Enhanced Cellular Coverage and Throughput using Rateless Codes

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Papers

- A. Rajanna and M. Haenggi, *Enhanced Cellular Coverage and Throughput using Rateless Codes*, IEEE Transactions on Communications, vol. 65, no. 5, pp. 1899 -1912, May 2017.

- A. Rajanna and M. Haenggi, *Downlink Coordinated Joint Transmission for Mutual Information Accumulation*, IEEE Wireless Communications Letters, vol. 6, no. 2, pp.198-201, Apr 2017.

Motivation

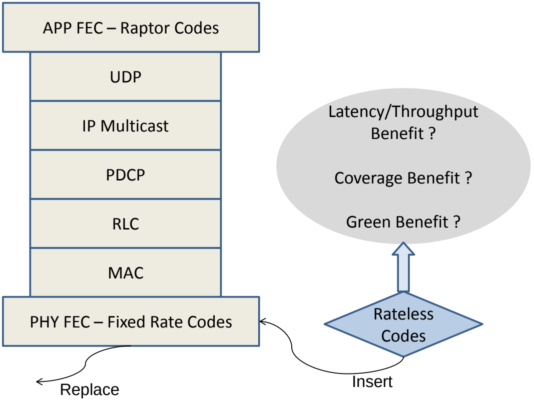

Figure: 3GPP MBMS Protocol Stack.

Motivation

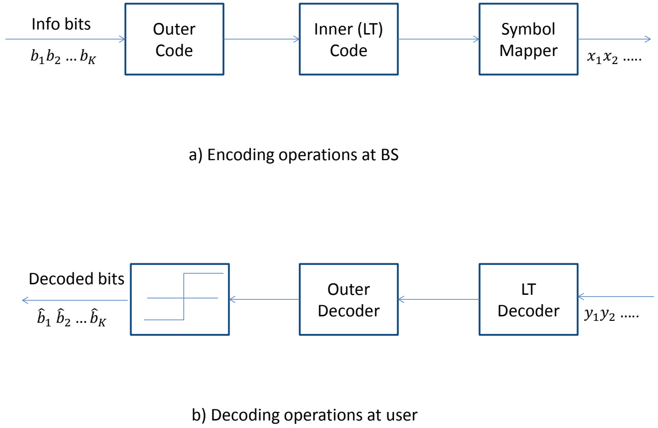

Figure: Block Diagram View

Motivation

Property	IR-HARQ/ Punctured Fixed-Rate Codes	Rateless/ Fountain Codes
Initial codeword	Generate a low rate mother codeword of the information (info) packet.	No predetermined codeword of info packet. Info bits are selected adaptively.
Parity generation and transmission	Puncture the codeword into multiple codeblocks and transmit blocks incrementally.	Fountain generation of parity bits, i.e., incrementally obtain *any* number of parity bits.
Adaptive nature	No adaptive generation of parity bits to channel variations.	Adapt the degree distribution (for parity bit generation) to channel variations.
Selection of info bits	No adaptive selection of info bits.	Adaptively select the info bits to generate parity bits through non-uniform selection.
Rates	Can realize finite discrete rates.	Can realize rates truly matched to the instantaneous channel.

Figure: Punctured Fixed-Rate Codes v/s Rateless Codes.

Rateless Transmission

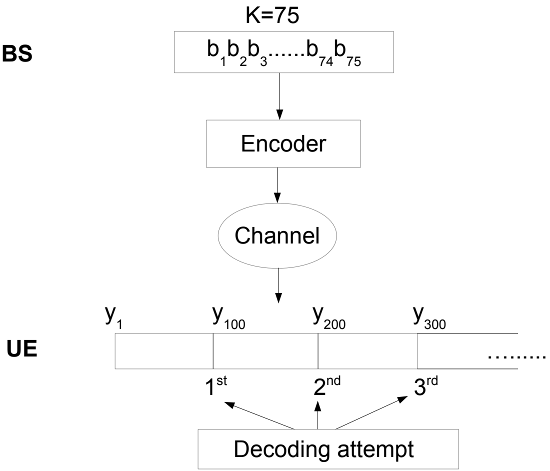

Figure: Rateless Transmission of K-bit packet.

Outline

System Model

- BS process is a homogeneous Poisson point process (PPP) Φ of intensity λ.
- Each BS $X_i \in \Phi$ communicates a K-bit packet to a user Y_i in its Voronoi cell.
- Encoding - Decoding operations
  - **BS**: Encodes K bits with a rateless code and sends Gaussian symbols incrementally over the channel.
  - **User**: For every L channel uses, makes an attempt to decode a subset of K bits.
  - Process continues until user decodes K bits, and sends an ACK to BS.

Time-varying Interference

- The interference power at user Y_i at time t is given by

$$I_i(t) = \sum_{k \neq i} |h_{ki}|^2 |X_k - Y_i|^{-\alpha} e_k(t). \tag{1}$$

$|h_{ki}|^2$: Fading from BS X_k to user Y_i
α : Path loss exponent
$e_k(t)$: MAC state of k^{th} BS.

- MAC state of BS X_k is

$$e_k(t) = 1\left(0 < t \leq T_k\right). \tag{2}$$

T_k: packet transmission time of BS X_k

- **Thinning** of the BS PPP Φ with time t.

Achievable Rate

- **Nearest-Neighbor Decoder**: Performs minimum Euclidean distance decoding for non-Gaussian noise. (only *CSIR*)
- Achievable rate at user Y_i is

$$C_i(t) = \log_2\left(1 + \frac{|h_{ii}|^2 D_i^{-\alpha}}{\hat{I}_i(t)}\right) \tag{3}$$

$\hat{I}_i(t)$- 2^{nd} moment of non-Gaussian noise.

- Time averaged interference up to time t

$$\hat{I}_i(t) = \frac{1}{t}\int_0^t I_i(\tau)\,\mathrm{d}\tau. \tag{4}$$

Packet Transmission Time

- The time to decode K information bits

$$\hat{T}_i = \min\{t : K < t \cdot C_i(t)\} \quad (5)$$

- Each packet transmission of K bits is subject to a delay constraint of N.

- Packet transmission time of user Y_i is

$$T_i = \min(N, \hat{T}_i) \quad (6)$$

- CCDFs of T and $\hat{T}$ are key $\longrightarrow$

  Quantify the performance advantages of rateless codes for PHY-FEC.

Analytical Model

- CCDF of $\hat{T}$

$$\mathbb{P}\left(\hat{T} > t\right) = \mathbb{P}\left(\frac{K}{t} \geq \log_2\left(1 + \frac{|h|^2 D^{-\alpha}}{\hat{I}(t)}\right)\right) \tag{7}$$

We let $\theta_t = 2^{K/t} - 1$, then (7) can be written out as

$$\begin{aligned}\mathbb{P}\left(\hat{T} > t\right) &= \mathbb{E}\left[1 - \mathbb{P}\left(\frac{|h|^2 D^{-\alpha}}{\hat{I}(t)} \geq \theta_t \middle| D\right)\right] \\ &\overset{(a)}{=} \mathbb{E}\left[1 - \mathbb{E}\left[\exp\left(-\theta_t D^{\alpha} \hat{I}(t)\right) \middle| D\right]\right] \\ &= \mathbb{E}\left[1 - \mathcal{L}_{\hat{I}(t)}\left(\theta_t D^{\alpha}\right)\right], \end{aligned} \tag{8}$$

where (a) follows from evaluating the tail of $|h|^2 \sim \mathrm{Exp}(1)$ at $\theta_t D^{\alpha} \hat{I}(t)$.

Analytical Model

- Interference

$$\hat{I}(t) = \sum_{k \neq 0} |h_k|^2 |X_k|^{-\alpha} \min\left(1, T_k/t\right) \quad (9)$$

Marks T_k are *correlated* for different $k \longrightarrow$ No characteristic function.

- **Approximation**:
  - Replace T_k in (9) by i.i.d. $\bar{T}_k$.
  - $\bar{T}_k$: Transmission duration of interferer X_k.
- $\bar{T}$: Distribution of Interferer transmission duration

$$\mathbb{P}\left(\bar{T} > t\right) = 1 - {}_2F_1\left([1, \delta]; 1 + \delta; -\theta_t\right), \quad (10)$$

where $\delta = 2/\alpha$ and $\theta_t = 2^{K/t} - 1$.

Analytical Model

- **Independent Thinning Model-** Let $\bar{I}(t)$ be the interference in (9) with $\bar{T}_k$ in place of T_k.

$$\bar{I}(t) = \sum_{k \neq 0} |h_k|^2 |X_k|^{-\alpha} \min\left(1, \bar{T}_k/t\right) \tag{11}$$

$\Rightarrow$ Closed form characteristic function.

- Typical user's packet transmission time

$$\hat{T} = \min\left\{t : K < t \cdot \log_2\left(1 + \frac{|h|^2 D^{-\alpha}}{\bar{I}(t)}\right)\right\}$$
$$T = \min(N, \hat{T}). \tag{12}$$

Analytical Study Feasible.

Distribution of T

Theorem

An upper bound on the CCDF of the typical user packet transmission time under the independent thinning model, T in (12), is given by

$$\mathbb{P}\left(T > t\right) \leq \begin{cases} P_{\mathrm{ub}}(t) & t < N \\ 0 & t \geq N, \end{cases} \tag{13}$$

where

$$P_{\mathrm{ub}}(t) = 1 - \frac{1}{{}_2F_1\left(\left[1, -\delta\right]; 1-\delta; -\theta_t \min\left(1, \mu/t\right)\right)}, \tag{14}$$

$\delta = 2/\alpha$, $\theta_t = 2^{K/t} - 1$, *and*

$$\mu = \mathbb{E}\left[\bar{T}\right] = \int_0^N \left(1 - {}_2F_1\left(\left[1, \delta\right]; 1+\delta; 1 - 2^{K/t}\right)\right) \mathrm{d}t. \tag{15}$$

Comparison

- **Fixed-Rate Coding**:
  - K info bits $\longrightarrow$ Transmit codeword of N parity symbols.
  - Rate of K/N if success or 0 if outage.

$$p_{\mathrm{s}}(N) \triangleq \mathbb{P}\left(\mathrm{SIR} > 2^{K/N} - 1\right) \tag{16}$$

$$R_N \triangleq p_{\mathrm{s}}(N)\frac{K}{N}. \tag{17}$$

- **Rateless Coding**:
  - Incrementally transmit up to N parity symbols.
  - Multiple decoding attempts.
  - K bits are decoded by variable number of parity symbols.

$$p_{\mathrm{s}}(N) \triangleq 1 - \mathbb{P}(\hat{T} > N) \tag{18}$$

$$R_N \triangleq \frac{Kp_{\mathrm{s}}(N)}{\mathbb{E}\left[T\right]}. \tag{19}$$

SIR Gain

- SIR Gain (Horizontal Gap): $\bar{F}_1$ and $\bar{F}_2$ are SIR CCDFs of two schemes. If

$$\bar{F}_2(\theta) \sim \bar{F}_1\left(\theta/\Gamma\right), \quad \theta \to 0, \tag{20}$$

then scheme 2 provides a SIR gain Γ over scheme 1.

- (20) $\Rightarrow \bar{F}_2(\theta) \approx \bar{F}_1\left(\theta/\Gamma\right)$ for all θ.
- Scheme 1: Fixed-Rate Coding & Scheme 2: Rateless Coding
- $\theta = 2^{K/N} - 1$ with $N \to \infty$ as $\theta \to 0$.
- Compare coverage probability $p_{\rm s}(N)$ for schemes 1 & 2.

SIR Gain

Proposition

Rateless coding in cellular downlink leads to a SIR gain of $\Gamma = \frac{N}{\mu}$ *relative to fixed-rate coding under the independent thinning model, where* $\mu = \mathbb{E}\left[\bar{T}\right]$ *is the mean interferer transmission duration given in (15).*

- Fixed-rate coding

$$p_{\mathrm{s}}(N) = \frac{1}{{}_2F_1\left(\left[1, -\delta\right]; 1-\delta; 1-2^{K/N}\right)}$$

- Rateless coding

$$\tilde{p}_{\mathrm{s}}(N) \geq \frac{1}{{}_2F_1\left(\left[1, -\delta\right]; 1-\delta; \left(1-2^{K/N}\right) \min\left(1, \mu/N\right)\right)}.$$

- μ decreases with path loss exponent α and monotonic with delay constraint N.

Rate Gain

Proposition

In cellular downlink, the rate gain g_{r} of rateless codes relative to fixed-rate codes is

$$g_{\mathrm{r}} = g_{\mathrm{s}} \, \frac{N}{\mathbb{E}\left[T\right]}. \tag{21}$$

- Success probability gain:

$$g_{\mathrm{s}} \geq \frac{{}_2F_1\left(\left[1, -\delta\right]; 1-\delta; 1-2^{K/N}\right)}{{}_2F_1\left(\left[1, -\delta\right]; 1-\delta; \left(1-2^{K/N}\right)\mu/N\right)} \geq 1. \tag{22}$$

- Transmission time gain:

$$\frac{N}{\mathbb{E}\left[T\right]} \geq \frac{N}{\int_0^N P_{\mathrm{ub}}(t)\,\mathrm{d}t} \geq 1. \tag{23}$$

- $\Rightarrow g_r \geq 1.$

Per-User Gain

- Typical User Performance $\longrightarrow$ Spatial Average

  Per-User Downlink Performance $\longrightarrow$ Conditioned on Φ.

- Coverage and Rate are random variables (RVs), achieved by *any* BS-UE pair in a given Φ realization.

- Fixed-rate coding:

$$R_N \triangleq \frac{K}{N}\, \mathbb{P}\left(\mathrm{SIR} > 2^{K/N} - 1 \mid \Phi\right). \tag{24}$$

- Rateless coding:

$$R_N \triangleq \frac{K\left[1 - \mathbb{P}(\hat{T} > N \mid \Phi)\right]}{\mathbb{E}\left[T \mid \Phi\right]}. \tag{25}$$

Per-User Gain

- Rate gain *conditioned on* Φ: Ratio of random rates of rateless and fixed-rate coding.

$$G_R = G_S \, \frac{N}{\mathbb{E}\left[T \mid \Phi\right]} \tag{26}$$

$$G_S = \frac{1 - \mathbb{P}(\hat{T} > N \mid \Phi)}{\mathbb{P}\left(\mathrm{SIR} > 2^{K/N} - 1 \mid \Phi\right)}. \tag{27}$$

Proposition

Every BS-UE pair in a cellular downlink with PPP Φ realization experiences a throughput gain due to rateless code PHY-FEC relative to fixed-rate codes, i.e., $G_R \geq 1$.

- Can show $G_S \geq 1$.

Coverage Probability

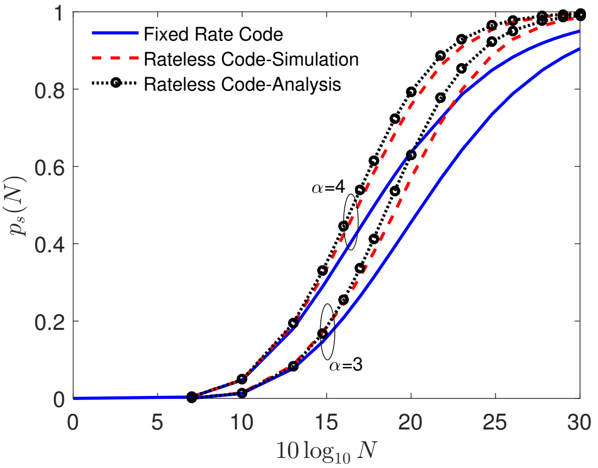

Figure: Success Probability $p_s(N)$ v/s Delay constraint N.

Rate

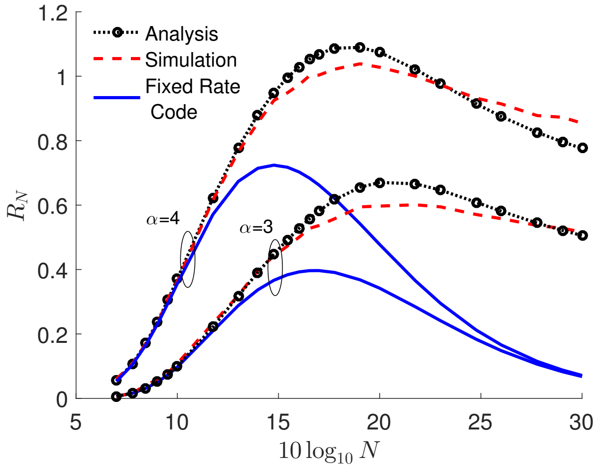

Figure: Rate R_N v/s Delay constraint N.

Rate

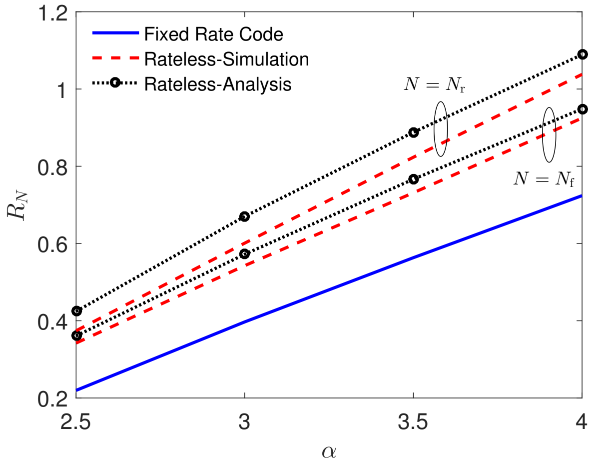

Figure: Rate R_N v/s path loss exponent α.

Per-User Case

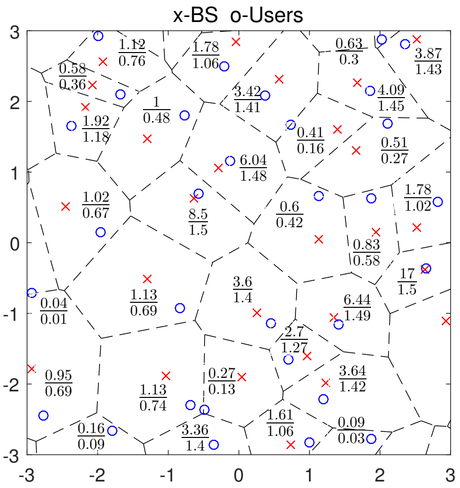

Figure: Ratio of Rates conditioned on Φ.

Per-User Rates

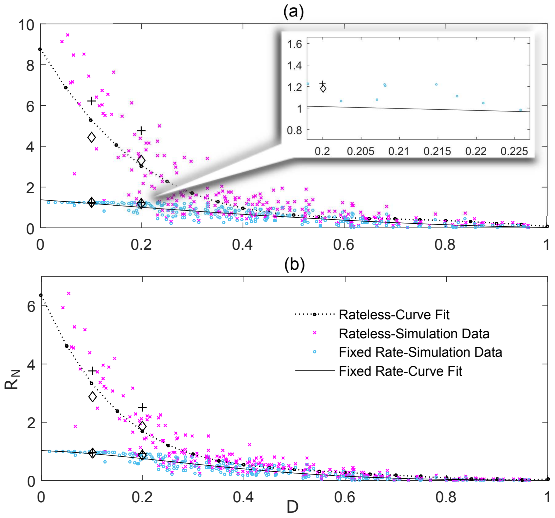

Figure: Per-User Rates v/s BS-UE Distance D.

Rate Gain

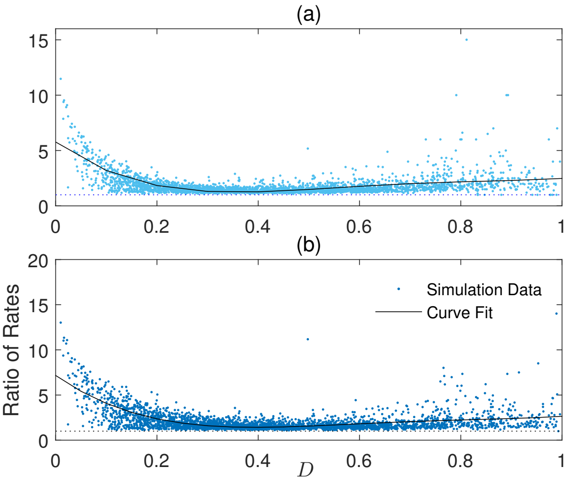

Figure: Rate Gain v/s BS-UE Distance D.

Outline

Assumptions

- Interfering BSs transmit *continuously*. MAC state of interfering BS X_k at time t is $e_k(t) = 1, \; t \geq 0$.
- Typical user receives a K-bit packet via a rateless code by one BS.
- Interference at typical user

$$I = \sum_{k \neq 0} |h_k|^2 |X_k|^{-\alpha}. \tag{28}$$

Simple characteristic function.

- CCDF of the typical user packet transmission time T

$$\mathbb{P}(T > t) = 1 - \frac{1}{{}_2F_1\left([1, -\delta]; 1 - \delta; -\theta_t\right)}, \quad t < N. \tag{29}$$

Performance Comparison

- No Coverage gain: $p_s(N)$ same for both rateless coding and fixed-rate coding. $\bar{g}_{\mathrm{s}} = 1$
- Rate gain: Ratio of rates

$$\bar{g}_{\mathrm{r}} = \frac{N}{\mathbb{E}\left[T\right]}. \tag{30}$$

- *The rate gain of rateless codes in the cellular downlink under the continuous transmission case is given by*

$$\bar{g}_{\mathrm{r}} = \left[1 - \frac{1}{N}\int_0^N \frac{1}{{}_2F_1\left(\left[1, -\delta\right]; 1-\delta; -\theta_t\right)}\,\mathrm{d}t\right]^{-1}. \tag{31}$$

Rate Gain

- Rate Gain due to **Interferer Activity**:

  The rate gains $\bar{g}_{\mathrm{r}}$ and g_{r} in the cellular downlink by using rateless codes for PHY-FEC satisfy the relation

  $$1 \leq \bar{g}_{\mathrm{r}} \leq g_{\mathrm{r}}. \tag{32}$$

- g_{r}: **Monotonically decreasing** interference due to *thinning* of interfering BSs.
- $\bar{g}_{\mathrm{r}}$: **Constant** interference due to *continuous* transmission of interfering BSs.
- Rate gain of a practical user:
  - $\bar{g}_{\mathrm{r}}$ - Lower bound
  - g_{r} - Upper bound.

Rate Gain

- Rate Gain due to **User Location**:
  - General User: close to *only one* BS or
  - Edge User: equidistant from *two* BSs or
  - Vertex User: equidistant from *three* BSs.
- General user was the focus till now.
- Vertex User: Resides on a vertex of the Voronoi tessellation of Φ.
- CCDF of the packet transmission time of the vertex user

$$\mathbb{P}(T > t) = 1 - \left[\frac{1/(1+\theta_t)}{{}_2F_1([1, -\delta]; 1-\delta; -\theta_t)} \right]^2, \quad t < N. \tag{33}$$

Rate Gain

- Rate gain of the typical vertex user is

$$\bar{g}_{\mathrm{rv}} = \left[1 - \frac{1}{N}\int_0^N \left(\frac{1/\left(1+\theta_t\right)}{{}_2F_1\left(\left[1,-\delta\right];1-\delta;-\theta_t\right)}\right)^2 \mathrm{d}t\right]^{-1}. \tag{34}$$

- Rate gains of the vertex user and general user satisfy

$$1 \leq \bar{g}_{\mathrm{rv}} \leq \bar{g}_{\mathrm{r}}. \tag{35}$$

- $\bar{g}_{\mathrm{r}}$: Interfering BSs are **all further away** than serving BS.
- $\bar{g}_{\mathrm{rv}}$: **Two** interfering BSs at **same distance** as serving BS while remaining are further away.

Rateless code adapts to changing channel conditions ⇒ Varied gains over fixed-rate code.

Coordinated Transmission (CoMP)

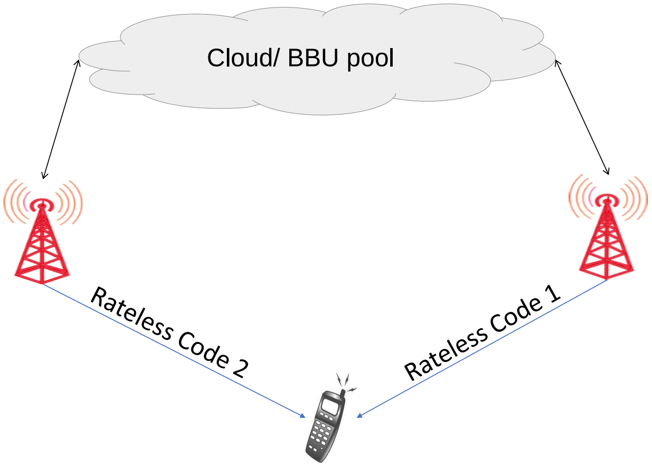

Figure: CoMP for Mutual Information (MI) Accumulation

MI Accumulation

- Cooperating BSs: Joint Transmission of K-bit packet to user.
- BSs access: K bits - X2/S1 interface of backhaul to cloud.
- Each BS uses a *unique* rateless code of K-bit packet.
- NOMA schemes resolve codewords.
- **Multiple codewords** input to iterative decoder at user.
- Achievable rate at user

$$C = \sum_{i=1}^{M} \log_2 \left(1 + \mathrm{SIR}_i\right). \tag{36}$$

System Model

- Two independent PPPs Φ_1 and Φ_2 of intensity $\lambda/2$.
- BSs $\in \Phi_k$: Use spreading code $k \in \{1, 2\}$.
- BSs in cellular downlink $\Phi = \Phi_1 \cup \Phi_2 = \{X_i\},\ i = 1, 2, \cdots$.
- Typical user receives a codeword from nearest BS in both Φ_1 and Φ_2.
- Time to decode K-bit packet and packet transmission time

$$\hat{T} = \min\{t : K < t \cdot C\} \tag{37}$$

$$T = \min(N, \hat{T}). \tag{38}$$

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General User

Theorem

The CCDF of the general user packet transmission time with MI accumulation, T in (38), is lower bounded as

$$\mathbb{P}\left(T > t\right) \geq \int_0^{\gamma} \left(G\left(\gamma - y\right) - 1\right) G'(y)\,\mathrm{d}y, \tag{39}$$

$$G(\nu) = \frac{1}{{}_2F_1\left([1, -\delta]\,; 1-\delta; -\nu\right)}. \tag{40}$$

where $\gamma = 2\left(2^{K/2t} - 1\right)$ *and* $\delta = 2/\alpha$.

Success Probability and Rate

$$p_s(N) \triangleq 1 - \mathbb{P}\left(\hat{T} > N\right) \tag{41}$$

$$R_N \triangleq \frac{Kp_s(N)}{\mathbb{E}\left[T\right]}. \tag{42}$$

Diversity Gain

- For rateless coding, the diversity gain is

$$g_{\mathrm{d}} \triangleq \lim_{N \to \infty} \frac{\log\left(1 - p_s(N)\right)}{-\log N}. \tag{43}$$

- No Cooperation

$$1 - p_s(N) \sim \frac{K \log 2}{N} \frac{\delta}{1-\delta}, \quad N \to \infty. \tag{44}$$

$$g_{\mathrm{d}} = 1.$$

- MI Accumulation

$$1 - p_s(N) \sim \frac{1}{2} \left(\frac{\delta}{1-\delta}\right)^2 \left(\frac{K \log 2}{N}\right)^2, \quad N \to \infty, \tag{45}$$

$$g_{\mathrm{d}} = 2.$$

Vertex User

Vertex user is served by the **two equidistant** BSs with unique spreading codes ($M = 2$) with the third equidistant BS as interferer.

Theorem

The CCDF of the vertex user packet transmission time with MI accumulation, T in (38), is lower bounded as

$$\mathbb{P}(T > t) \geq \int_0^{\infty}\int_0^{\gamma} \left(U(\gamma - y) - 1\right) \tilde{G}'(y) f_D(r)\,\mathrm{d}y\,\mathrm{d}r \tag{46}$$

$$\tilde{G}(y) = \exp\left(-\pi \frac{\lambda}{2} r^2 \left({}_2F_1\left([1, -\delta]; 1 - \delta; -y\right) - 1\right)\right) \tag{47}$$

$$U(y) = \frac{\tilde{G}(y)}{1 + y}, \quad \gamma = 2\left(2^{K/2t} - 1\right). \tag{48}$$

Diversity Gain

- Vertex User: CoMP achieves diversity gain - Analysis *tricky though*
- No Cooperation

$$1 - p_s(N) \sim \frac{K \log 2}{N} \left(2 + \frac{2\delta}{1-\delta}\right), \quad N \to \infty. \tag{49}$$

$$g_{\mathrm{d}} = 1.$$

- MI Accumulation

$$1 - p_s(N) \sim \left(\frac{K \log 2}{N\sqrt{2}}\right)^2 \int \left(\pi \frac{\lambda}{2} r^2 \frac{\delta}{1-\delta}\right)^2 f_D(r)\,\mathrm{d}r, \;\; N \to \infty. \tag{50}$$

$$g_{\mathrm{d}} = 2.$$

Coverage Probability

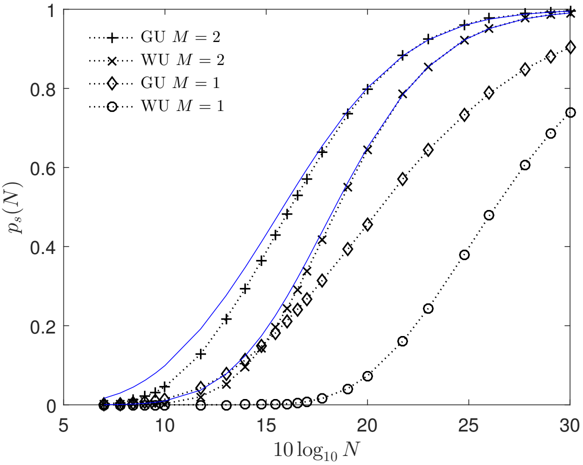

Figure: Success Probability $p_s(N)$ v/s Delay constraint N.

Rate

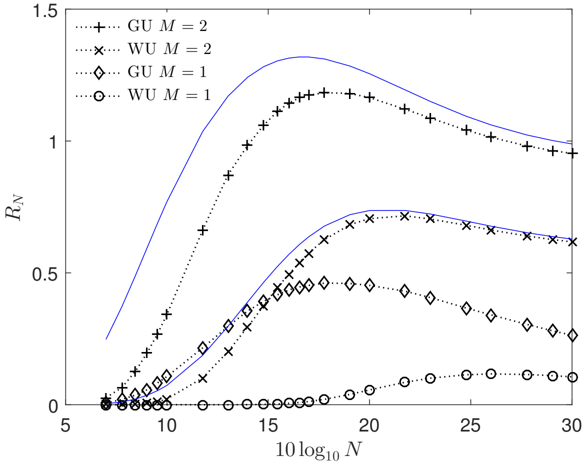

Figure: Rate R_N v/s Delay constraint N. Rate gain of 2.6 and 6.12 for general and vertex user.

Conclusion

- Stochastic geometry model for downlink:

  Rateless codes for PHY-FEC $\Longrightarrow$ Enhanced cellular coverage and throughput.

- Typical user: Rateless PHY relative to fixed-rate PHY $\Rightarrow$
  - SIR gain (Horizontal Gap): *Coverage* improvement
  - Rate gain: *Throughput* improvement

- Per-user case
  - *Every BS-UE pair in a cellular network realization has a throughput gain* ≥ 1 *by rateless codes.*
  - Achieve per-user rates $\Rightarrow$ Efficient network operation.

- CoMP with Rateless codes:
  - Achieves coverage and rate improvement relative to no-cooperation.
  - Vertex user benefits more from CoMP.