

Optimal Access Point deployment in Ultra-dense Networks

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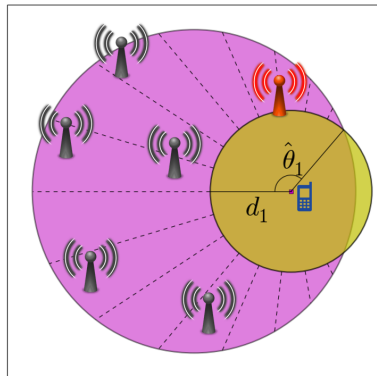
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Motivation

Spatial Densification: The heterogenisation of the existing network architecture by deploying smaller base stations, Access Points (APs).

What is the probability that a Mobile User (MU) is in the downlink of its nearest AP?

How should the APs be deployed for a given distribution of mobile users (MU) in order to maximise overall network coverage?



How should APs be deployed in a finite domain when MUs are distributed non-uniformly.

- * Nearest Neighbour distribution for a non-uniform PPP
- * A nearest neighbour connection model with interference.
- * MU Coverage in an interference limited environment.
- * Optimal distribution of APs

Nearest Neighbour Distribution (NND)

Poisson Point Process Φ emits a density,

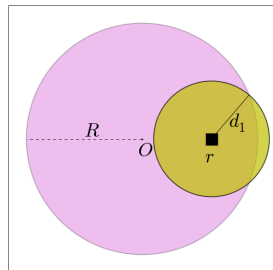
$$\lambda(t) = \lambda_0 \left(1 - \frac{bR^2}{2} + bt^2 \right), \quad \text{where } 2\pi \int_0^R \lambda(t)t dt = \lambda_0 |\mathcal{V}|$$

Complementary cdf of d_1 , is the probability there are no points within $B_r(d_1)$

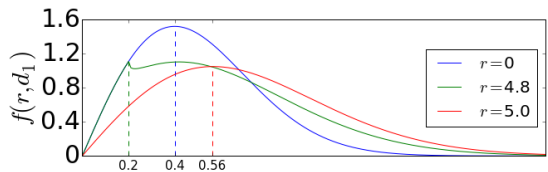
$$\bar{F} = \mathbb{P}[N(B_r(d_1)) = 0] = e^{\left(-\int_{\mathcal{V} \setminus B_r(d_1)} \lambda(\mathbf{y}) d\mathbf{y}\right)}$$

Thus the Nearest neighbour distribution (NND) is defined as,

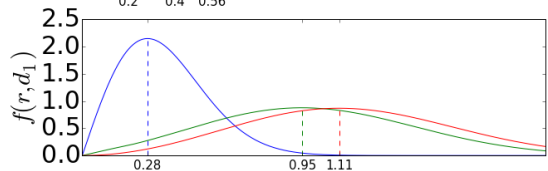
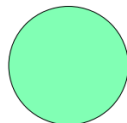
$$\begin{aligned} f(r, d_1) &= \frac{d}{dd_1} (1 - \mathbb{P}[N(B_r(d_1)) = 0]) \\ &= -\frac{d}{dd_1} \exp \left(-\int_{\mathcal{V} \setminus B_r(d_1)} \lambda(\mathbf{y}) d\mathbf{y} \right) \end{aligned}$$



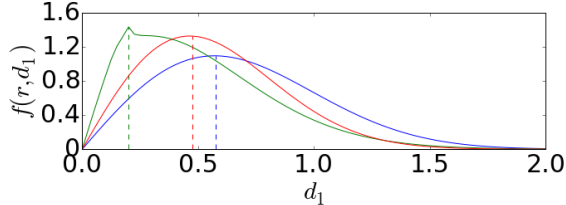
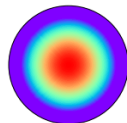
Nearest Neighbour Distribution



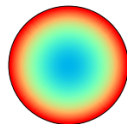
$a=1$
 $b=0$



$a=2$
 $b=-2/R^2$



$a=1/2$
 $b=1/R^2$



Nearest Neighbour Connection Model

$\mathcal{H}_1(d_1, \mathbf{r})$: The probability that a receiver at \mathbf{r} can successfully decode a signal from its nearest AP separated by a distance d_1 .

$$\mathcal{H}_1(\mathbf{r}, d_1) = \mathbb{P} \left[\frac{g(d_1)|h_1|^2}{\mathcal{N} + \gamma \sum_{k>1} g(d_k)|h_k|^2} \geq q | \mathbf{r}, d_1 \right]$$

- $g(d_1) = d_1^{-\eta}$; path loss typically $\eta \in [2, 6]$
- $|h_1|^2 \sim \exp(1)$; channel gain
- $\gamma \in [0, 1]$; captures the orthogonality in the code
- \mathcal{N} ; Noise
- q ; threshold value

$$\mathcal{H}_1(d_1, \mathbf{r}) = \mathbb{P} \left[\frac{|h_1|^2 g(d_1)}{\mathcal{N} + \gamma \sum_{k>1} |h_k|^2 g(d_k)} \geq q \mid \mathbf{r}, d_1 \right]$$

$$\mathcal{H}_1(d_1, \mathbf{r}) = \mathbb{E}_{\mathcal{I}_1} \left[\mathbb{P} \left[|h_1|^2 \geq \frac{q(\mathcal{N} + \gamma \sum_{k>1} |h_k|^2 g(d_k))}{g(d_1)} \mid \mathbf{r}, d_1, \mathcal{I}_1 \right] \right]$$

$$\mathcal{H}_1(d_1, \mathbf{r}) = \exp \left[-\frac{q\mathcal{N}}{g(d_1)} \right] \mathcal{L}_{\mathcal{I}}(q \gamma d_1^\eta),$$

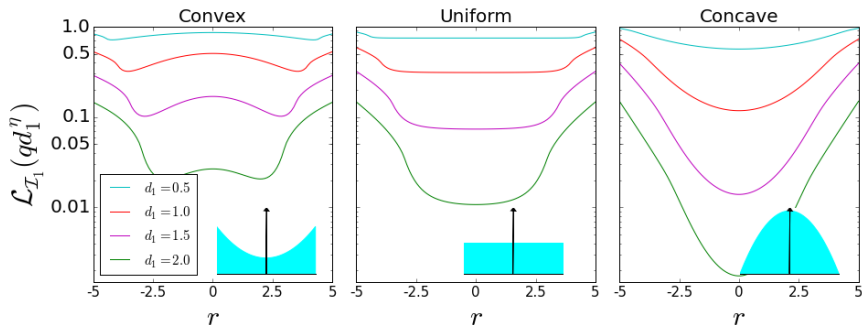
where,

$$\mathcal{L}_{\mathcal{I}_1}(q \gamma d_1^\eta) = \exp \left[-\int_{\mathcal{V} \setminus B_r(d_1)} \frac{\lambda(z)}{1 + \frac{d_k^\eta}{q \gamma d_1^\eta}} d_k d_k d\theta \right]$$

$$z = \sqrt{r^2 + d_k^2 - 2rd_k \cos \theta}$$

Interference Continued

The interference component of the connection function.



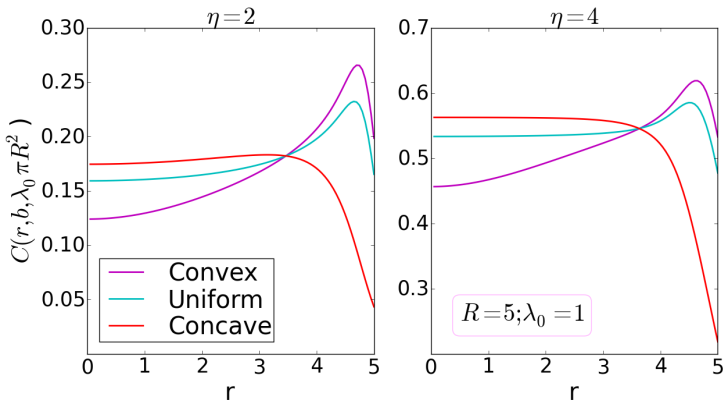
Parameters: $R = 5$; $\eta = 6$; $q = 1$; $\gamma = 1$

Conditioning on a larger d_1 , $\mathbb{E} \left[\frac{d_1}{d_k} \right] \rightarrow 1$

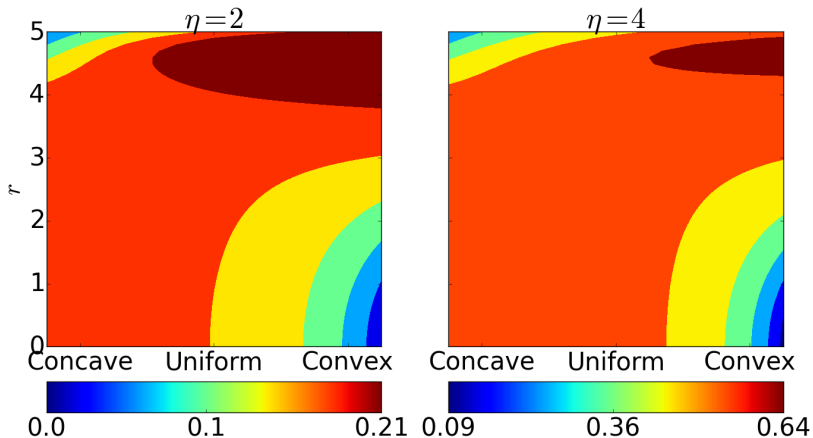
Network Coverage

The probability that a mobile user receiver located at \mathbf{r} can decode a message from its nearest transmitter.

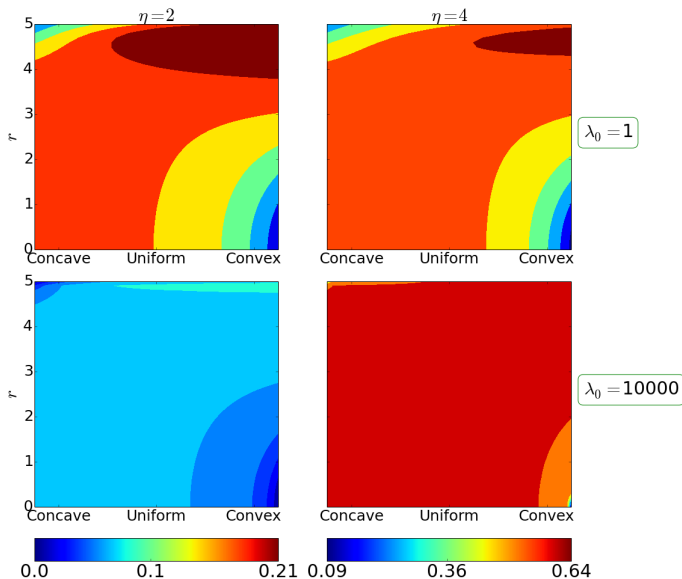
$$C(\mathbf{r}, b, \lambda_0 \pi R^2) = \int_0^{d_{\max}} f(\mathbf{r}, d_1, b, \lambda_0 \pi R^2) \mathcal{H}_1(d_1, \mathbf{r}) dd_1$$



Network Coverage



High Densities - Coverage



Olbers' Paradox

Olbers' paradox summary: *"The paradox is that a static, infinitely old universe with an infinite number of stars distributed in an infinitely large space would be bright rather than dark."*



Comparison: The paradox is that for an infinite density of devices, a user located anywhere would expect to be able to connect to its nearest neighbour regardless of the interference. For $\eta = 2$ the aggregate interference causes an *"interference storm"*.

Namely, $\eta = \text{dim. of the network}$ is the transition point between local and global behaviour.

Optimal Distribution of APs

Aim: Find the optimal distribution of APs so that we can maximise the number of MUs we can serve.

Define the distribution of MUs as

$$\rho(r, \beta) = \rho_0 \left(1 - \frac{\beta R^2}{2} + \beta r^2 \right)$$

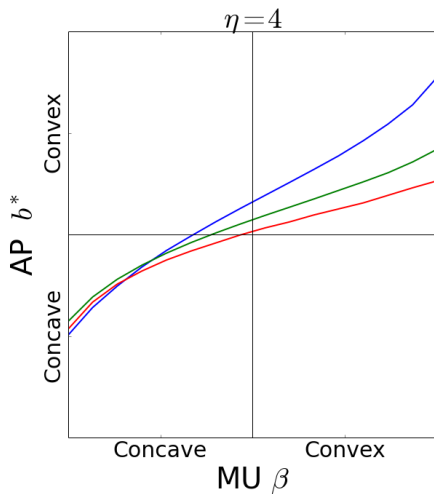
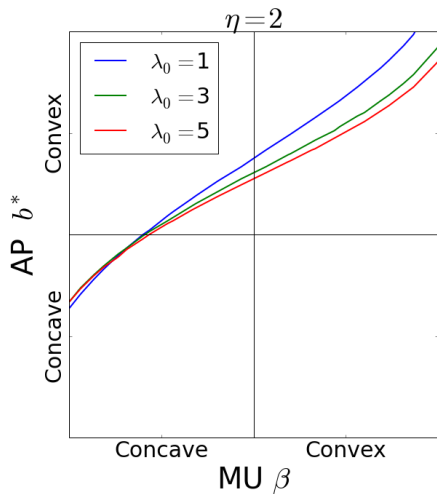
The average coverage is,

$$\bar{C}(b, \beta, \lambda_0 \pi R^2) = \frac{2}{R^2} \int_0^R \rho(r, \beta) C(r, b, \lambda_0 \pi R^2) r dr$$

Find b^* such that \bar{C} is maximised,

$$b^*(\eta, \lambda_0 \pi R^2) = \arg \max_b \bar{C}$$

Optimal Distribution of APs



Conclusions and Applications

- Trade-off between border effects and distribution of APs.
- User coverage depends on location, distribution of APs, border effects and the pathloss model in interference limited environments.
- Pathloss exponent captures the global or local behaviour of the network.
- Adaptive transmission schemes could be used to provide optimal MU Coverage.

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