The evaluation of the complexity of complex networks

Presented by : Raihana MOKHLISSI
Supervised by : D.Lotfi, J.Debnath, M.El Marraki

mokhlissiraihana@gmail.com

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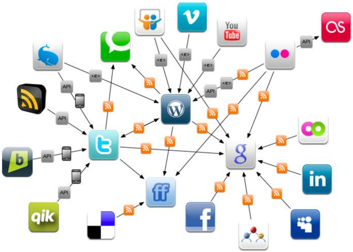
Laboratoire de Recherche en
et Télécommunications
éme
URAC'15
l'Institut scientifique de Rabat
AJCIT

Introduction

What is a complex network ?

- Complex Networks are large networks of real life.

- Social networks :
    - Facebook, Twitter...
    - phone-call networks



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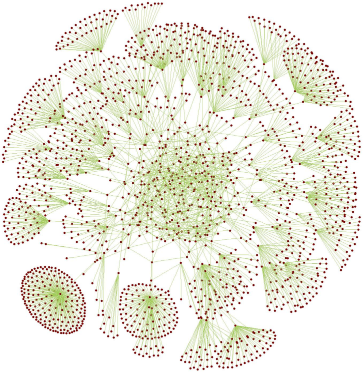

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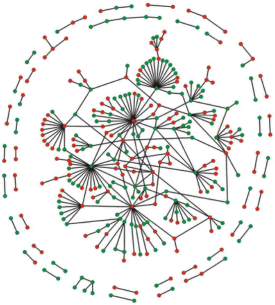
- Biological networks :
  - protein interaction networks
  - the food web



Different types of a complex network

- Three well-known and much studied classes of complex networks are :

- Scale-free networks
  - have power law degree distribution (WWW, semantic maps, electronic circuits).
- Small-world networks
  - have generalized binomial distribution (neural networks in the brain).
- Random networks
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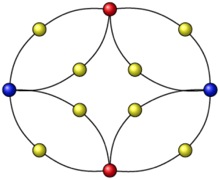
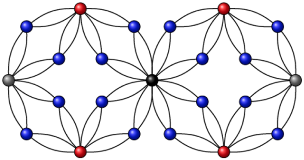


Flower Network



Mosaic Network

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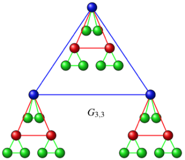
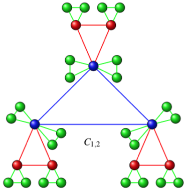


Small World Exponential Network



Koch network

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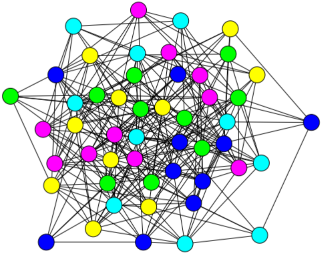

random graph

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What is a robustness of a network ?

- The robustness of a network is the ability to withstand failures and perturbations, the ability to adapt random changes in its structure and the capacity to tolerate changes over evolutionary time.
- In this work, we suggest a structural characterization of robustness in terms of **spanning trees entropy**.

- The entropy of spanning trees of a network or the asymptotic complexity is a quantitative measure of the number of spanning trees to evaluate the robustness of networks.

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with $\tau(G_n)$ is the number of spanning trees of G_n or what is called the complexity of a network G_n.

Theorem :

The Kirchhoff Matrix Tree Theorem computes the complexity of a graph G by

$$\tau(G) = (-1)^{i+j} det \quad L^*(G)$$

$L^*(G)$ is a matrix obtained by deleting row i and column j of the Laplacian matrix $L(G)$.

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  The use of Kirchhoff's theorem is not practical in the case of large and complex networks.
- **Solution :**
  The application of combinatorial methods that facilitate the calculation of the number of spanning trees of a large and complex network.

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  - **Reduction and Bipartition approaches** to construct two scale free networks : Flower network and Mosaic network and calculate their complexity.
  - **Decomposition methods** to calculate the complexity of two small world networks : The Exponential small world network, and the Koch network.

I-Reduction and Bipartition approaches

Definition :

- A reduced graph $R_2(G)$ is obtained when every edge of G is multiplied.
- A k-reduced graph $R_k(G)$ is obtained when we add k multiple edges connecting two existing vertices of G.

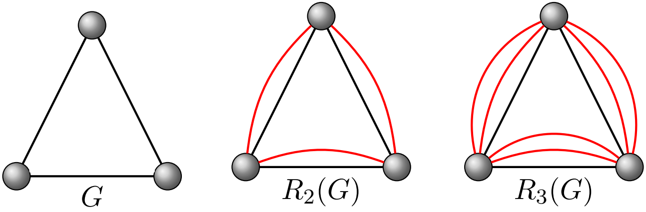

FIGURE 1 – A graph G, its reduced graph and its 3-reduced graph

The properties of the k-reduced graph $R_k(G)$:

- The number of vertices is : $|V_{R_k(G)}| = |V_G|$
- The number of edges is : $|E_{R_k(G)}| = k|E_G|$
- The number of faces is : $|F_{R_k(G)}| = |F_G| + (k-1)|E_G|$
- The average degree is : $< z >_{R_k(G)} = \frac{2|E_{R_k(G)}|}{|V_{R_k(G)}|} = \frac{2k|E_G|}{|V_G|}$.

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- The main objective of the reduction approach is to make the complexity of G having a large number of edges easy for computation.

Definition :

- A bipartite graph $B_2(G)$ is obtained by adding a new vertex between two connected vertices.
- A k-partite graph $B_k(G)$ is obtained when we add $k-1$ new vertices in each edge of planar graph G.

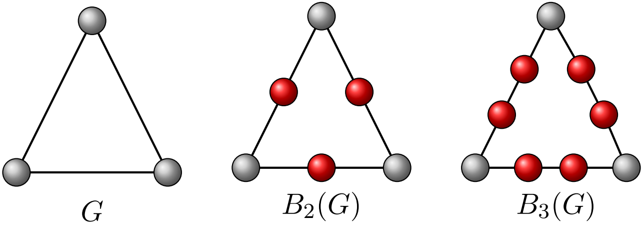

FIGURE 2 – A graph G, its bipartite graph and its 3-partite graph

The properties of the k-partite graph $B_k(G)$:

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- The main goal of the bipartite approach is to simplify the calculation of the complexity of G having a large number of vertices.

- To show the effectiveness of our approaches, we combine them to study two types of scale free networks that are characterized by the self-similarity :

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  - The Flower network.
  - The Mosaic network.

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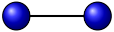

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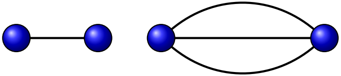

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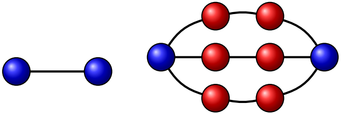

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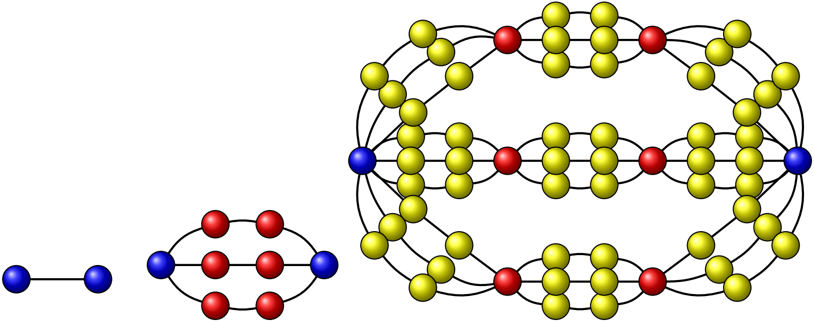

G_0 G_1 G_2

FIGURE 3 – The first three iterations of the 3-Flower network.

*Mokhlissi, Raihana, et al. "An Innovative Combinatorial Approach for the Spanning Tree Entropy in Flower Network." International Conference on Networked Systems. Springer,2017.

The topological properties of G_n are presented as follows :

- The number of vertices of G_n : $|V_{G_n}| = 2 + \frac{k(k^{2n}-1)}{k+1}$
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The complexity of the k-Flower network G_n :

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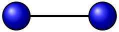

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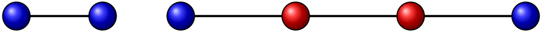

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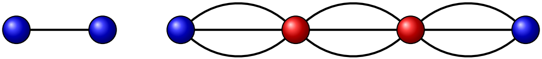

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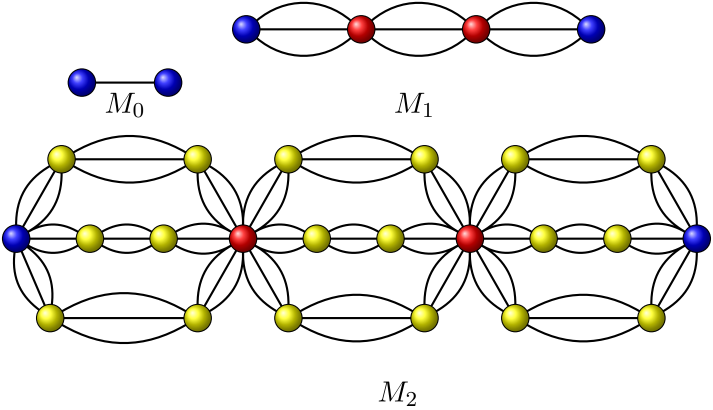

FIGURE 4 – The first three iterations of the 3-Mosaic network

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The complexity of the k-Mosaic network M_n :

$$\tau(M_n) = k^{k[\frac{k^{2n}-1}{k^2-1}]} \tag{4}$$

*Mokhlissi, Raihana, et al. "Spanning tree entropy of mosaic network." Proceedings of the 32nd International Conference on Computers and Their Applications, CATA 2017 .

- The k-Flower network and the k-Mosaic network have the same complexity :

$$\tau(M_n) = \tau(G_n) = \tau(R_k \circ B_k(M_{n-1})) = \tau(B_k \circ R_k(G_{n-1}))$$

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- The combination of the k-partition and the k-reduction approaches with two different ways leads to the same complexity in spite of the difference of the structure and the properties of our complex networks.

*Mokhlissi, Raihana, Dounia Lotfi, and Mohamed El Marraki. "A theoretical study of the complexity of complex networks." Information and Digital Technologies (IDT), 2017 International Conference on. IEEE, 2017.

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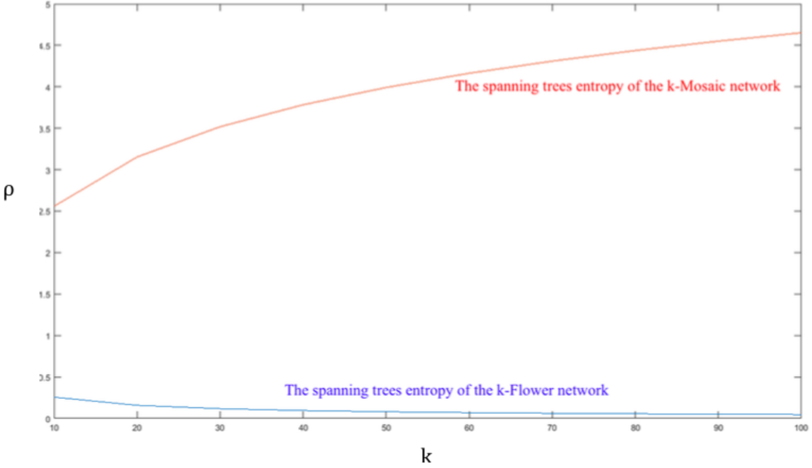

FIGURE 5 – The spanning tree entropy of the k-Flower network and the k-Mosaic network

Comparison the Flower network with other networks having the same average degree

Type of network	$< z >$	ρ
Koch network	3	0.549
Hanoi networks	3	0.677
2-Flower networks	**3**	**0.6931**
The 3-2-12 lattices	3	0.721
The 4-8-8 bathroom tile	3	0.787
Honeycomb lattice	3	0.807

TABLE 1 – The entropy of several networks having the same average degree.

- The 2-Flower network has an average spanning tree entropy rate compared to other networks with the same average degree.

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- The 2-Flower network has an average spanning tree entropy rate compared to other networks with the same average degree.
- The 2-Flower network is more robust than the Koch networks and the Hanoi networks, on the other hand, It is less robust than the 3-2-12 lattices, the 4-8-8 bathroom tile and Honeycomb lattice.

Type of network	$< z >$	ρ
Pseudofractal web	4	0.8959
Fractal scale-freelattice	4	1.0397
The 2-dimensional Sierpinskigasket	4	1.0486
Square lattice	4	1.1662
The 2-Mosaic networks	**4**	**1,3862**

Table 2 – The entropy of several networks having the same average degree.

- This table proves that the value of the entropy of the spanning trees of the 2-Mosaic networks is the biggest known for networks with average degree 4.

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- This table proves that the value of the entropy of the spanning trees of the 2-Mosaic networks is the biggest known for networks with average degree 4.
- This result proves that the 2-Mosaic network is more robust than other networks having the same average degree.

II-Decomposition method

Principle of the decomposition method : “Divide and Conquer”

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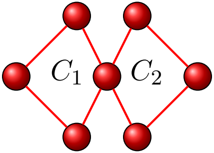

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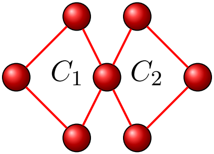

FIGURE 6 – A network $G = C_1 \bullet C_2$

$$\tau(G) = \tau(C_1 \bullet C_2) = \tau(C_1) \times \tau(C_2). \tag{5}$$

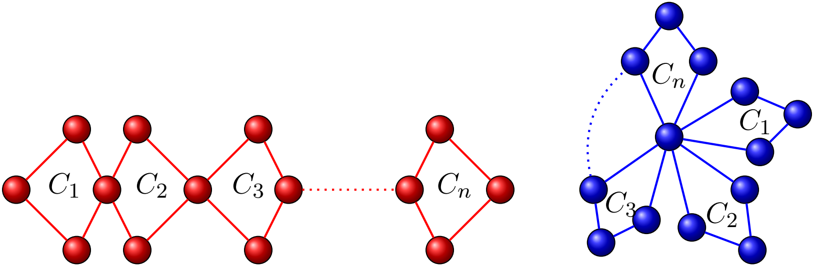

FIGURE 7 – Star network and chain network

$$\tau(G) = \prod_{i=1}^{n} \tau(C_i). \qquad (6)$$

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  - The Exponential Small World Network.
  - The Koch network.

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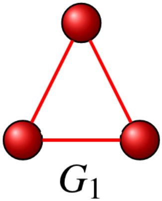
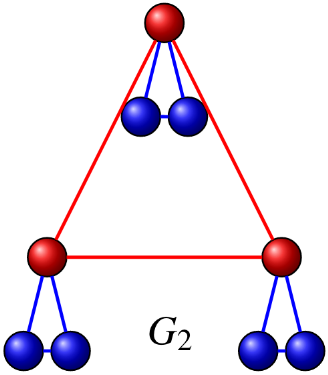


- To construct the Exponential Small World Network, we follow an iterative algorithm :

- At $n = 0$, we have a simple node.
- At $n = 1$, G_1 is a simple triangle.
- For $n > 1$, each node in the network of the previous iteration is replaced by a new triangle.



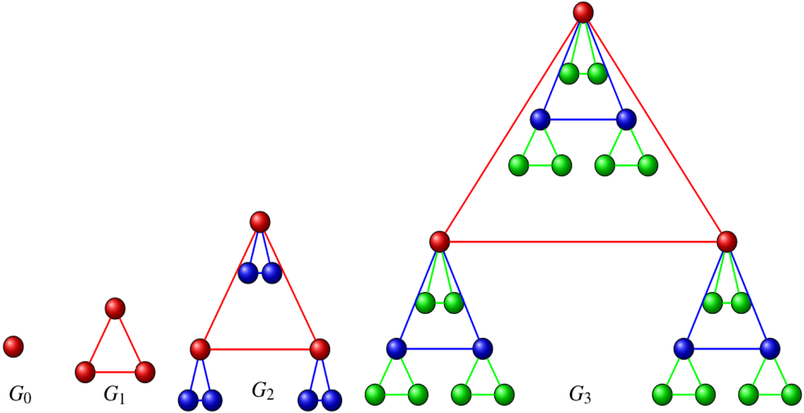


FIGURE 8 – The first 4 generations of the exponential Small World Network G_n

The topological properties of G_n are presented as follows :

- The number of nodes of G_n is : $V_{G_n} = 3^n$
- The number of edges of G_n is : $E_{G_n} = 3(\frac{3^n-1}{2})$
- The number of faces of G_n is : $F_{G_n} = 3^n - \frac{3^n-1}{2}$
- the average degree of G_n is : $< z >_{G_n} = \frac{3^n-1}{3^{(n-1)}}$

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The complexity of the Exponential Small World Network G_n :

$$\tau(G_n) = 3^{\frac{3^n-1}{2}}, n \geq 1. \qquad (7)$$

*Mokhlissi, Raihana, et al. "Complexity Analysis of "Small-World Networks" and Spanning Tree Entropy." International Workshop on Complex Networks and their Applications. Springer International Publishing, 2016.

- To construct the Koch Network, we follow an iterative algorithm :

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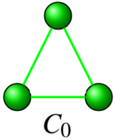
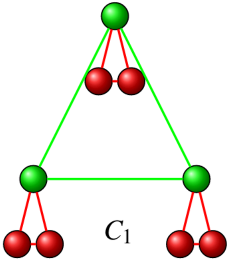


- To construct the Koch Network, we follow an iterative algorithm :

- At $n = 0$, C_0 is a simple triangle.
- For $n \geq 1$, C_n is obtained from C_{n-1} by adding one triangle for each of the three nodes of every existing triangles in C_{n-1}.



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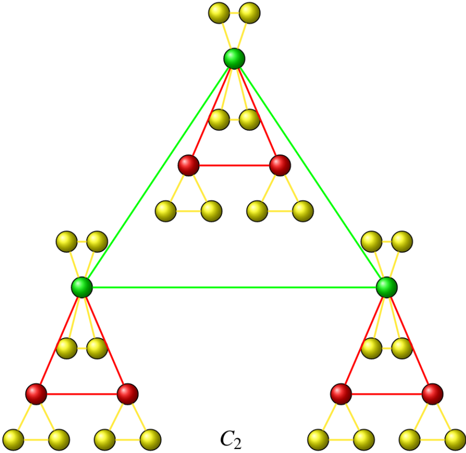
- The growth process to the next iteration continues in a similar way.



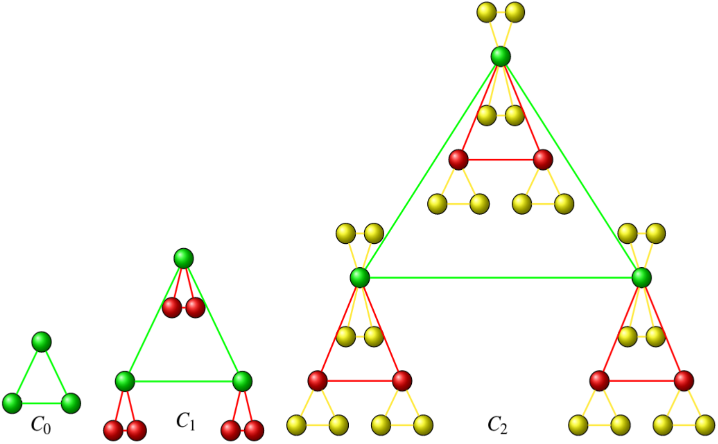


Figure 9 – The first 3 generations of the koch network C_n

The topological properties of C_n are presented as follows :

- The number of nodes of C_n is : $V_{C_n} = 2 \times 4^n + 1$
- The number of edges of C_n is : $E_{C_n} = 3 \times 4^n$
- The number of faces of C_n is : $F_{C_n} = 4^n + 1$
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- the average degree of C_n is : $< z >_{C_n} = \frac{6\times 4^n}{2\times 4^n+1}$.

The complexity of the Koch Network C_n :

$$\tau(C_n) = 3^{4^n}, n \geq 1. \tag{8}$$

*Zhang, Zhongzhi, et al. "Mapping Koch curves into scale-free small-world networks." Journal of Physics A : Mathematical and Theoretical 43.39 (2010) : 395101.

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$$\rho_{G_n} = \frac{ln(3)}{2} = 0.549$$

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- The Exponential small world network and the Koch network have the same entropy of spanning tree.
- The Exponential small world network and the Koch network have the same robustness in spite of the difference of their structure and their properties.

Type of network	$<z>$	ρ
The Koch network	**3**	**0.549**
The Exponential small world network	**3**	**0.549**
Hanoi networks	3	0.677
The 2-Flower networks	3	0.6931
The 3-2-12 lattices	3	0.721
The 4-8-8 bathroom tile	3	0.787
Honeycomb lattice	3	0.807

TABLE 3 – The entropy of several networks having the same average degree.

- This table proves that the value of the entropy of the spanning trees of the Koch network and the Exponential small world network is the smallest known for networks with average degree 3.

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TABLE 3 – The entropy of several networks having the same average degree.

- This table proves that the value of the entropy of the spanning trees of the Koch network and the Exponential small world network is the smallest known for networks with average degree 3.
- The Koch network and the Exponential small world network are less robust than other networks having the same average degree.

Conclusion

- Among the applications of the evaluation of the complexity of a complex network, we opt to use the entropy of spanning tree to quantify the robustness of a network.

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- This measure gives us an idea of the capacity of a network to adapt random changes in its structure.
- In this work, we calculated and compared the spanning tree entropy of four complex networks with other networks having the same average degree and we deduced the most robust network between them.

- Gao, Jianxi, et al. "Robustness of a network of networks." Physical Review Letters 107.19 (2011) : 195701.
- Lin, Yuan, et al. "Counting spanning trees in self-similar networks by evaluating determinants." Journal of Mathematical Physics 52.11 (2011) : 113303.
- Lotfi, Dounia, Mohamed El Marraki, and Driss Aboutajdine. "The Enumeration of Spanning Trees in Dual, Bipartite and Reduced Graphs." Journal of Discrete Mathematical Sciences and Cryptography 18.6 (2015) : 673-687.
- Amaral, Luıs A. Nunes, et al. "Classes of small-world networks." Proceedings of the national academy of sciences 97.21 (2000) : 11149-11152.

Thank You