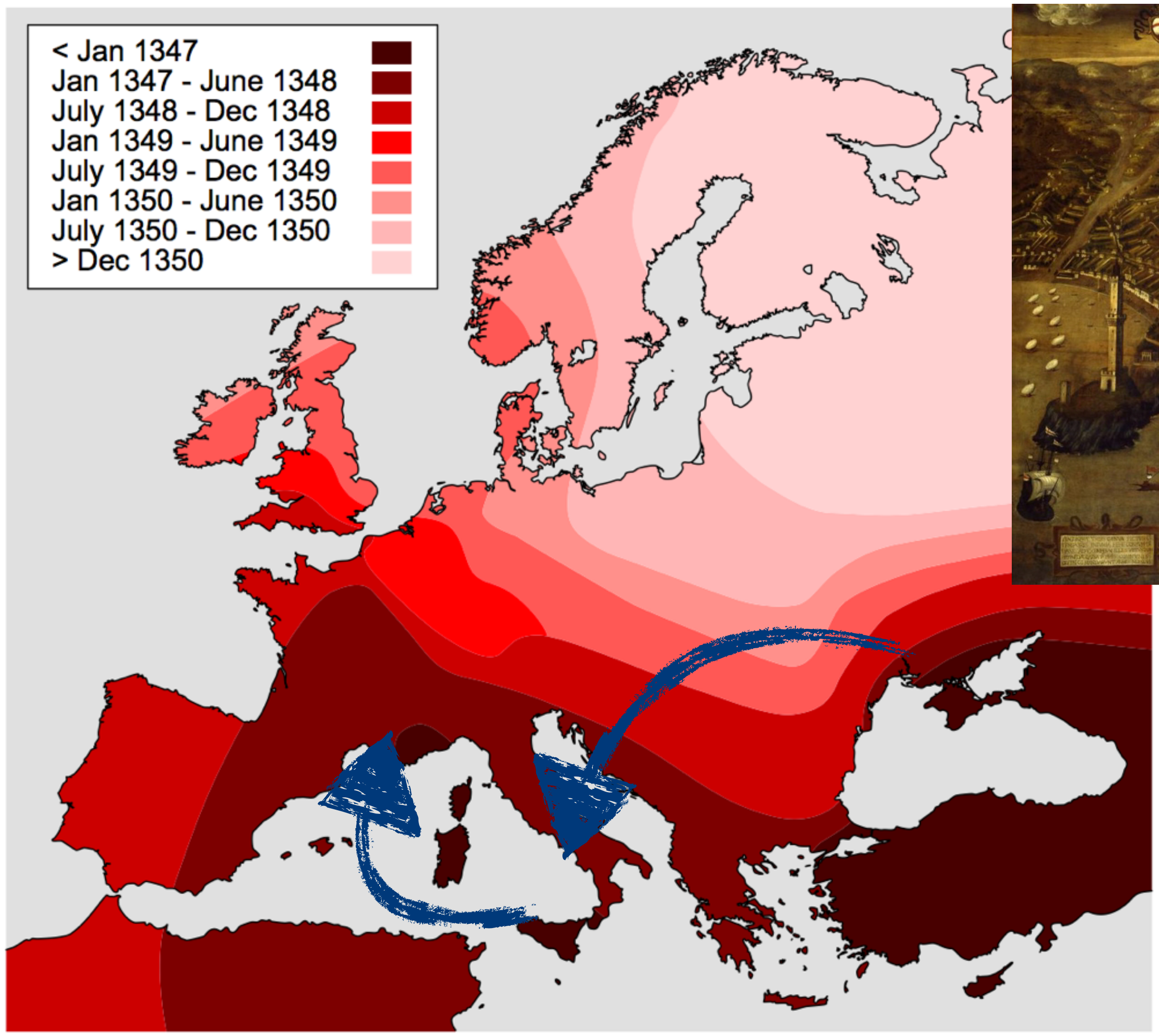
- Epidemics historically described by wave front propagation (WFP)



Black death
-Marvel et al (2014) arXiv 1310.2636

Epidemics: Then and Now

- Epidemics historically described by wave front propagation (WFP)
- but they have shortcuts as teleconnections

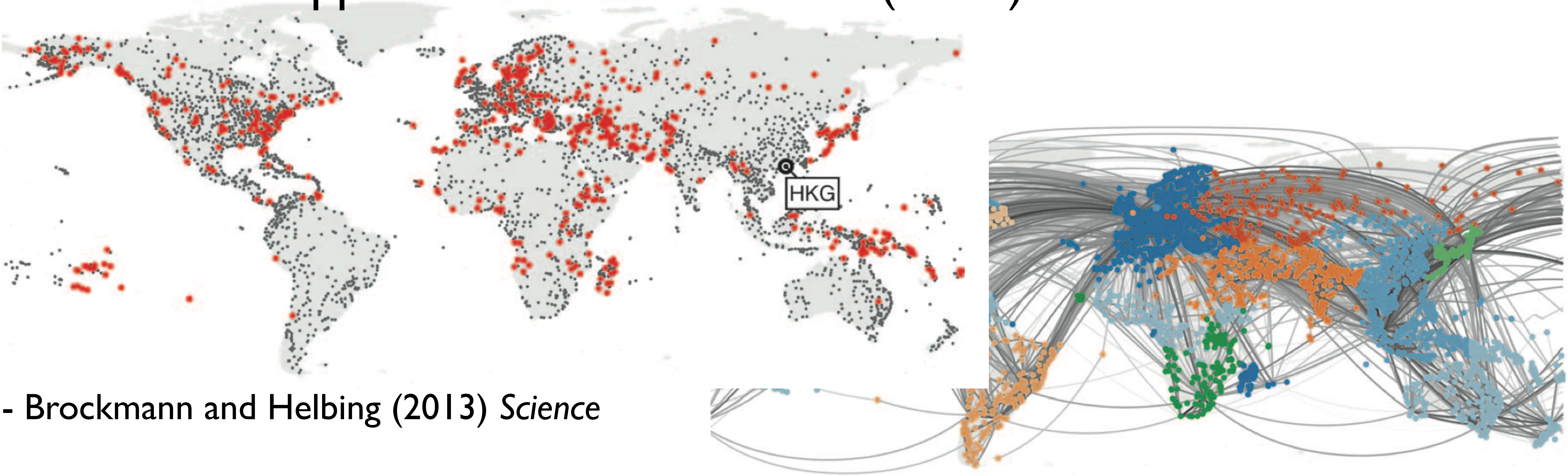


Genoese traders

Black death
-Marvel et al (2014) arXiv 1310.2636

Epidemics: Then and Now

- Epidemics historically described by wave front propagation (WFP)
- but they have shortcuts as teleconnections
- Modern epidemics dominated by airline network
- leads to appearance of new clusters (ANC)



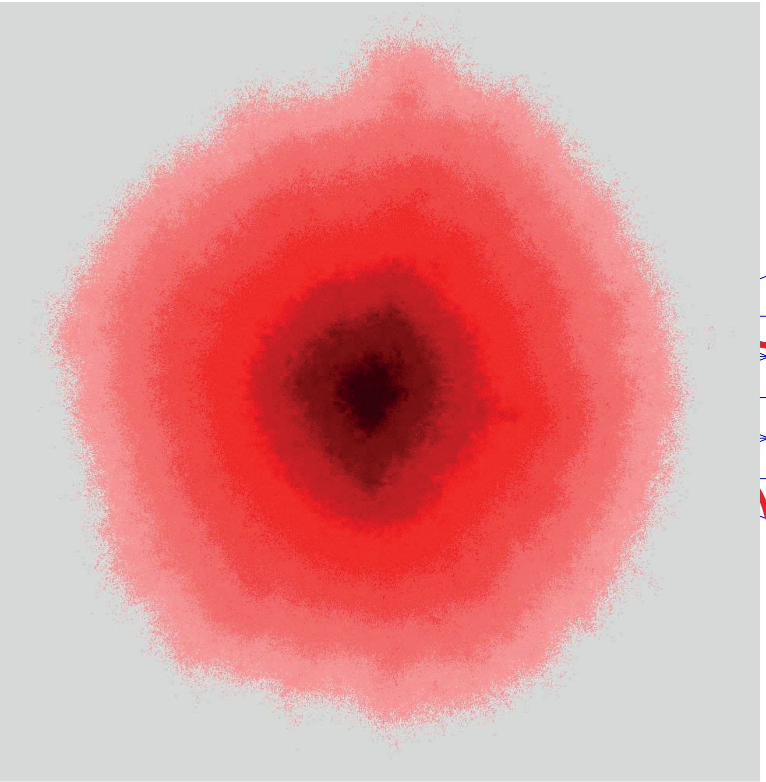
- Brockmann and Helbing (2013) *Science*

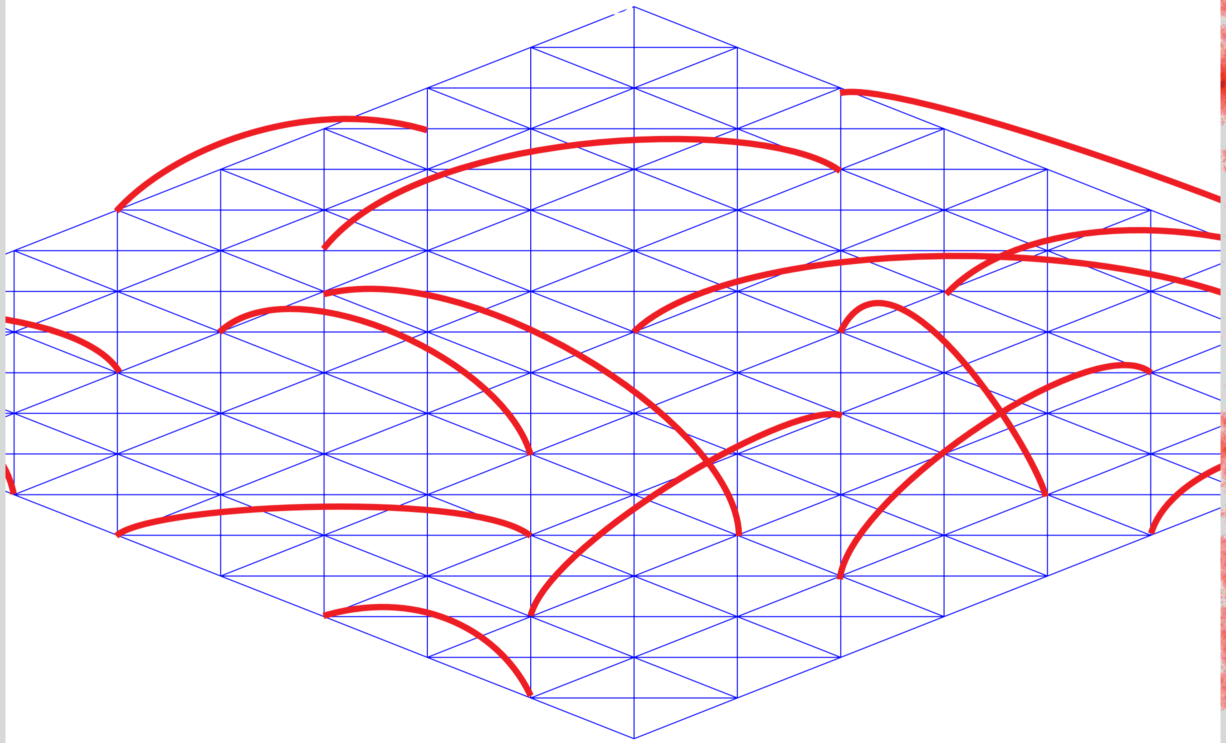
Goal of our work

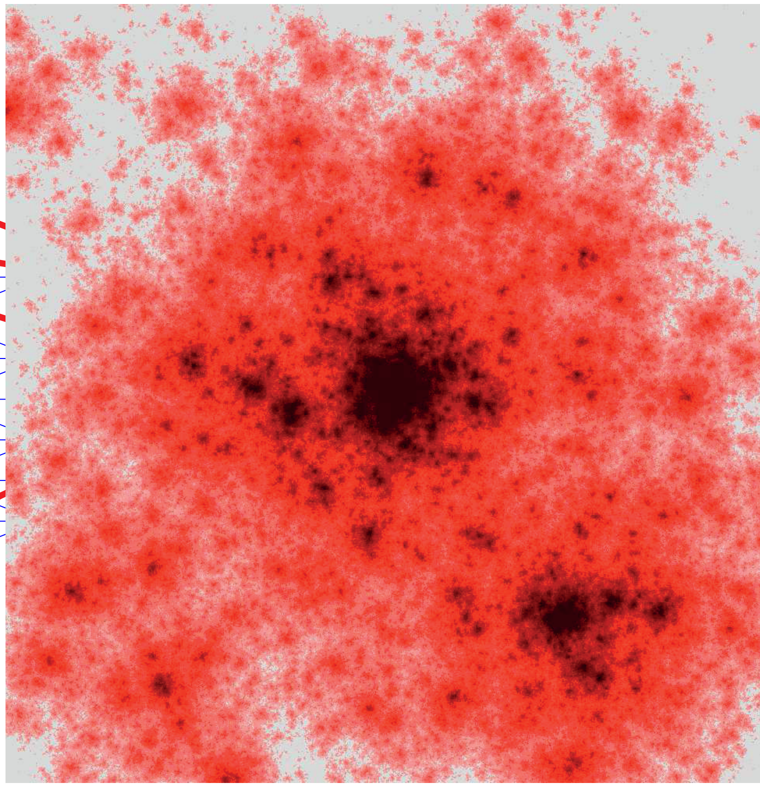
- Under which conditions does only WFP and no ANC occur?
- **Can we use complex contagions to get insights into the underlying geometry/manifold?**

WFP

ANC







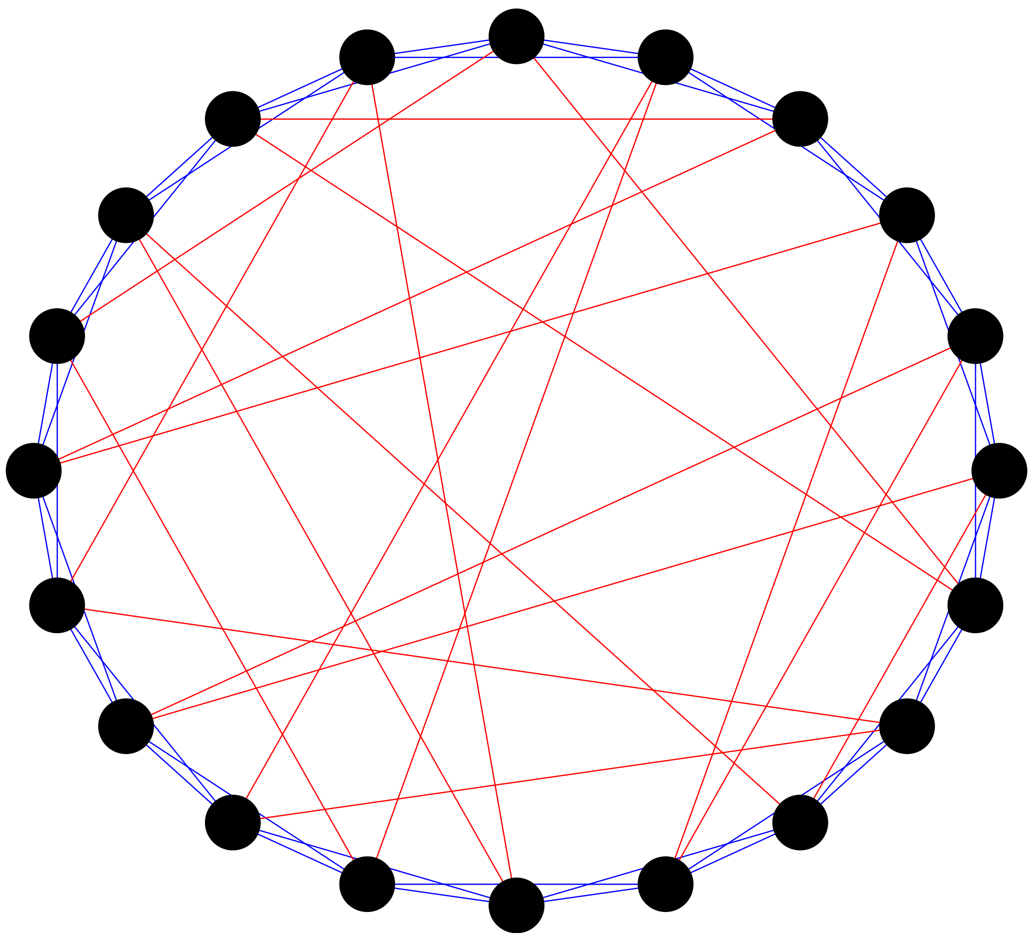
Noisy ring lattices

(=Newman variant of Watts-Strogatz graph)

Three given parameters

- Number of nodes N
- Geometric degree d^G
- Non-Geometric degree d^{NG}

-> Ratio of non-geometric to geometric edges, $\boldsymbol{\alpha} = d^{NG}/d^G$



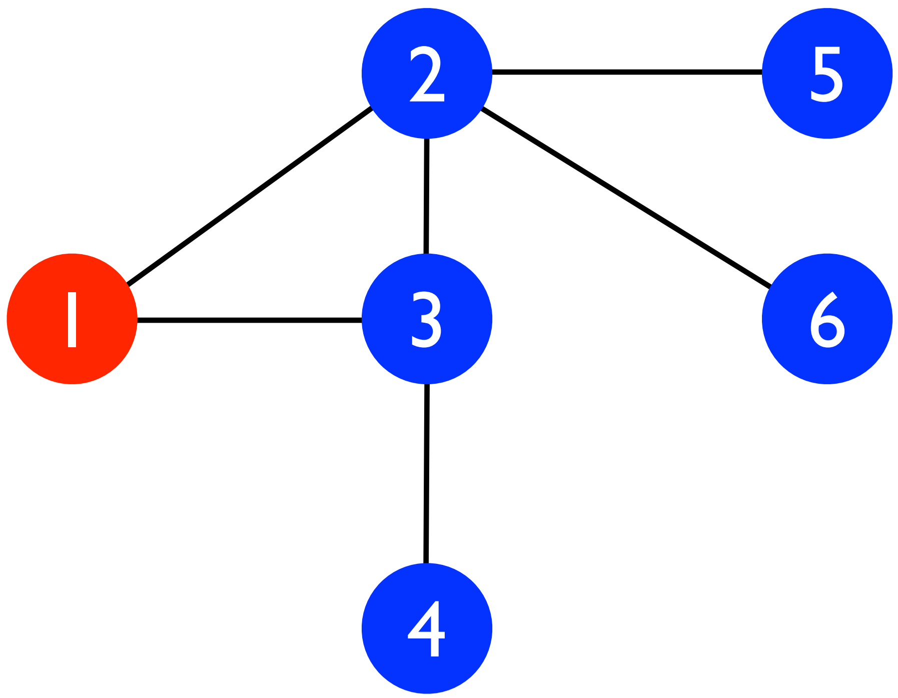
$N=20$, $d^G=4$, $d^{NG}=2$, $\boldsymbol{\alpha}=1/2$

Watts' model

- simple dynamic for information spread
- binary state (active/inactive)
- node gets activated if at least a fraction of *T* of its neighbors are active -> deterministic
- active nodes stay active
- in the beginning a random perturbation of seed node(s)

Watts, Duncan J. "A simple model of global cascades on random networks." PNAS

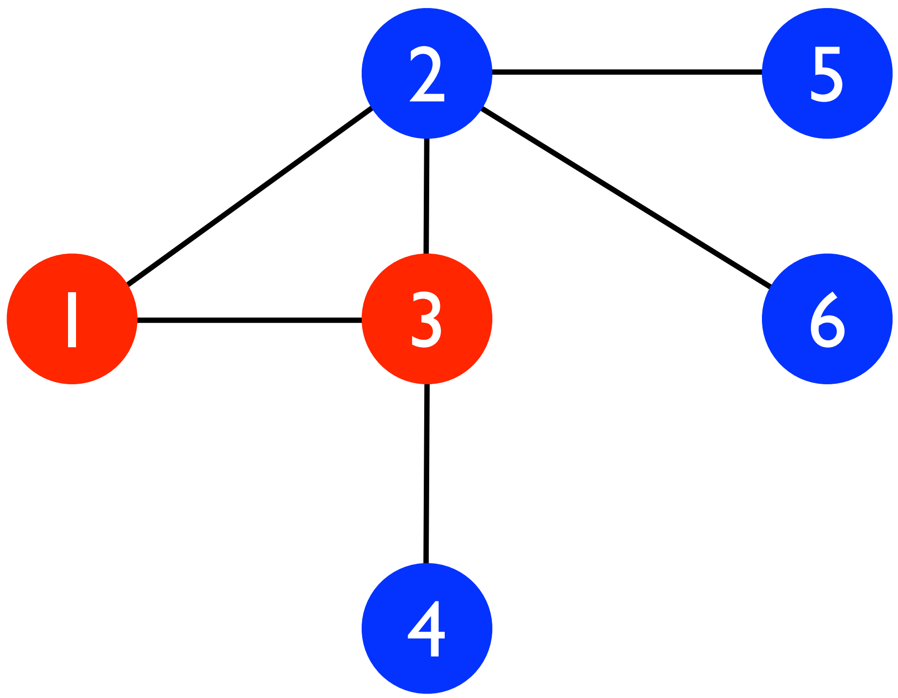
An example with $T=0.3$



Timestep 0

Node	1	2	3	4	5	6
Activation time t	0					

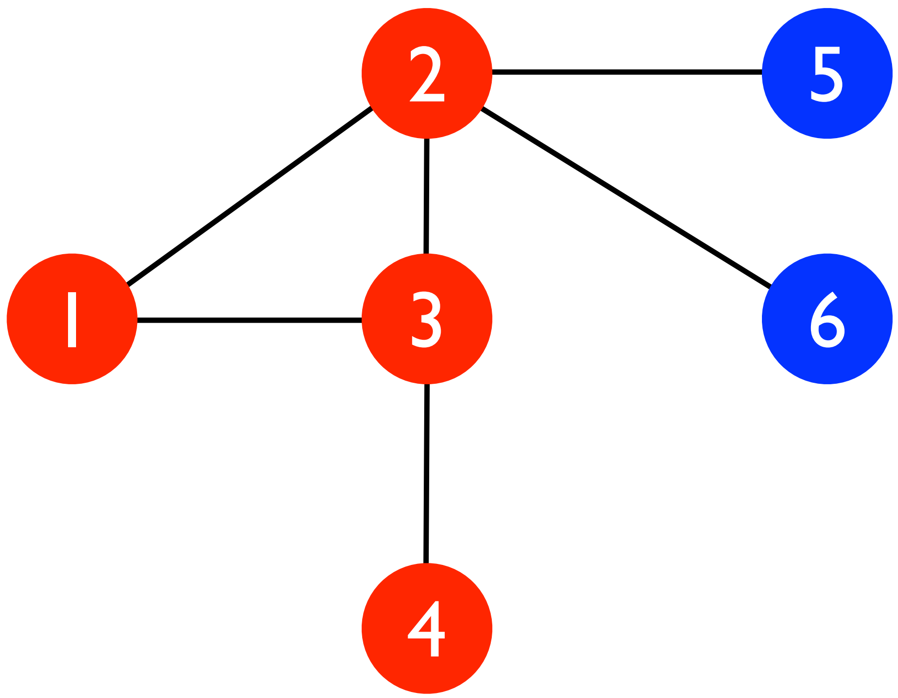
An example with *T=0.3*



Timestep 1

Node	1	2	3	4	5	6
Activation time t	0		1			

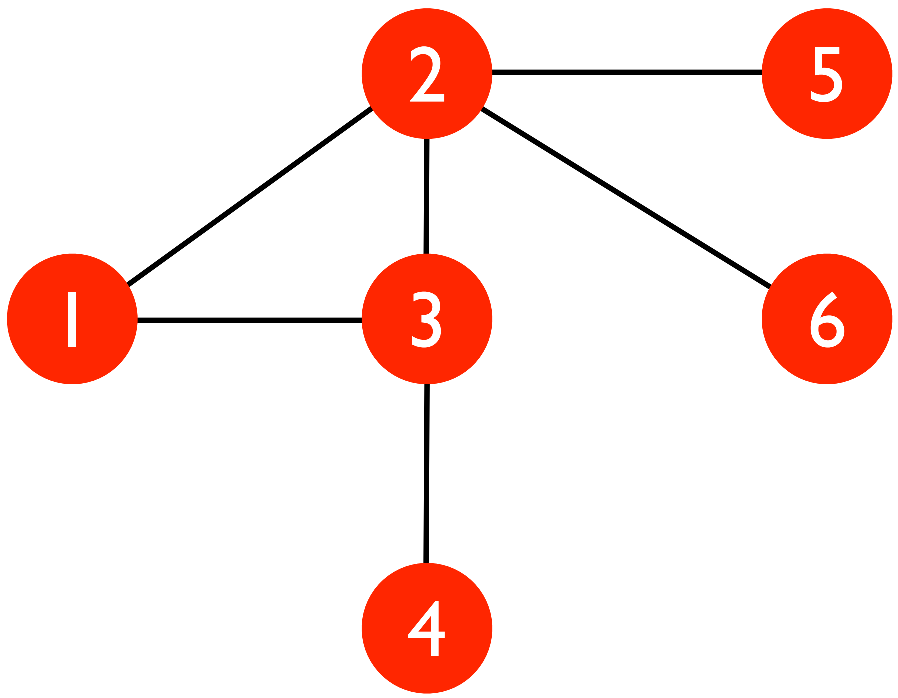
An example with T=0.3



Timestep 2

Node	1	2	3	4	5	6
Activation time t	0	2	1	2		

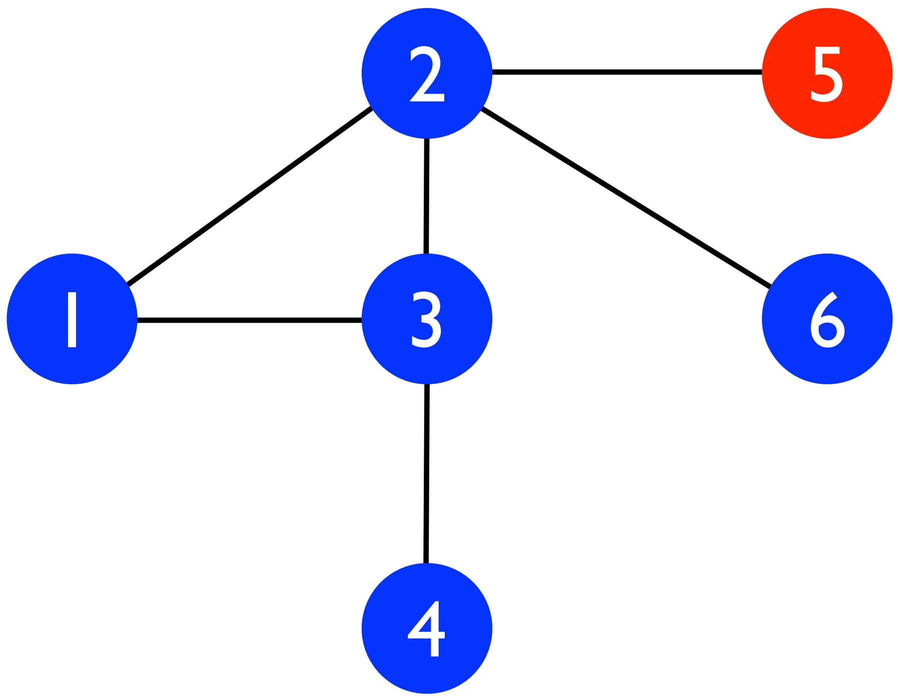
An example with *T=0.3*



Timestep 3

Node	1	2	3	4	5	6
Activation time t	0	2	1	2	3	3

Initial condition sensitive



T=0.3

Timestep 0

=> nothing happens

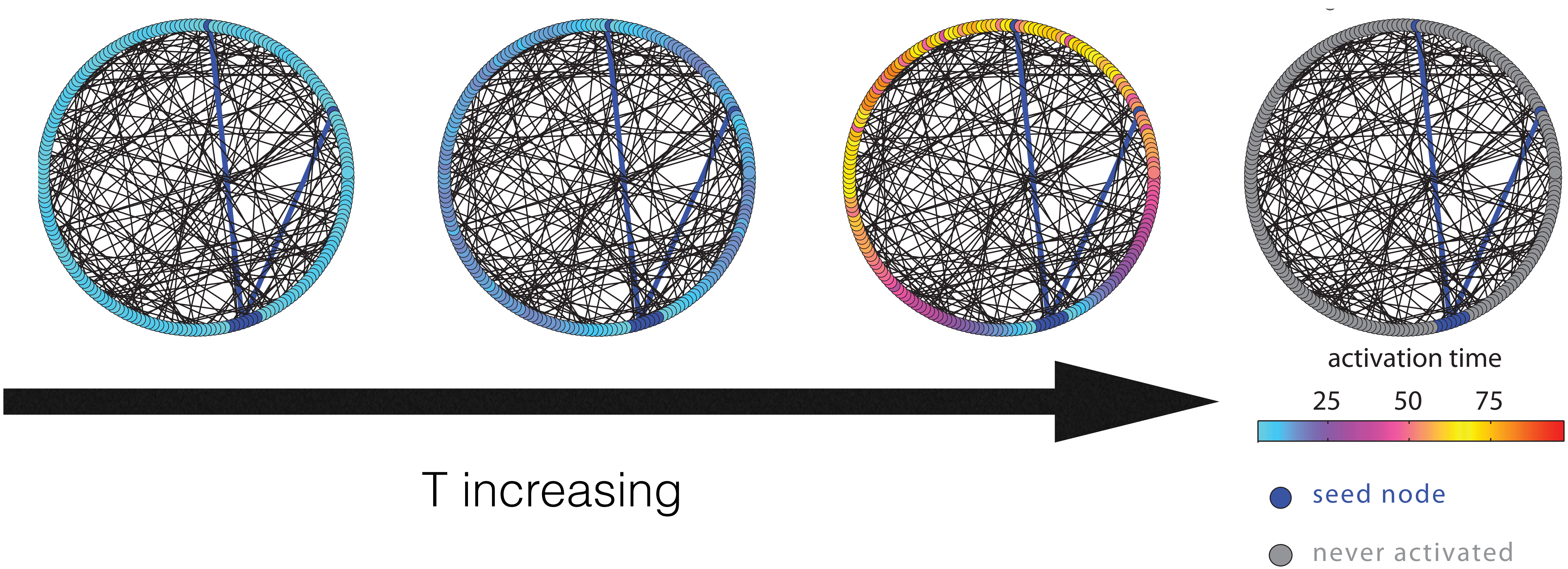
Node	1	2	3	4	5	6
Activation time t	∞	∞	∞	∞	0	∞

A single WTM realisation

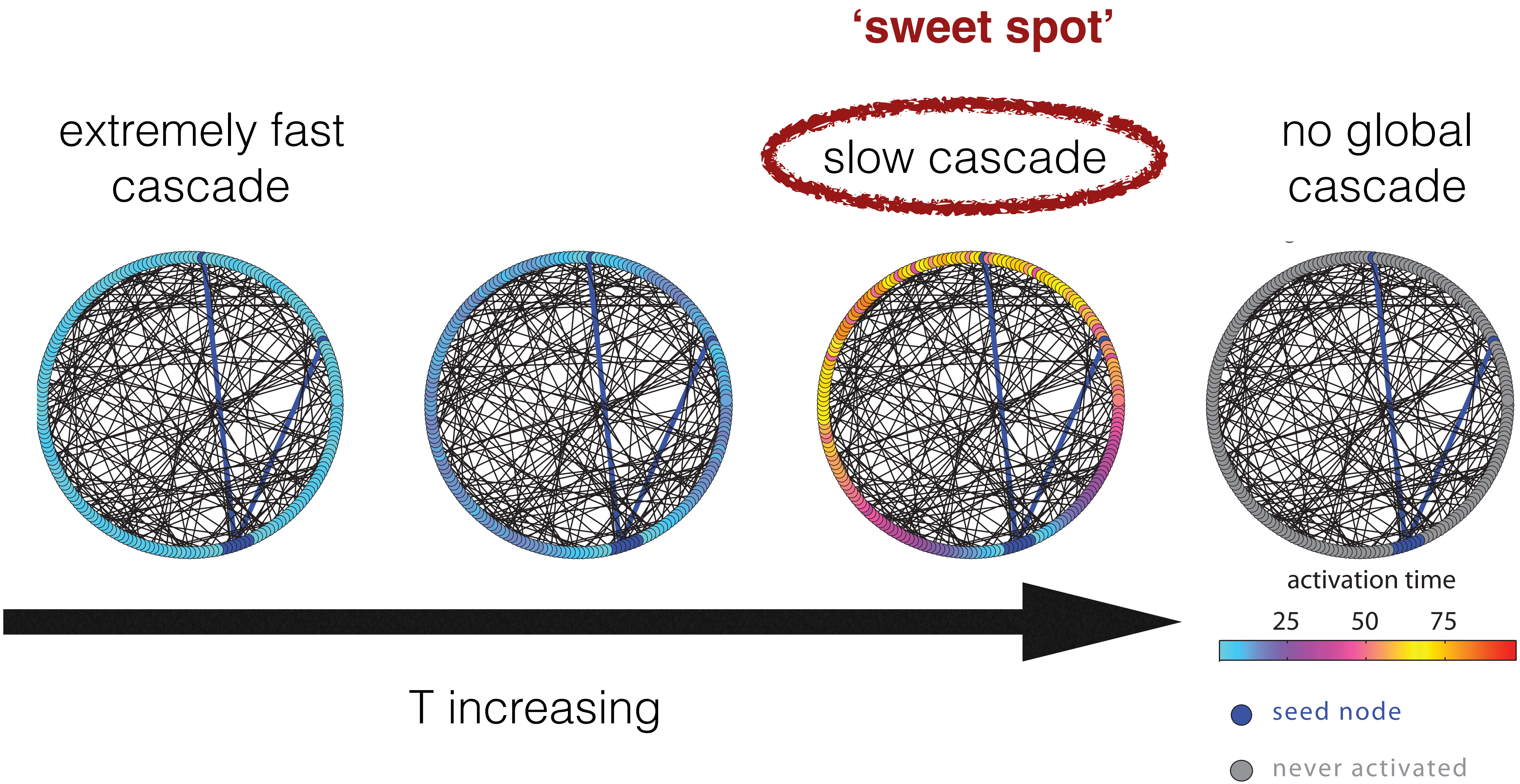
extremely fast cascade

slow cascade

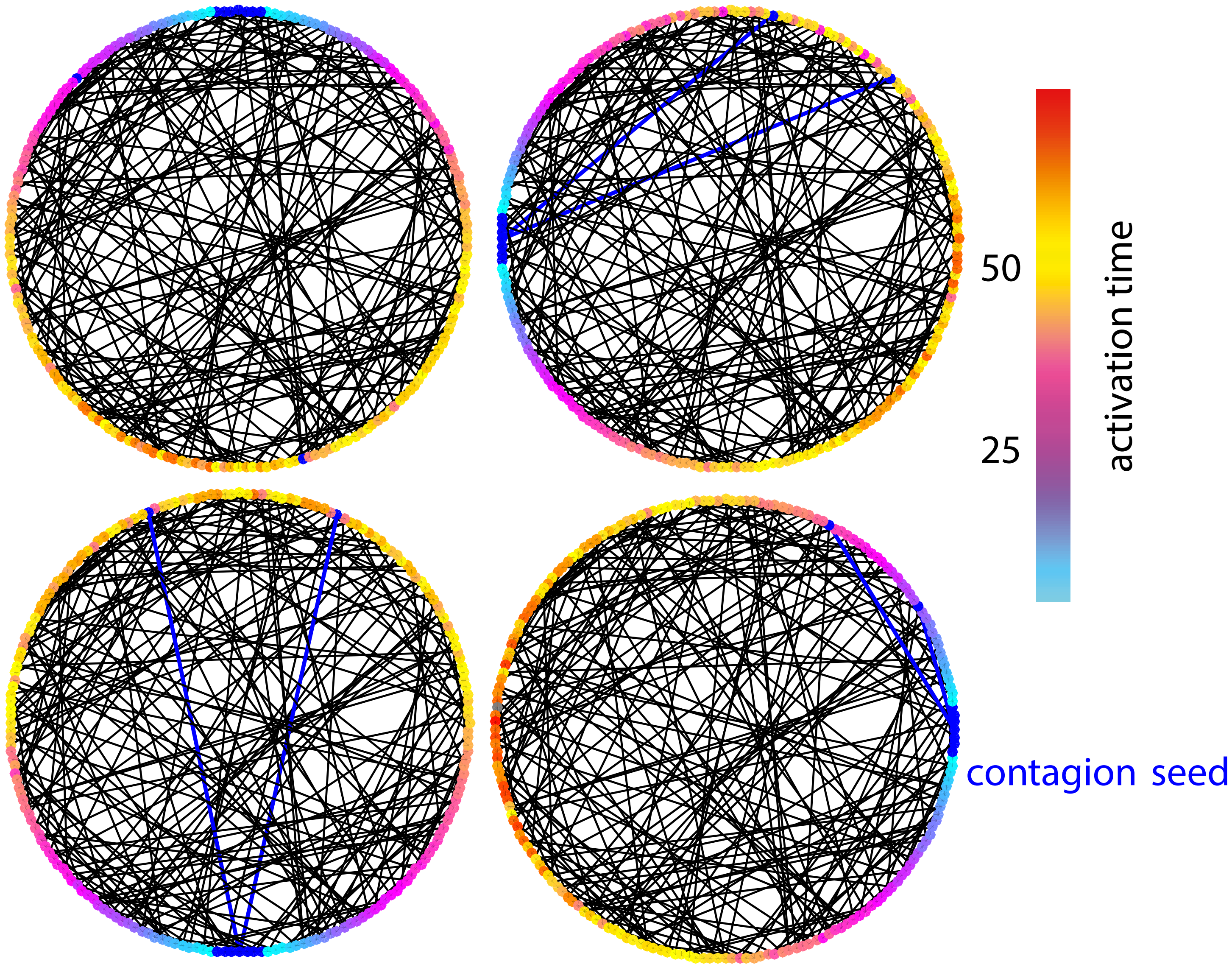
no global cascade



A single WTM realisation



Multiple contagions with fixed threshold T



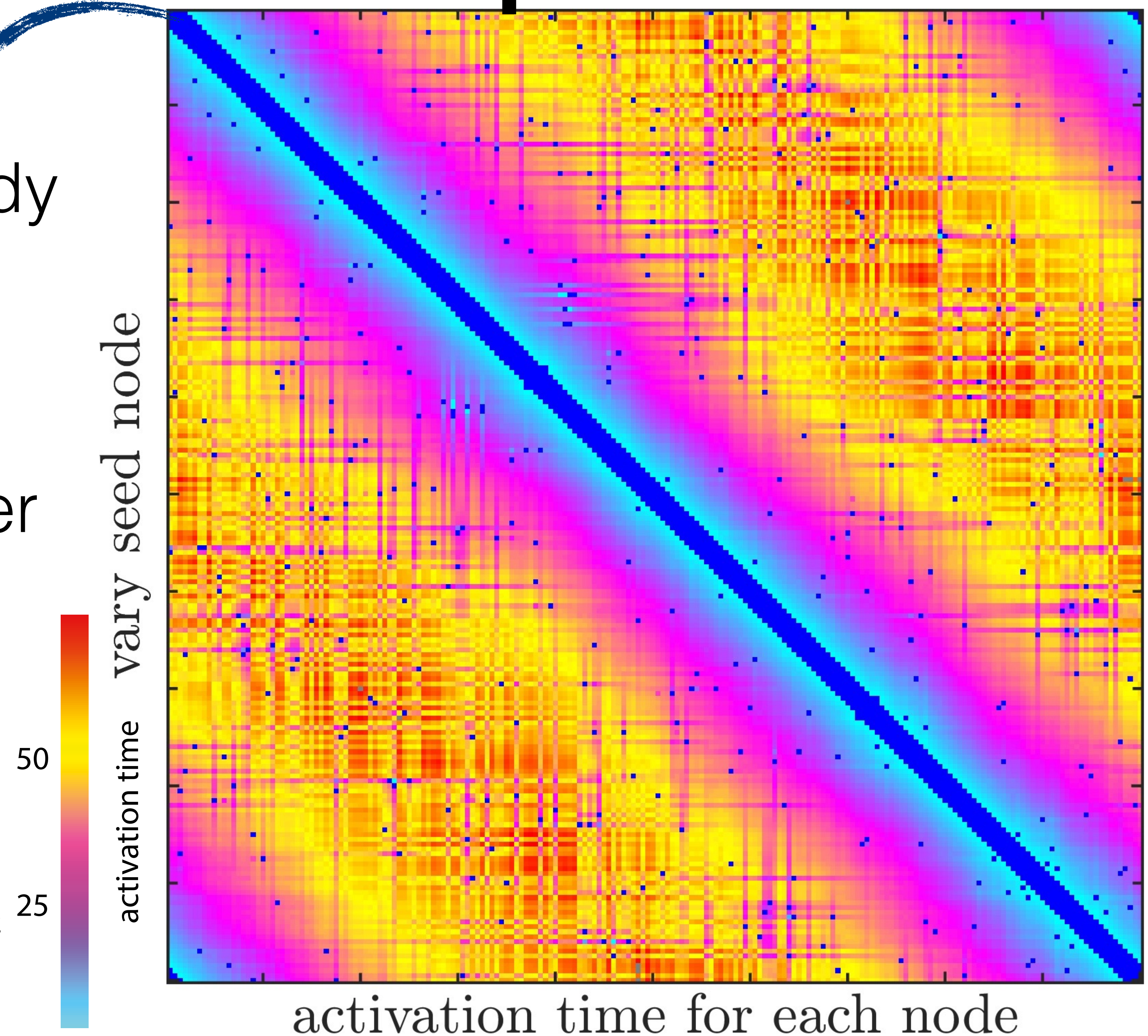
- each one starting from different seeds nodes -> N possibilities

WTM maps

ring structure already observable

for each node activation time under each possible contagion with different seed node

NxN non-symmetric matrix



extremely fast cascade

slow cascade

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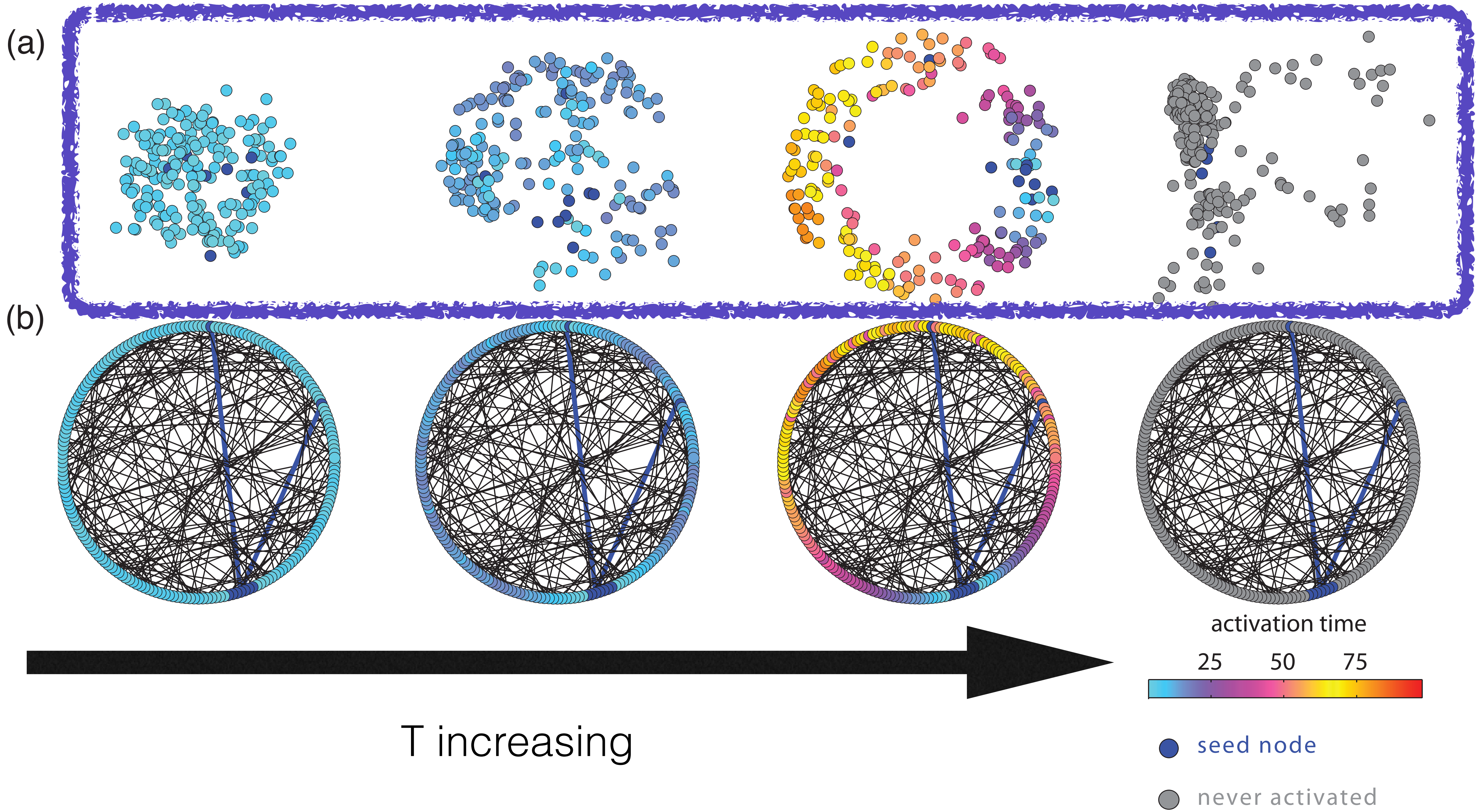
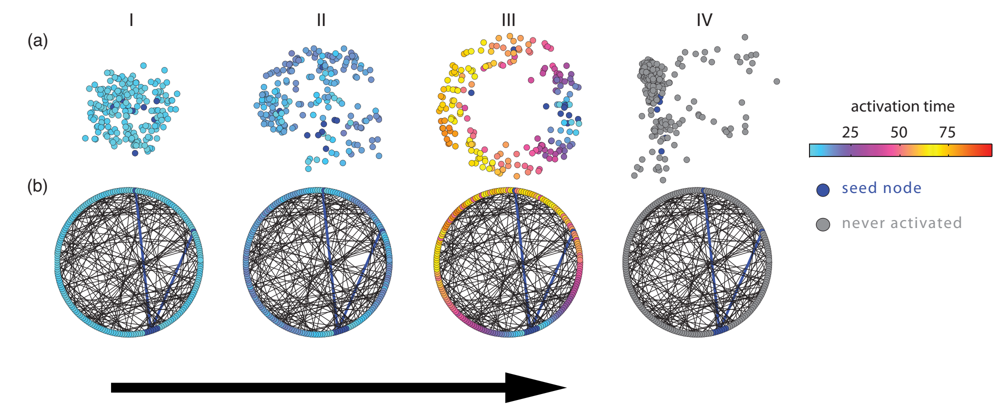
N dimensional WTM projected onto $\Re^2$



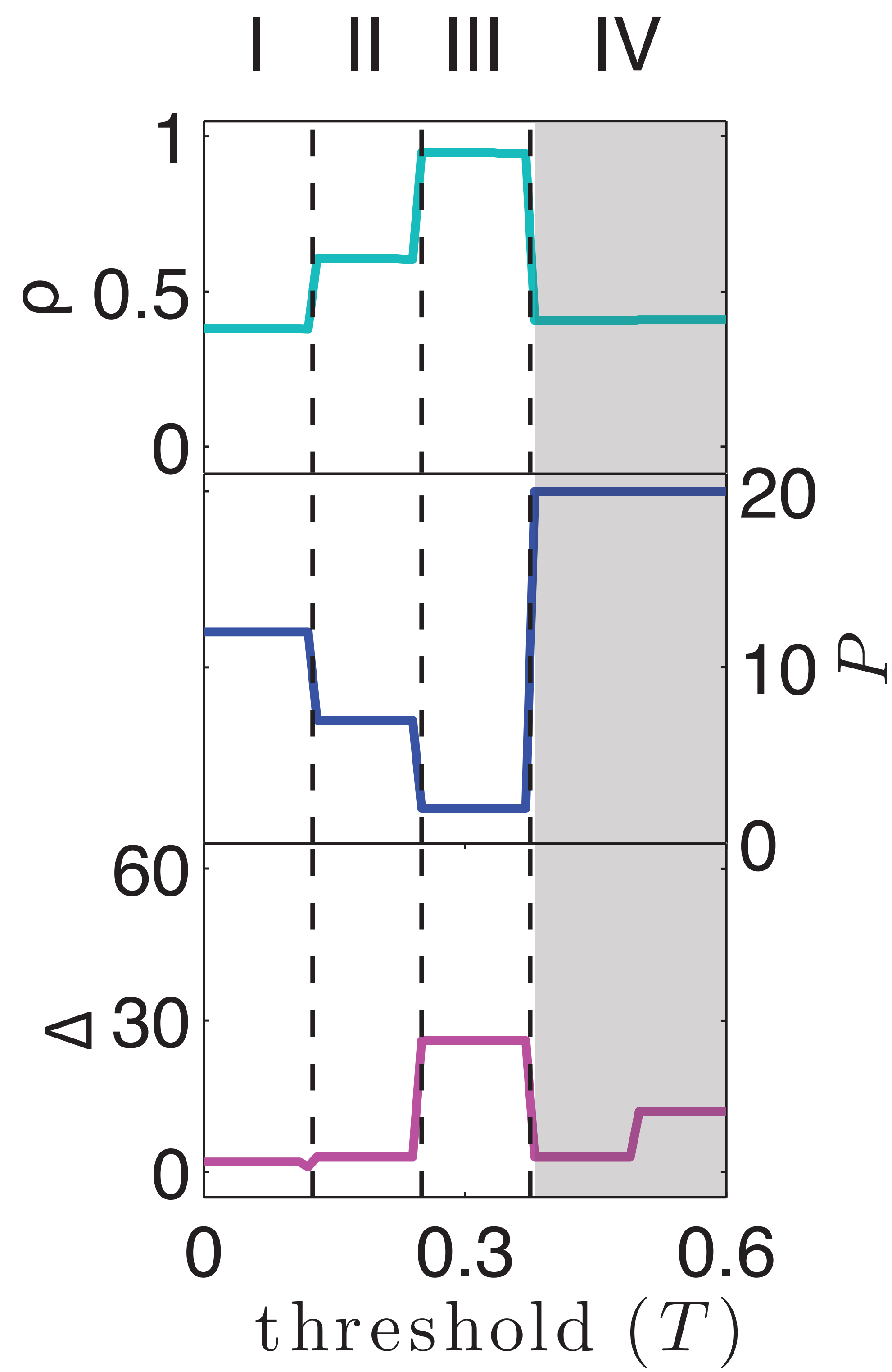
Validate the WTM



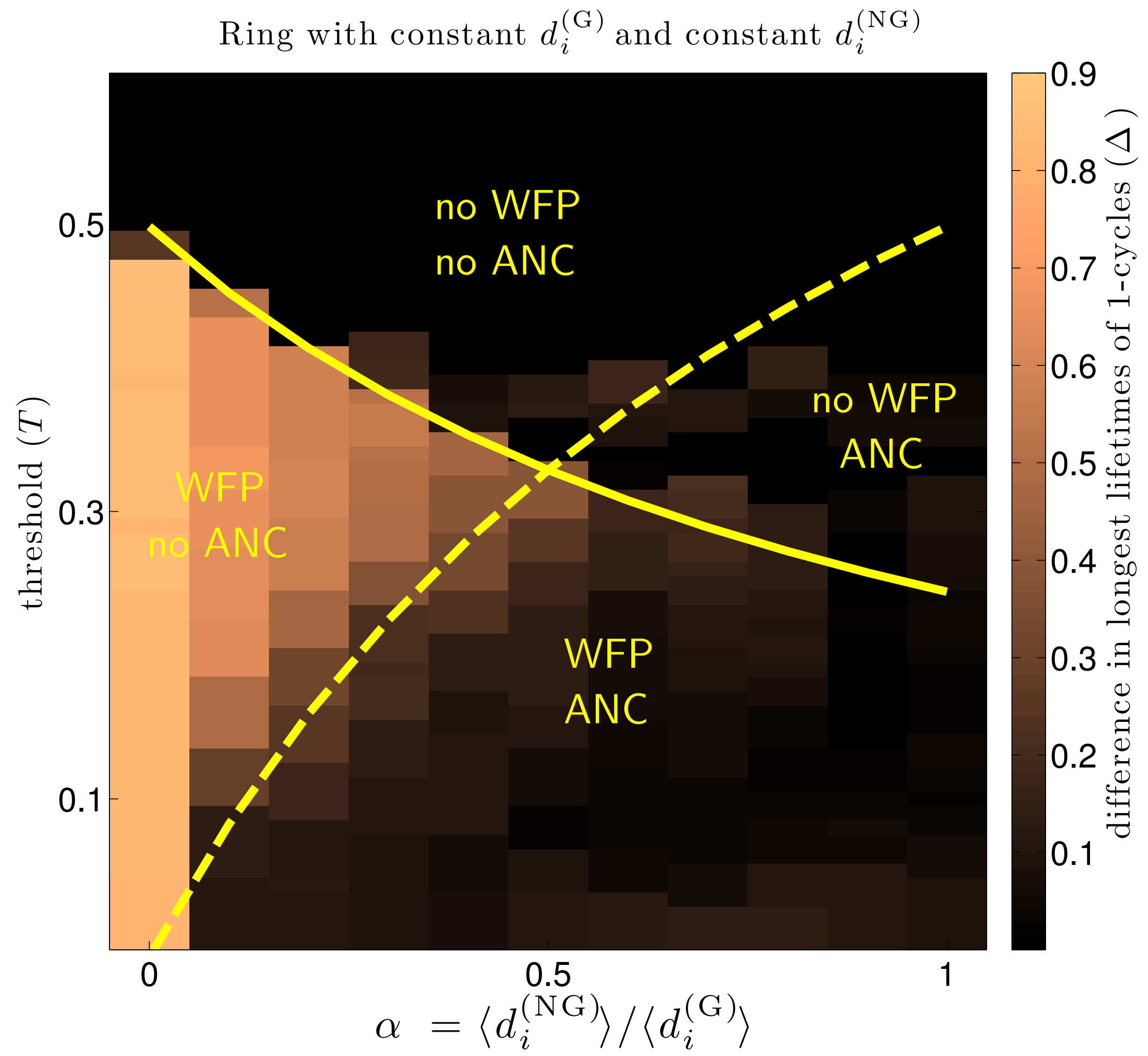
geometry: correlation between real node-to-node-distance and WTM node-to-node-distance

dimensionality: smallest dimension needed to explain 95 % of variance of the WTM

topology: strength of a single ring structure in the high-dimensional WTM (use of persistence homology)



WTM on noisy rings



even for high noise levels WFP dominates for well-chosen T

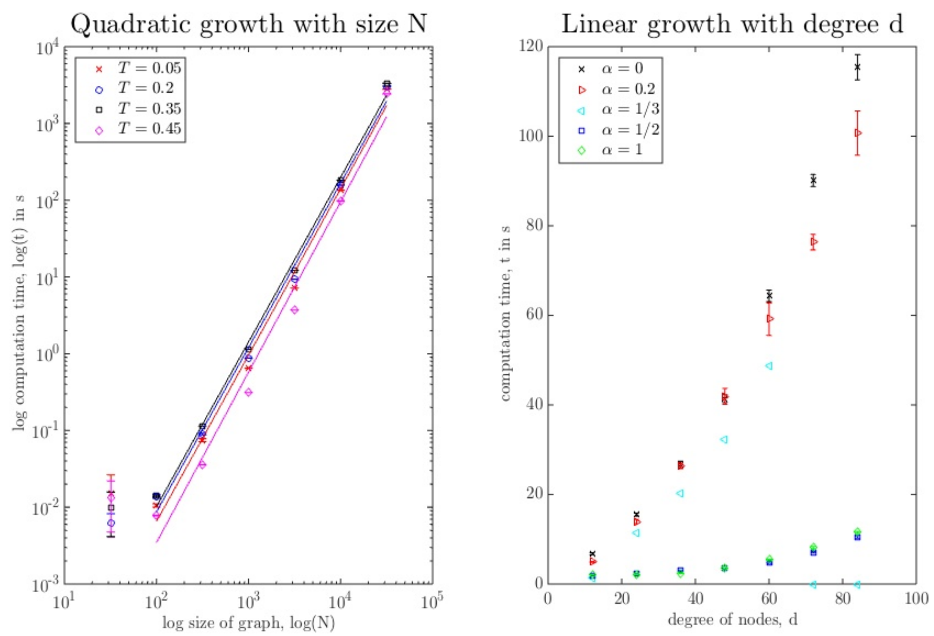
Watts' threshold model allows analytic bifurcation analysis

Conclusions

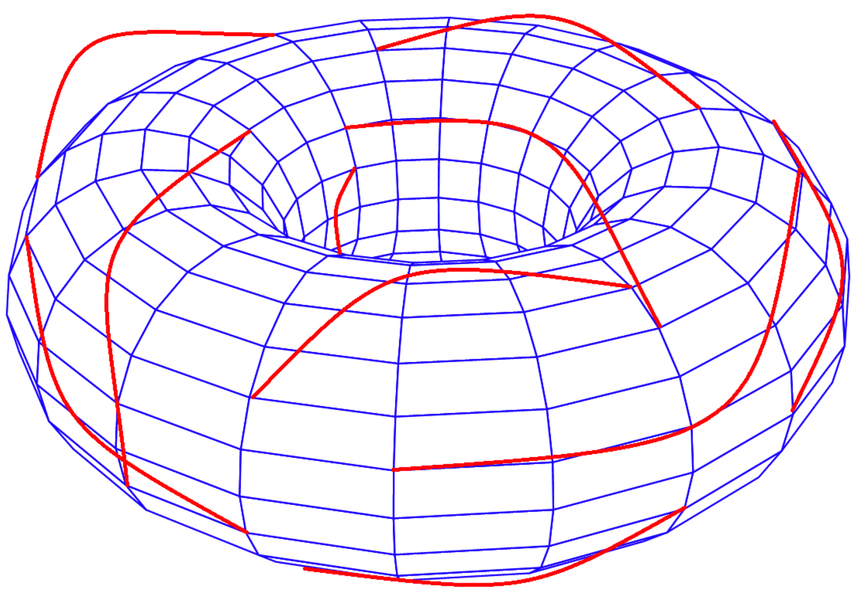
- Wave-Front Propagation (WFP) and Appearance of New Clusters (ANC) often coexist in complex contagions on noisy networks
- Watts' threshold mapping (WTM) uses multiple contagions to create a high-dimensional map
- if the contagions are largely dominated by WFP the mapping gives us insights on the underlying manifold

Outlook

speed: can we improve the complexity $\mathcal{O}(dN^2)$ of the contagion maps?



manifolds: does the mapping generalise to other, more complex, structures



http://arxiv.org/abs/1408.1168

Taylor, D., Klimm, F., Harrington, H. A., Kramár, M., Mischaikow, K., Porter, M. A., & Mucha, P. J. (2015). Topological data analysis of contagion maps for examining spreading processes on networks. *Nature Communications*, *6*, 7723.

Thank you!

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Collaborative work with:

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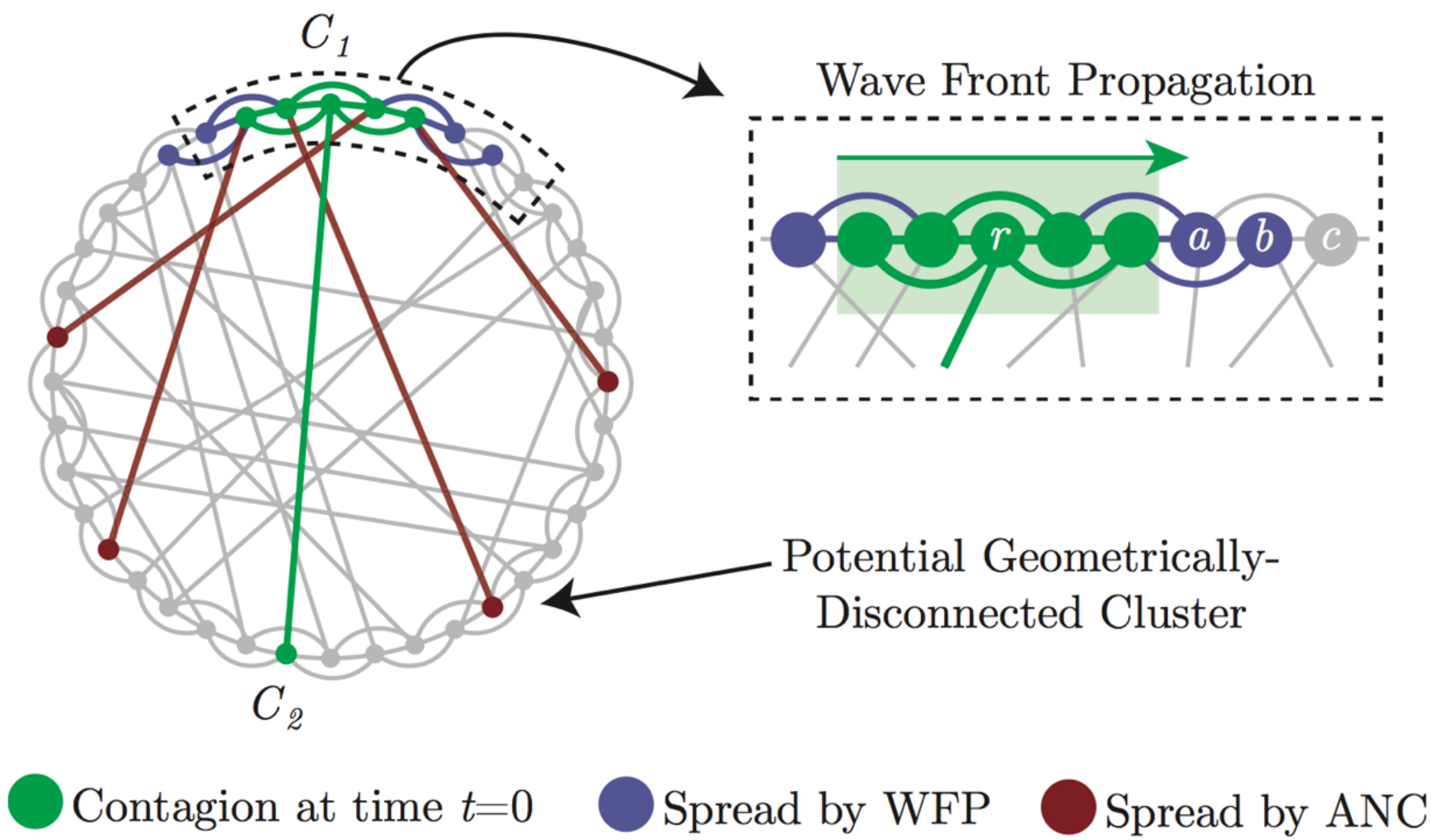
Konstantin Mischaikow, Rutgers University
Mason A. Porter, University of Oxford
Peter J. Mucha, University of North Carolina at Chapel Hill
Barbara Mahler, University of Oxford

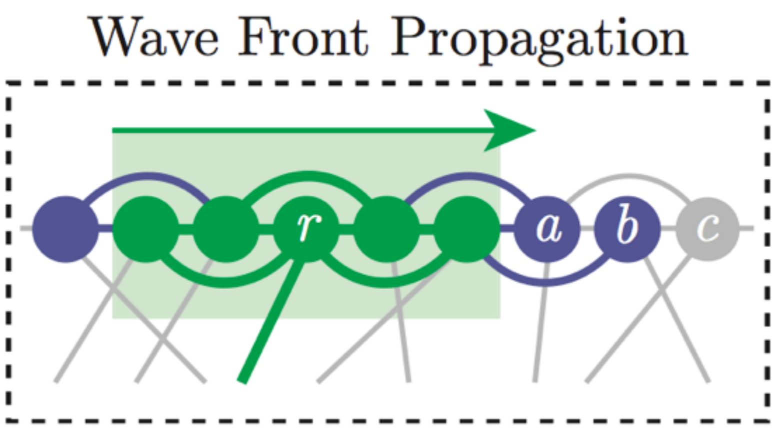
Contagion Phenomena

- Wave front propagation (WFP)
- Geometrically distant clusters and the appearance of new clusters (ANC)

Cluster seeding: the contagion starts at a node and its neighbors



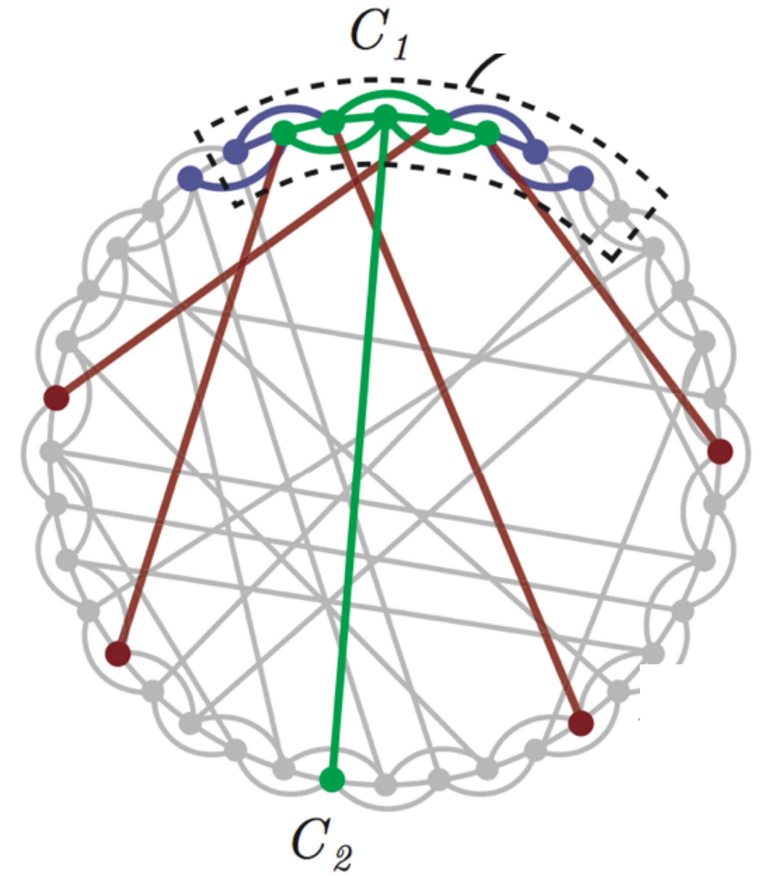
Wave Front Propagation



- Spreading across geometric edges
- Wave front propagation travels with rate k nodes per time step when $T_{k+1}^{WFP} \leq T < T_k^{WFP}$
- There is no wave front propagation if $T \geq T_0^{WFP}$
- Critical thresholds:

$$T_k^{WFP} = \frac{d^G/2 - k}{d^G + d^{NG}} \qquad k = 0, 1, \ldots, d^G/2$$

Appearance of New Clusters



- Spreading across a non-geometric edges
- If $T_{k+1}^{ANC} \leq T < T_k^{ANC}$ then a node must have at least $d^{NG} - k$ non-geometric neighbors that are adopters to adopt
- Critical thresholds:

$$T_k^{ANC} = \frac{d^{NG} - k}{d^G + d^{NG}} \qquad k = 0, 1 \ldots, d^{NG}$$

Bifurcation Analysis

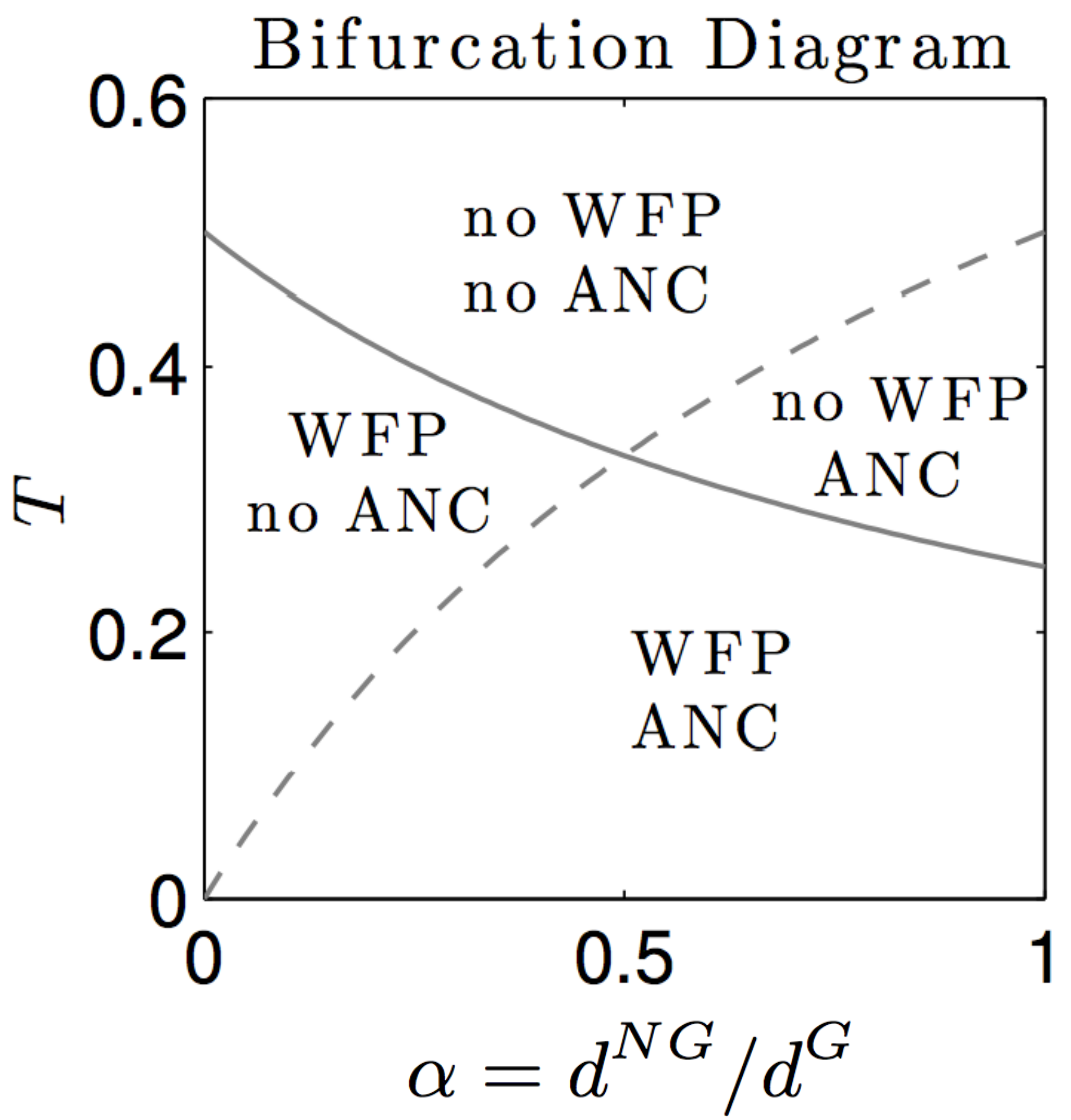
- We first examine the regions of similar contagion dynamics, i.e., the absence/presence of wave front propagation (WFP) and the appearance of new clusters (ANC)
- Given critical thresholds

$$T_k^{WFP} = \frac{d^G/2 - k}{d^G + d^{NG}} \qquad T_k^{ANC} = \frac{d^{NG} - k}{d^G + d^{NG}}$$

- We examine $k = 0$ and express results based on the ratio of non-geometric versus geometric edges, $\alpha = d^{NG}/d^G$

$$T_0^{WFP} = \frac{1}{2 + 2\alpha} \qquad T_0^{ANC} = \frac{\alpha}{\alpha + 1}$$

Bifurcation Analysis



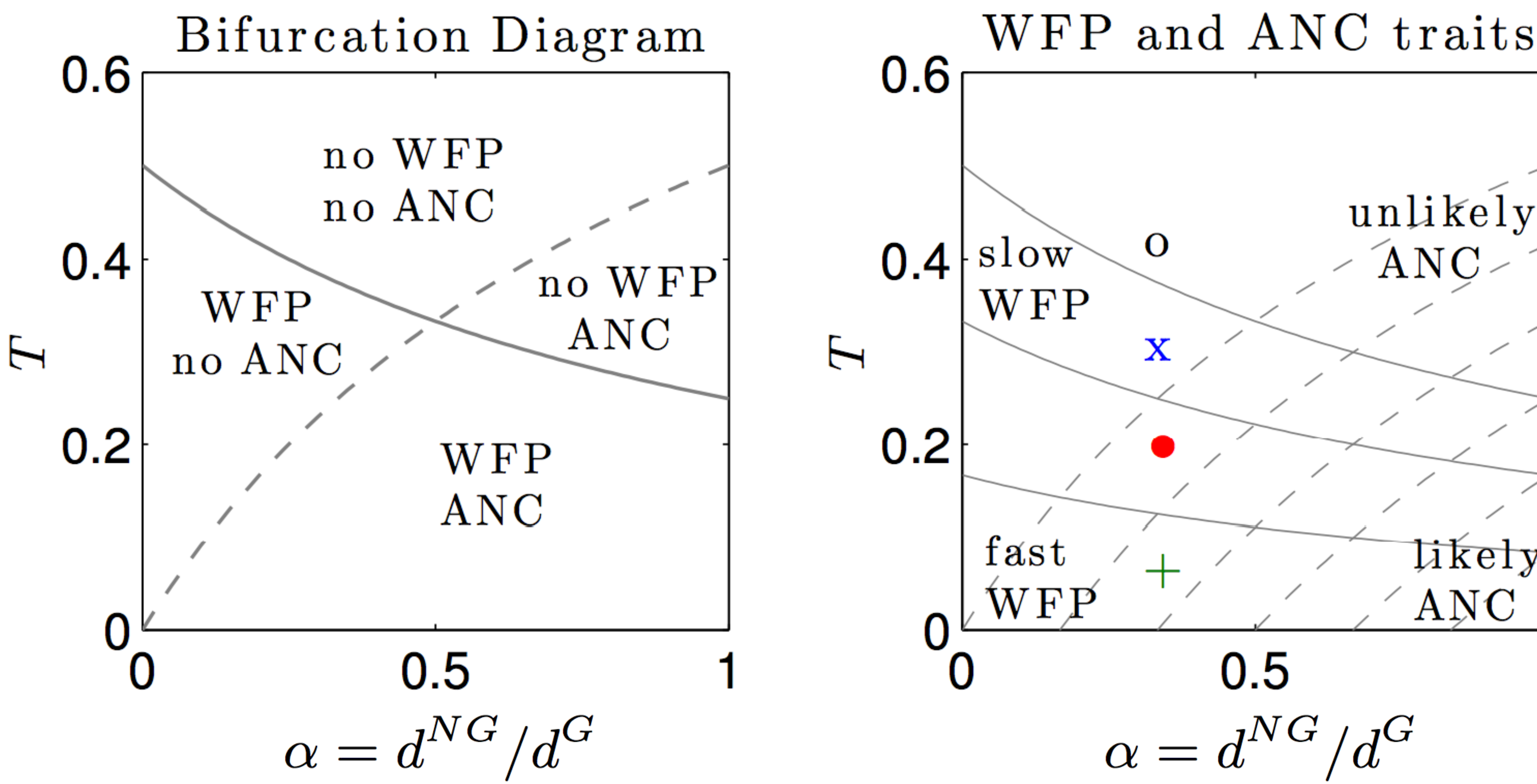
$$T_0^{WFP} = \frac{1}{2 + 2\alpha}$$

$$T_0^{ANC} = \frac{\alpha}{\alpha + 1}$$

Intersection at

$$\left(\alpha, T\right) = \left(\frac{1}{2}, \frac{1}{3}\right)$$

Bifurcation Analysis



- results shown for $d^G = 6$