Temporal Correlation of Interference and Outage in Mobile Networks With Correlated Mobility in Finite Regions

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Abstract—In practice, wireless networks are deployed over finite domains, the level of mobility is different at different locations, and user mobility is correlated over time. All these features have an impact on the temporal properties of interference which is often neglected. In this paper, we show how to incorporate correlated user mobility into the interference and outage correlation models. We use the Random Waypoint Mobility Model as an example model inducing correlation, and we calculate its displacement law at different locations. Based on that, we illustrate that the temporal correlations of interference and outage are indeed location-dependent, being lower close to the centre of the domain, where the level of mobility is higher than near the boundaries. Close to the boundaries, more time is also needed to see uncorrelated interference at the receiver. Our findings suggest that an accurate description of the mobility pattern is important, because it leads to more accurate understanding/modeling of interference and receiver performance.

Index Terms—Correlation, Interference, Mobility, Stochastic geometry, Wireless networks.

1 Introduction

The performance of wireless networks is limited by interference. Interference is correlated over time when there are temporal correlations in the propagation channel, the user traffic and the user location [1]. Interference correlation is directly related to the correlation of outage and because of that, it also affects other network performance metrics, e.g., the temporal diversity gain, the multi-hop delay, etc., thus becoming essential in the design of routing protocols, retransmission and Medium Access Control (MAC) schemes. The temporal correlation of interference has received some attention in the literature, however, under the assumption of infinite networks, where the locations of interferers are modelled by a Poisson Point Process (PPP) in the infinite plane [1], [2], [3], [4], [5], [6], [7], [8], [9]. Also, the impact of mobility on the interference correlation has been studied for mobility models which do not introduce correlation in the locations of a user over time [4], [5], [6], [7], [8], [9].

In [1], the interference correlation is investigated for static Poisson networks, i.e., the user locations are fixed but unknown, with slotted ALOHA. These sorts of networks can be modelled by a single realization of a PPP, where every user transmits independently in each time slot with some probability. In that case, the interferers are always selected from the same set of users, making the locations of interferers, and subsequently the interference pattern, look correlated over time. It is shown in [1] that a high transmission probability is associated with a high interference correlation, and the temporal diversity gain due to retransmissions may completely vanish [2]. Obviously, the correlation becomes higher in a block fading channel and under correlated user traffic [3].

If the locations of interferers in each time slot are drawn from a new and independent realization of the PPP, an infinite user velocity is essentially modelled [4]. With infinite velocities, the interference pattern becomes uncorrelated over time, irrespective of the fading channel and the user traffic [3]. The local delay, i.e., the mean time needed to connect to the nearest neighbor, is always finite [4], [5] while, in the static case, it may have a heavy tail. In relay chains under a Poisson field of interferers, the mean and variance of the end-to-end delay become smaller, when a new realization of the PPP is drawn in each time slot [6].

The impact of finite mobility on the interference correlation is studied in [7] for Poisson networks and various mobility models, i.e., constrained independent and identically distributed (i.i.d.) mobility, random walk and discrete-time Brownian motion. For all the considered models, the location changes are i.i.d. over time, and the uniform distribution of users is preserved between the time slots. It is shown in [7] that the interference correlation decreases inversely proportional with the mean speed of the users. In [8], the results are extended to illustrate that the interference and outage are positively correlated, when the desired transmitter is placed at a fixed and known distance from the receiver. Different levels of mobility are studied in [9], where it is assumed that in each time slot, only a fraction of users remains static, and the rest move according to a mobility model preserving uniformity. A higher fraction of mobile users is associated with a lower correlation in the outage probability resulting in higher diversity gains [9].

In practice, networks have finite boundaries and the concept of a typical receiver which is placed at the origin and where the network performance is assessed is not always realistic. This issue has already been pointed out in [8], [11], [12], where single-snapshot analysis of interference and outage are carried out at different locations. For a Poisson network deployed over a convex domain, the receivers

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close to the boundaries experience more outage due to isolation [10], but less due to interference [11]. The location of the receiver becomes more important in non-uniform deployments, where the interference would naturally vary more. In mobile networks following the Random Waypoint Mobility (RWPM) model, see for instance [13], the users tend to concentrate close to the center of the area. Because of that, the interference over there becomes significantly higher than at the borders, motivating the use of location-aware routing protocols and MAC schemes [12]. Under the RWPM model, the mean interference at the center becomes at least twice the mean interference generated by the equi-dense PPP [8].

The temporal correlation of interference and outage for networks with finite boundaries is yet to be studied. If the users are distributed according to a general inhomogeneous PPP and remain static, the analysis in [1] still holds, i.e., the correlation coefficient will be proportional to the random access probability, and inversely proportional to the second moment of the fading Random Variable (RV). In this paper, we take a step further and consider the case where the users are mobile, and the mobility induces correlation in the user location over time. We will show how to incorporate the correlation due to mobility into the interference model, and we will illustrate that the temporal correlations of interference and outage are in general location-dependent. As compared to the i.i.d. mobility models, e.g. [7], [8], [9], the interference correlation under correlated mobility should decay more slowly. We will examine how quickly the interference correlation decays at different locations. Note that we do not treat in this paper group mobility models, where the locations of different users are correlated [14].

We need a mobility model which is defined over a finite area and it introduces temporal correlation in the user locations. The RWPM model has both of these features and it has been widely-used in the performance assessment of mobile wireless ad hoc networks [8], [15], [16]. In this paper, we consider a discretized version of it in space and time over one-dimensional lattices. Even though this may seem to be overly simplistic, it suffices to illustrate how to incorporate the mobility correlations into the interference model. Other mobility models over one-dimensional lattices can be treated in the same way, provided that the displacement laws at every vertex can be computed. Extending the results of this paper to continuous-domain mobility models and higher dimensions have been left as future topics, but the main outcomes of this paper will still hold. To motivate a little bit further the use of RWPM model, we point out that the model allows studying: (i) different levels of mobility by varying the think time, and (ii) a static network with a uniform user distribution in the limit of infinite think time. The contributions of this paper are summarized below:

- For the RWPM model, we show how to compute the steady state probabilities for the displacement of users over one-dimensional lattice after $t = 1$ and $t = 2$ time slots. If the think time is equal to zero, i.e., fast mobility, we also provide an approximation for the displacement probabilities for $t > 2$. We find that the displacement law is location-dependent, due to the existence of boundaries and different levels of mobility at different locations.
- We show how to incorporate the displacement law into the description of the temporal correlation of interference at different locations. For the RWPM model, we illustrate that the interference correlation is in general higher close to the boundaries, and it decays faster close to the middle of the lattice. With fast mobility, we illustrate how many time slots it takes for the interference to become uncorrelated at different locations.
- We show how to incorporate the probabilities for the user displacement into the calculation of the conditional outage probability at different locations, i.e., the probability of being in outage after $t = \tau$ time slots, given that an outage occurs at $t = 0$. For the RWPM model, we illustrate that a receiver located close to the border is unlikely to be in outage, however, the conditional outage probability in the subsequent time slot might be significantly higher than the unconditional. On the other hand, close to the middle of the lattice, the outage events are more probable but also less correlated.

The rest of the paper is organized as follows. In Section 2 we present the system model assumptions including the mobility model, and in Section 3 we identify the steady state distribution of users. In Section 4 we compute the displacement probabilities. With these probabilities at hand, we calculate the interference correlation coefficient in Section 5, and the conditional outage probability in Section 6. Numerical examples are available in Section 7, and conclusions in Section 8.

2 System model

We consider one-dimensional lattice of size N, $n = 1, 2, \dots N$, and K users moving across the lattice. Initially, the users are allocated uniformly. Then, each user selects uniformly at random a destination point, and travels towards it with a constant speed v. The time is discretized in time slots $t \in \mathbb{N}$. Since the user speed is common for all the users, we can normalize it, so that the distance covered in a time slot is equal to the distance between two lattice points. The user updates its location in the beginning of a time slot, and keeps on doing so till it reaches the destination. Then, it pauses and thinks for a number of time slots, m_i, selected from the discrete uniform distribution $m_i \in \{0, 1, 2, \dots, M\}$ $\forall i$, where M is the maximum think time in time slots. Then, the same procedure is repeated.

The described mobility model is a modified version of the RWPM model in the discrete time-space. In the continuous-domain model [13], every user may select its speed uniformly at random from the interval $[v_{\min}, v_{\max}]$. We could utilize a discrete distribution for the user velocity, $v \in \{v_{\min}, v_{\min} + 1, \dots, v_{\max} - 1, v_{\max}\}$, where $v_{\min}$ must be an integer multiple of the distance between two lattice points. However, we decided to keep the same speed for all the users to avoid unnecessary complexity in the system model. Recall that the aim of this paper is not to quantify the precise impact of the RWPM model on the interference, but rather to show how to incorporate the impact of correlated mobility into the interference and outage correlation models over areas with finite boundaries.

We are interested to quantify the interference correlation at different locations in the steady state. To avoid unnecessary complexity in the notation, we denote $t = 0$ as the time slot when the steady state starts. We place a virtual receiver at each lattice point and denote its location by $x_p \in \{1, 2, \ldots, N\}$. We assume a common transmit power level P_t for all the users. The MAC scheme is slotted ALOHA where each user, at the beginning of a time slot, decides whether to transmit or not, independently of its own activity at previous time slots, and the activities of other users. The transmission probability is ξ. The generated interference at point x_p and time slot $t \geq 0$ is

$$\mathcal{I}(x_p, t) = P_t \sum_{i=1}^{K} \xi_i(t)\, h_i(t)\, g(x_i(t) - x_p), \tag{1}$$

where ξ_i is a Bernoulli RV describing the i-th user activity, h_i is an exponential RV modeling Rayleigh fast fading, $x_i \in \{1, 2, \ldots, N\}$ is a RV describing the location of the i-th user with Probability Distribution Function (PDF) calculated in the next section, and $g(\cdot)$ is the distance-based propagation pathloss function, $g(x) = \frac{1}{\epsilon + |x|^a}$, where ϵ is used to avoid singularity at distance $x = 0$ and it normally takes a small positive value.

In the steady state, the moments of interference become independent of the time t we take the measurements. The Moment Generating Function (MGF) of interference distribution is denoted by $\Phi_{\mathcal{I}}$, and the correlation coefficient, $\rho(x_p, \tau)$, at time $t = 0$, time-lag τ and point x_p is

$$\rho(x_p, \tau) = \frac{\mathbb{E}\{\mathcal{I}(x_p, \tau)\,\mathcal{I}(x_p)\} - \mathbb{E}\{\mathcal{I}(x_p)\}^2}{\mathbb{E}\left\{\mathcal{I}(x_p)^2\right\} - \mathbb{E}\{\mathcal{I}(x_p)\}^2}, \tag{2}$$

where the notation $\mathcal{I}(x_p, 0)$ is shortened to $\mathcal{I}(x_p)$ for brevity.

While studying the correlation of outage, we will assume that the desired transmitter is placed at a fixed and known distance from the associated receiver. Unlike the users generating interference, the desired transmitter is static, and it transmits in every time slot at power level P_t with probability one. It is not part of the mobile users generating interference. The location of the desired transmitter is denoted by x_t. The channel between the transmitter and the receiver is also subject to Rayleigh fading h_{tx}, and the distance-based propagation pathloss model is also $g(\cdot)$. The noise power level at the receiver is P_N.

3 User distribution in the steady state

Since the users move independently of each other, it is sufficient to identify the spatial distribution for a single user. Let $\mathcal{N}$ denote the RV whose value n is the location of the user at a randomly selected time slot in the steady state. In order to identify the Cumulative Distribution Function (CDF) of the user location, $\mathbb{P}(\mathcal{N} \leq n)$, one has to monitor the user for a sufficiently large number of time slots. The CDF is simply equal to the number of time slots spent at the lattice points $\{1, 2, \ldots, n\}$, divided by the number of time slots the user is monitored.

Let ignore for the moment the thinking time, i.e., $M = 0$, and assume that the user completes $J \to \infty$ travels in the monitoring time. We denote by T_j the number of time slots spent on the j-th travel and by $T_{n,j}$ the number of time slots spent at the lattice points $1, 2, \ldots, n$ during the j-th travel. Then, the CDF is

$$\mathbb{P}(\mathcal{N} \leq n) = \lim_{J\to\infty} \frac{\sum_{j=1}^{J} T_{j,n}}{\sum_{j=1}^{J} T_j} = \frac{\mathbb{E}\{T_n\}}{\mathbb{E}\{T\}},$$

where T is a RV describing the number of time slots spent on a travel, and T_n is a RV describing the number of time slots spent at the lattice points $\{1, 2, \ldots, n\}$ during a travel.

Since the user speed is fixed and constant, the travel time is proportional to the distance covered. Hence,

$$\mathbb{P}(\mathcal{N} \leq n) = \frac{\mathbb{E}\{L_n\}}{\mathbb{E}\{L\}},$$

where L is a RV describing the distance, or equivalently, the number of lattice points visited during a travel, and L_n is a RV whose value is equal to the number of lattice points from the set $\{1, 2, \ldots, n\}$ visited during a travel.

In order to compute the expected value of the RV L, one has to consider all possible travel paths over the lattice and compute their expected length. Assuming that the source and the destination points are different, there are $N(N-1)$ paths in total. Note that a user selects its destination independent of the source, thus all paths become equally probable. Therefore the expected length of a path is simply the arithmetic average of all the path lengths. Let denote the source of a path by s and its destination by d. The average path length is

$$\mathbb{E}\{L\} = \frac{2}{N(N-1)} \sum_{s=1}^{N-1} \sum_{d=s+1}^{N} (d-s) = \frac{N+1}{3}. \tag{3}$$

Similarly, in order to compute the expected value of the RV L_n, one may consider all $N(N-1)$ paths, count the number of lattice points from the set $\{1, 2, \ldots, n\}$ that the user visits in each path, and take the arithmetic average. The computation of $\mathbb{E}\{L_n\}$ can be split into four independent cases depending on the relative location of n, s and d. Let denote by $L_n^{(j)}, j = 1, 2, 3, 4$ the RV describing the number of lattice points from the set $\{1, 2, \ldots, n\}$ visited during a travel for the j-th case.

- $d > s$ and $d \leq n$. In that case, the user travels over $(d-s)$ lattice points which all contribute to the value of the RV L_n.

$$\mathbb{E}\left\{L_n^{(1)}\right\} = \frac{1}{N(N-1)} \sum_{s=1}^{n-1} \sum_{d=s+1}^{n} (d-s).$$

- $d > s$ and $d > n$. The user initially travels over $(n-s)$ points which contribute to the value of the RV L_n but then, the travel from $(n+1)$ to d does not make any contribution.

$$\mathbb{E}\left\{L_n^{(2)}\right\} = \frac{1}{N(N-1)} \sum_{s=1}^{n} \sum_{d=n+1}^{N} (n-s).$$

- $d < s$ and $s \leq n$. This case is similar to the first case with reversed source and destination points.

$$\mathbb{E}\left\{L_n^{(3)}\right\} = \frac{1}{N(N-1)} \sum_{d=1}^{n-1} \sum_{s=d+1}^{n} (s-d).$$

- $d < s$ and $s > n$. This case is similar to the second case with reversed source and destination points. However, one has to note that both lattice points n and d contribute to the value of the RV L_n because the user location is updated in the beginning of a time slot.

$$\mathbb{E}\left\{L_n^{(4)}\right\} = \frac{1}{N(N-1)}\sum_{d=1}^{n}\sum_{s=n+1}^{N}(n-d+1).$$

After computing the expected number of lattice points from the set $\{1,2,\ldots,n\}$ visited during a travel as the sum of the four individual cases, $\mathbb{E}\{L_n\} = \sum_{i=1}^{4}\mathbb{E}\left\{L_n^{(i)}\right\}$, and dividing it by the expected length of a path $\mathbb{E}\{L\}$ computed in equation (3), the CDF of the user location for the mobile component of the network, $F_m(n)$, can be read as

$$F_m(n) = \frac{\mathbb{E}\{L_n\}}{\mathbb{E}\{L\}} = \frac{\left(3Nn-2n^2-1\right)n}{N(N^2-1)},\ \forall n. \quad (4)$$

For comparison purposes, the CDF of the user location in the continuous-domain RWPM model is [13]

$$F_m(x) = \frac{3x^2}{x_0^2} - \frac{2x^3}{x_0^3},\ 0 \le x \le x_0.$$

If we consider a positive maximum think time, $M > 0$, the probability p that the user is static at a randomly selected time slot in the steady state is [13]

$$p = \frac{\mathbb{E}\{m\}}{\mathbb{E}\{m\}+\mathbb{E}\{T\}} = \frac{M/2}{M/2+(N+1)/3},$$

where the index i in the RV m has been dropped since the users are indistinct, the term $M/2$ is the expected think time, and the term $(N+1)/3$ describes the expected time of a travel for user speed $v=1$ lattice point per time slot.

Since all the paths are equally probable, the distribution of destination points is uniform. Therefore the distribution of static users is uniform too. The CDF of user location can be expressed as the weighted sum of a uniform distribution describing the static component of the network, $F_s(n) = \frac{n}{N}\forall n$, and the distribution given in (4) describing the mobile component

$$F(n) = pF_s(n) + (1-p)\,F_m(n),\ \forall n.$$

The PDF of the user location can be computed as $f(n) = F(n) - F(n-1), n \ge 2$ and $f(1) = F(1)$. Finally,

$$f(n) = \frac{p}{N} + (1-p)\,\frac{3N\,(2n-1) - 6n\,(n-1) - 3}{N(N^2-1)}, \forall n. \quad (5)$$

We note that for a zero think time, in the continuous-domain model, the probability that a user is located exactly at the borders is zero, while in the discrete-domain model, the same probability is $f(1) = f(N) = \frac{3}{N(N+1)}$.

4 Displacement probabilities

Let assume that the network has already reached the steady state, and the user which we monitor is located at the lattice point n. In this section, we show how to compute the displacement probability, $\mathbb{P}(n+k,\tau)$, i.e., the probability that the user moves to the lattice point $(n+k)$ after τ time slots. Since the user speed is fixed to one point per time slot, the possible displacement is $k \in \{-\tau,\ldots,0,\ldots,\tau\}$. We show how to compute $\mathbb{P}(n+k,\tau)$ for $\tau = 1$ and $\tau = 2$. Also, we show how to approximate $\mathbb{P}(n+k,\tau)$ for $\tau > 2$, assuming a zero think time.

4.1 Change of location after $\tau = 1$ time slots

As discussed in Section 3, the distribution of users which are thinking at a randomly selected time slot is uniform. On the other hand, from equation (5) we have that $f(n) > f(n-1), n = 2,3,\ldots,\lceil N/2\rceil$, which means that the distribution of users that are moving at a randomly selected time slot is non-uniform. Therefore the probability that the user does not change its location at $t = 1$, $\mathbb{P}(n,1)$, is location-dependent. It can be expressed as the fraction of the static component in the PDF given in (5)

$$\mathbb{P}(n,1) = \frac{p}{Nf(n)}, \forall n. \quad (6)$$

Given that the user changes its location with probability $1 - \mathbb{P}(n,1)$, the probability it moves to the right can be computed as the fraction of paths crossing the lattice point n while moving to the right, divided by the total number of paths crossing that point. The paths with source $s \in \{1,2,\ldots,n\}$ and destination $d \in \{n+1, n+2, \ldots, N\}$ cross the point n to the right while the paths with source $s \in \{n, n+1, \ldots, N\}$ and destination $d \in \{1,2,\ldots,n-1\}$ cross the point n to the opposite direction. Hence,

$$\mathbb{P}(n+1,1) = \frac{(1-\mathbb{P}(n,1))\,n\,(N-n)}{n(N-n)+(n-1)(N-n+1)}, n < N. \quad (7)$$

Obviously, $\mathbb{P}(n+1,1) = 0$ for $n = N$. Also, the probability that the user moves to the left is the complementary probability, $\mathbb{P}(n-1,1) = 1 - \mathbb{P}(n,1) - \mathbb{P}(n+1,1)$ for $n > 1$, and $\mathbb{P}(n-1,1) = 0$ for $n = 1$.

Remark 4.1 (Thinking at the border). After substituting $n = 1$ into (6), the probability that a user which is located at the border stays there and thinks at $t = 1$ is $\mathbb{P}(1,1) = \frac{M}{M+2}$, thus it is irrespective of the lattice size N.

Remark 4.2 (Thinking at the center). Let consider a large lattice $N \gg 2$. The probability that a user which is located at the center, $n = \lceil N/2\rceil$, stays there and thinks at $t = 1$ converges to $\lim_{N\to\infty}\mathbb{P}\left(\left\lceil\frac{N}{2}\right\rceil, 1\right) = \frac{M}{M+N}$, thus the user will rather move with a high probability. Also, starting from (7), one can show that for a large N, the probabilities to move left or right have equal limits, i.e., $\lim_{N\to\infty}\mathbb{P}\left(\left\lceil\frac{N}{2}\right\rceil - 1, 1\right) = \lim_{N\to\infty}\mathbb{P}\left(\left\lceil\frac{N}{2}\right\rceil + 1, 1\right) = \frac{1}{2}\frac{N}{N+M}$.

4.2 Change of location after $\tau = 2$ times lots

Due to the correlated user mobility in the RWPM model, the displacements at subsequent time slots are not independent. The probability that the user moves to the point $(n+2)$ after two time slots, $\mathbb{P}(n+2,2)$, can be expressed as the sum of the probabilities of two disjoint events: (i) The user travels over a path with source $s \in \{1,2,\ldots,n\}$ and destination $d \in \{n+2,\ldots,N\}$. (ii) The user travels over a path with source $s \in \{1,2,\ldots,n\}$ and destination $d = n+1$. After reaching the destination at $t = 1$, the user selects a zero think time with probability $\frac{1}{M+1}$ and then, it selects a new destination $d \in \{n+2, \ldots, N\}$. Hence, we have

$$\mathbb{P}(n+2,2) = (1-\mathbb{P}(n,1))\,\frac{n\,(N-n-1)+n\,\frac{1}{M+1}\,\frac{N-n-1}{N-1}}{n(N-n)+(n-1)(N-n+1)}, \tag{8}$$

for $n < N-1$, and $\mathbb{P}(n+2,2) = 0,\ n \geq N-1$.

In a similar manner, one can compute the probability that the user moves to the point $(n-2)$ after two time slots.

$$\begin{aligned}\mathbb{P}(n-2,2) \quad = \quad & (1-\mathbb{P}(n,1))\left(\tfrac{(n-2)(N-n+1)}{n(N-n)+(n-1)(N-n+1)}+\right.\\ & \left.\tfrac{(N-n+1)\frac{1}{M+1}\,\frac{n-2}{N-1}}{n(N-n)+(n-1)(N-n+1)}\right),\end{aligned} \tag{9}$$

for $n > 2$, and $\mathbb{P}(n-2,2) = 0,\ n \leq 2$.

The probability that the user moves to the point $(n+1)$ after two time slots is equal to the sum of the probabilities of the following events: (i) The user travels over a path with source $s \in \{1, 2, \ldots, n\}$ and destination $d = n+1$. After reaching its destination, the user selects a nonzero think time with probability $\frac{M}{M+1}$, and thinks over there at $t = 2$. (ii) The user thinks at $t = 1$ with probability $\mathbb{P}(n,1)$. Then, it selects a new destination $d \in \{n+1, \ldots, N\}$. Hence,

$$\begin{aligned}\mathbb{P}\,(n+1,2) \quad = \quad & (1-\mathbb{P}\,(n,1))\,\tfrac{n\,\frac{M}{M+1}}{n(N-n)+(n-1)(N-n+1)}+\\ & \mathbb{P}(n,1)\,(1-\mathbb{P}(n,2|n,1))\,\tfrac{N-n}{N-1},\end{aligned} \tag{10}$$

for $n < N$, and $\mathbb{P}\,(n+1,2) = 0,\ n = N$.

In the above equation, the term $\mathbb{P}(n,2|n,1)$ describes the conditional probability that a user which thinks at $t = 1$ at the lattice point n, keeps on thinking over there at $t = 2$. In order to compute this probability, we use the fact that in the steady state, the fraction of users which are thinking at point n is $\mathbb{P}(n,1)$ at any time slot. At $t = 2$, the users which are thinking at point n can be one of the following types: (i) Users which have been thinking over there at $t = 1$ and $t = 2$. (ii) Users which arrived at point n at $t = 1$, and stay there and think at $t = 2$. In the steady state, the users that arrive at a lattice point are equal, on average, to the users that move from that point to other lattice points, i.e., $(1 - \mathbb{P}(n,1))$ for lattice point n. Out of the users which arrived at point n at $t = 1$, the fraction of users which stay there at $t = 2$ is equal to the fraction of users whose destination is the point n, times the probability $\frac{M}{M+1}$ to select a nonzero think time. Hence,

$$\mathbb{P}\,(n,1)\,\mathbb{P}\,(n,2|n,1)+(1-\mathbb{P}\,(n,1))\,\frac{qM}{M+1} = \mathbb{P}\,(n,1)\,,$$

where $q = \frac{N-1}{n(N-n)+(n-1)(N-n+1)}$ is the fraction of paths with destination the lattice point n. After solving for $\mathbb{P}\,(n,2|n,1)$ in the above equation we get

$$\mathbb{P}\,(n,2|n,1) = 1-\frac{qM}{M+1}\,\frac{1-\mathbb{P}(n,1)}{\mathbb{P}(n,1)}. \tag{11}$$

The probability that the user moves to the point $(n-1)$ after two time slots can also be expressed in terms of the conditional probability $\mathbb{P}\,(n,2|n,1)$

$$\begin{aligned}\mathbb{P}\,(n-1,2) \quad = \quad & (1-\mathbb{P}\,(n,1))\,\tfrac{(N-n+1)\frac{M}{M+1}}{n(N-n)+(n-1)(N-n+1)}+\\ & \mathbb{P}(n,1)\,(1-\mathbb{P}(n,2|n,1))\,\tfrac{n-1}{N-1},\end{aligned} \tag{12}$$

for $n > 1$, and $\mathbb{P}\,(n-1,2) = 0,\ n = 1$.

Finally, the probability that the user is located at the lattice point n after two time slots is equal to the sum of the probabilities of the following events: (i) The user thinks over there at $t = 1$ and $t = 2$. (ii) The user travels over a path with source $s \in \{1, 2, \ldots, n\}$ and destination $d = n+1$. After reaching its destination at $t = 1$, the user selects a zero think time with probability $\frac{1}{M+1}$ and then, a new destination $d \in \{1, 2, \ldots, n\}$. (iii) The user travels over a path with source $s \in \{n, \ldots, N\}$ and destination $d = n-1$. After reaching its destination, the user selects a zero think time and then, a new destination $d \in \{n, \ldots, N\}$.

$$\begin{aligned}\mathbb{P}\,(n,2) \quad = \quad & (1-\mathbb{P}(n,1))\,\tfrac{\frac{1}{M+1}\,\frac{n^2+(N-n+1)^2}{N-1}}{n(N-n)+(n-1)(N-n+1)}+\\ & \mathbb{P}(n,1)\mathbb{P}\,(n,2|n,1)\,,\end{aligned} \tag{13}$$

for $1 < n < N$, and $\mathbb{P}\,(n,2) = \frac{(1-\mathbb{P}(n,1))(N^2+1)}{(N-1)^2(M+1)} + \mathbb{P}(n,1)\mathbb{P}(n,2|n,1)$, for $n = \{1, N\}$.

4.3 Change of location after $\tau > 2$ slots

The analysis of Section 4.2 could be extended to more time slots, $\tau > 2$, however, incorporating all possible user moves in the computation of the displacement probabilities $\mathbb{P}(n+k,\tau)$ will be cumbersome. One way to get around this issue, is to consider only a limited range of moves, the most probable ones, and obtain approximations for the $\mathbb{P}(n+k,\tau)$. For instance, the probabilities could be estimated assuming at most one directional change in the user mobility. Even under this assumption, the amount of possible user moves remains high for a large value of time-lag τ and a positive maximum think time M. Also, the computation of the conditional probabilities, see equation (11) for $\tau = 2$, becomes highly-nested for $\tau > 2$. Limited by this kind of constraints, we show how to approximate the probabilities $\mathbb{P}(n+k,\tau)$ only for a zero think time, $M = 0$.

Studying the long-term correlation of user location under fast mobility, i.e., a high τ and $M = 0$, can be used as one extreme case, which will be compared to the other extreme involving no mobility at all. Note that a static network with a uniform density of users can be obtained in the limit of $M \rightarrow \infty$. Using these two extremes, we will infer the impact of correlated mobility on the long-term correlation of interference.

4.3.1 *Zero think time*

Let consider a positive $k \geq 0$, and approximate the probability to move right, $\mathbb{P}(n+k,\tau),\ n \leq N,\ k \leq N-n$. The probability to move left can be obtained as $\mathbb{P}(n-k,\tau) = \mathbb{P}(l+k,\tau),\ l = N-n+1,\ k < n$. We start with the special case where $k = \tau$. The user may reach to the point $(n+\tau)$ in one of the following ways: (i) The user is on a path with source $s \in \{1, 2, \ldots, n\}$ and destination $d \in \{n+\tau, \ldots, N\}$. (ii) The user is on a path with source $s \in \{1, 2, \ldots, n\}$ and destination $d \in \{n+1, \ldots, n+\tau-1\}$. After reaching its destination, the user selects a new destination $d \in \{n+\tau, \ldots, N\}$ with probability $\frac{1}{N-1}$. Hence, for $k = \tau$,

$$\mathbb{P}(n+k,\tau) = \frac{n\,(N-n-\tau+1)+(\tau-1)n\frac{N-n-\tau+1}{N-1}}{n(N-n)+(n-1)(N-n+1)}. \tag{14}$$

For $k < \tau$, the most probable paths ending at location $(n+k)$ after τ time slots are: (i) The user is on a path with source $s \in \{1, 2, \ldots, n\}$ and destination $d = n+k+\frac{\tau-k}{2}$. After reaching its destination, the user selects a new destination $d \in \{1, \ldots, n+k\}$ with probability $\frac{1}{N-1}$. (ii) In a similar manner, the user first moves to the point $(n-\frac{\tau-k}{2})$, then changes its direction and returns to point $(n+k)$. Hence,

$$\mathbb{P}(n+k, \tau) = \frac{n\frac{n+k}{N-1} + (N-n+1)\frac{N-n-k+1}{N-1}}{n(N-n)+(n-1)(N-n+1)}, \quad (15)$$

for $n+k+\frac{\tau-k}{2} \leq N, n-\frac{\tau-k}{2} \geq 1$.

When $n-\frac{\tau-k}{2} < 1$, only the paths reaching first to $d = n+k+\frac{\tau-k}{2}$, and then returning to $(n+k)$ should be considered

$$\mathbb{P}(n+k, \tau) = \frac{n\frac{n+k}{N-1}}{n(N-n)+(n-1)(N-n+1)}, \quad (16)$$

for $n+k+\frac{\tau-k}{2} \leq N, n-\frac{\tau-k}{2} < 1$.

Similarly, when $n+k+\frac{\tau-k}{2} > N$, only the paths reaching first to $d = n-\frac{\tau-k}{2}$, and then returning to $(n+k)$ should be counted

$$\mathbb{P}(n+k, \tau) = \frac{(N-n+1)\frac{N-n-k+1}{N-1}}{n(N-n)+(n-1)(N-n+1)}, \quad (17)$$

for $n+k+\frac{\tau-k}{2} > N, n-\frac{\tau-k}{2} \geq 1$.

Obviously, $\mathbb{P}(n+k, \tau) = 0$ when $n+k+\frac{\tau-k}{2} > N$, and $n-\frac{\tau-k}{2} < 1$. Also, due to the fact that the think time is zero, one has to note that for an even τ, $\mathbb{P}(n+k, \tau) = 0$ for $k = 2l+1, l \in \{-\frac{\tau}{2}, -\frac{\tau}{2}+1, \ldots, \frac{\tau}{2}-1\}$, and similarly, for an odd τ, $\mathbb{P}(n+k, \tau) = 0$ for $k = 2l, l = \{-\frac{\tau-1}{2}, -\frac{\tau-3}{2}, \ldots, \frac{\tau-1}{2}\}$.

The approximations for the user displacement presented in this section are meant to estimate the number of time slots needed to see uncorrelated interference at the receiver. In the numerical examples, we will see that the interference becomes uncorrelated for values of τ significantly smaller than the lattice size. When the time-lag τ is comparable to the lattice size, the user may change its direction more than once with a non-negligible probability, and the approximation accuracy of equations (14)−(17) may degrade. However, in this order of time-lags, the interference correlation is already negligible.

5 Interference correlation

In order to compute the correlation coefficient of interference, see equation (2), one has to compute the first two moments of the interference distribution, and the first-order cross-moment. The MGF of the interference is

$$\Phi_{\mathcal{I}} = \int_{\mathrm{h}} \sum_{\Xi} \sum_{\mathrm{X}} e^{P_t \sum_{i=1}^{K} s_i h_i \xi_i g(x_i - x_p)} f_{\mathrm{X}}(\mathrm{x})\, f_{\Xi}(\xi)\, f_{\mathrm{H}}(\mathrm{h})\, \mathrm{dh},$$

where ξ, h and x are vectors of RVs with K elements each, ξ_i, h_i and x_i respectively, and $f_{\Xi}, f_{\mathrm{H}}, f_{\mathrm{X}}$ are the joint user PDFs of activity, fading and location.

The moments of the interference can be computed by differentiating the MGF. Assuming i.i.d. fading, activity and location among the users, the first two moments of the interference are computed in equations (18) and (19) on the top of next page. Note that the last term $\frac{1}{K}\mathbb{E}\{I(x_p)\}^2$ in equation (19) essentially describes the difference in the variances of a PPP and a Binomial Point Process (BPP) with the same density of users. The variance of the BPP is smaller, because there is no randmomness in the number of users at each realization. One may argue that the last term becomes negligible for a large number of users K, thus propose to approximate the variance of a BPP with the variance of the equi-dense PPP. As we will shortly see, the same term will appear in the calculation of the first-order cross-moment of the interference generated by a BPP. While it might be accurate to approximate the variance of a BPP by the variance of the equi-dense PPP for some large K, one should be aware that the same approximation might induce non-negligible errors in the approximation of the cross-moments of the interference. In the numerical examples we will illustrate that for some large K, even though the PPP provides a good approximation for the standard deviation of the interference generated by a BPP, it results in significant approximation errors for the correlation coefficients.

The interference cross-correlation at time-lag τ can be computed following similar steps to those in equation (19). One has to note that the vectors ξ, h and x have now $2K$ elements each, describing the activity, fading and location for K users at the two different time slots, $t = 0$ and $t = \tau$, e.g., $\mathrm{x} = (x_1, x_1(\tau), \ldots, x_K, x_K(\tau))^T$, where the notation $x_i(0)$ is shortened to x_i for brevity, and $(\,)^T$ denotes the transpose matrix. Also, let denote by $\mathrm{x_i}$ the vector of RVs describing the locations of the i-th user at the two different time slots, $\mathrm{x}_i = (x_i, x_i(\tau))^T$. The fading and activity are assumed i.i.d. over different time slots and users. On the other hand, the location of a user over different time slots is correlated due to the mobility model. In equation (20), the cross-correlation of the distance-based propagation pathoss g at time-lag τ, $\sigma_g(\tau) \triangleq \mathbb{E}_{\mathrm{x}_i}\Big\{g(x_i - x_p) g(x_i(\tau) - x_p)\Big\}$, can be computed after averaging the propagation pathloss over all possible user locations and user displacements

$$\sigma_g(\tau) = \sum_{n=1}^{N} g(n-x_p) f(n) \sum_{k=-\tau}^{\tau} g(n+k-x_p)\mathbb{P}(n+k, \tau). \quad (21)$$

Equations (18)−(21) are one of the main results of this paper showing how to incorporate the displacement law under any mobility model over one-dimensional lattice, into the interference correlation model. For the mobility model considered in this paper, the displacement probabilities are location-dependent because of the existence of borders, and the correlated user mobility, i.e., a randomly selected user is more probable to be in the middle of a flight when it is located close to the center. After substituting equations (18)−(21) into (2), one can compute the correlation coefficient which is location-dependent too.

In the limit of a large think time, $M \to \infty$, the static component in the CDF of the user location, $F_s(n)$, dominates, and the user distribution degenerates to uniform, $f(n) = \frac{1}{N}, \forall n$. After replacing $f(n) = \frac{1}{N}$ into equations (18)−(21), and $x_i = x_i(\tau)$, the correlation coefficient for infinite think time, $\rho_\infty(x_p)$, becomes

$$\rho_\infty(x_p) = \frac{\frac{K}{N} P_t^2 \mathbb{E}\{h\}^2 \mathbb{E}\{\xi\}^2 \sum_{n=1}^{N} g(n-x_p)^2 - \frac{\mathbb{E}\{\mathcal{I}(x_p)\}^2}{K}}{\frac{K}{N} P_t^2 \mathbb{E}\{h^2\} \mathbb{E}\{\xi^2\} \sum_{n=1}^{N} g(n-x_p)^2 - \frac{\mathbb{E}\{\mathcal{I}(x_p)\}^2}{K}}. \quad (22)$$

Therefore for a static network with a uniform density of users, the correlation coefficient does not depend on the

$$\begin{aligned}\mathbb{E}\left\{\mathcal{I}\left(x_p\right)\right\} &= \sum_{i=1}^{K} \frac{\partial}{\partial s_i} \Phi_{\mathcal{I}}\left(s_i=0\right) \\ &= P_t \sum_{i=1}^{K} \int_{\mathrm{H}} \sum_{\Xi} \sum_{\mathrm{X}} \xi_i\, h_i\, g(x_i-x_p)\;\; f_{\Xi}(\xi) f_{\mathrm{H}}(\mathrm{h}) f_{\mathrm{X}}(\mathrm{x})\, \mathrm{dh} \\ &= P_t \sum_{i=1}^{K} \mathbb{E}\left\{h_i\right\}\, \mathbb{E}\left\{\xi_i\right\} \sum_{X_i} g(x_i - x_p)\, f_{X_i}(x_i) \\ &= K P_t\, \mathbb{E}\left\{h\right\}\, \mathbb{E}\left\{\xi\right\} \sum_{n=1}^{N} g(n-x_p)\, f(n).\end{aligned} \tag{18}$$

$$\begin{aligned}\mathbb{E}\left\{\mathcal{I}\left(x_p\right)^2\right\} &= \sum_{i=1}^{K} \frac{\partial^2}{\partial s_i^2} \Phi_{\mathcal{I}}\left(s_i=0\right) + \sum_{i=1}^{K}\sum_{j\neq i} \frac{\partial^2}{\partial s_i \partial s_j} \Phi_{\mathcal{I}}\left(s_i=0, s_j=0\right) \\ &= P_t^2 \sum_{i=1}^{K} \int_{\mathrm{H}} \sum_{\Xi}\sum_{\mathrm{X}} \xi_i^2\, h_i^2\, g(x_i-x_p)^2\;\; f_{\Xi}(\xi)\, f_{\mathrm{H}}(\mathrm{h})\, f_{\mathrm{X}}(\mathrm{x})\, \mathrm{dh} + \\ &\quad P_t^2 \sum_{i=1}^{K}\sum_{j\neq i} \int_{\mathrm{H}} \sum_{\Xi}\sum_{\mathrm{X}} \xi_i \xi_j\, h_i h_j\, g(x_i-x_p)\, g(x_j-x_p)\;\; f_{\Xi}(\xi)\, f_{\mathrm{H}}(\mathrm{h})\, f_{\mathrm{X}}(\mathrm{x})\, \mathrm{dh} \\ &= P_t^2 \sum_{i=1}^{K} \mathbb{E}\left\{\xi_i^2\right\} \mathbb{E}\left\{h_i^2\right\} \sum_{X_i} g\left(x_i-x_p\right)^2 f_{X_i}(x_i) + \\ &\quad P_t^2 \sum_{i=1}^{K}\sum_{j\neq i} \mathbb{E}\left\{\xi_i\xi_j\right\} \mathbb{E}\left\{h_i h_j\right\} \sum_{X_i} g(x_i-x_p) f_{X_i}(x_i) \sum_{X_j} g(x_j-x_p) f_{X_j}(x_j) \\ &= K P_t^2\, \mathbb{E}\left\{\xi^2\right\}\, \mathbb{E}\left\{h^2\right\} \sum_{n=1}^{N} g(n-x_p)^2 f(n) + \\ &\quad K\left(K-1\right) P_t^2\, \mathbb{E}\left\{\xi\right\}^2\, \mathbb{E}\left\{h\right\}^2 \left(\sum_{n=1}^{N} g(n-x_p) f(n)\right)^2 \\ &= \mathbb{E}\left\{I\left(x_p\right)\right\}^2 + K P_t^2\, \mathbb{E}\{\xi^2\}\, \mathbb{E}\{h^2\} \sum_{n=1}^{N} g(n-x_p)^2 f(n) - \frac{1}{K}\mathbb{E}\left\{I\left(x_p\right)\right\}^2.\end{aligned} \tag{19}$$

$$\begin{aligned}\mathbb{E}\left\{\mathcal{I}(x_p,\tau)\mathcal{I}(x_p)\right\} &= P_t^2 \sum_{i=1}^{K} \int_{\mathrm{H}} \sum_{\Xi}\sum_{\mathrm{X}} \xi_i \xi_i(\tau) h_i h_i(\tau) g(x_i-x_p)\, g(x_i(\tau)-x_p) f_{\Xi}(\xi) f_{\mathrm{H}}(\mathrm{h}) f_{\mathrm{X}}\left(\mathrm{x}\right) \mathrm{dh} + \\ &\quad P_t^2 \sum_{i=1}^{K}\sum_{j\neq i} \int_{\mathrm{H}} \sum_{\Xi}\sum_{\mathrm{X}} \xi_i \xi_j(\tau) h_i h_j(\tau) g(x_i-x_p)\, g(x_j(\tau)-x_p) f_{\Xi}(\xi) f_{\mathrm{H}}(\mathrm{h}) f_{\mathrm{X}}(\mathrm{x}) \mathrm{dh} \\ &= P_t^2 \sum_{i=1}^{K} \mathbb{E}\left\{\xi_i\right\}^2 \mathbb{E}\left\{h_i\right\}^2 \sum_{\mathrm{X}_i} g\left(x_i - x_p\right) g\left(x_i(\tau) - x_p\right) f_{\mathrm{X}_i}\left(\mathrm{x}_i\right) + \\ &\quad P_t^2 \sum_{i=1}^{K}\sum_{j\neq i} \mathbb{E}\left\{\xi_i \xi_j(\tau)\right\} \mathbb{E}\left\{h_i h_j(\tau)\right\} \sum_{X_i} g\left(x_i-x_p\right) f_{X_i}(x_i) \sum_{X_j} g\left(x_j-x_p\right) f_{X_j}(x_j) \\ &= P_t^2\, \mathbb{E}\left\{\xi\right\}^2\, \mathbb{E}\left\{h\right\}^2 \sum_{i=1}^{K}\sum_{\mathrm{X}_i} g\left(x_i - x_p\right) g\left(x_i(\tau) - x_p\right) f_{\mathrm{X}_i}\left(\mathrm{x}_i\right) + \\ &\quad K(K-1) P_t^2\, \mathbb{E}\left\{\xi\right\}^2 \mathbb{E}\left\{h\right\}^2 \left(\sum_{n=1}^{N} g\left(n - x_p\right) f(n)\right)^2 \\ &= K P_t^2 \mathbb{E}\left\{\xi\right\}^2 \mathbb{E}\left\{h\right\}^2 \mathbb{E}_{\mathrm{x}_i}\Big\{g(x_i-x_p) g(x_i(\tau)-x_p)\Big\} + \mathbb{E}\left\{I\left(x_p\right)\right\}^2 - \frac{1}{K}\mathbb{E}\left\{I\left(x_p\right)\right\}^2.\end{aligned} \tag{20}$$

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time-lag τ, however, it is still location-dependent. It becomes independent of the location, only if the number of users at each time slot is generated according to the Poisson distribution. In that case, the second terms in the nominator and denominator of equation (22) would vanish, and the correlation coefficient becomes equal to $\rho_\infty = \frac{\mathbb{E}\{h\}^2 \mathbb{E}\{\xi\}^2}{\mathbb{E}\{h^2\}\mathbb{E}\{\xi^2\}}$ at all the locations. Note that this result agrees with the result in [1], where the correlation coefficient is calculated for a static network modelled by a PPP in the infinite plane.

6 OUTAGE PROBABILITY

Let $\mathcal{E}_\tau$ describe the outage event at time slot $t=\tau$ in the steady state. Firstly, we show how to compute the probability of outage at an arbitrarily selected time slot, referred to as the unconditional outage probability and denoted by $\mathbb{P}(\mathcal{E})$. Secondly, in order to examine the impact of interference correlation on the outage correlation, we show how to compute the probability of outage at time slot $t=\tau$ given that an outage occurs at time slot $t=0$. This is referred to as the conditional outage probability and it is denoted by $\mathbb{P}(\mathcal{E}_\tau|\mathcal{E}_0)$.

6.1 Unconditional outage probability

It is assumed that when the Signal-to-Interference-and-Noise-Ratio (SINR) falls under a target level q, the receiver is not able to decode the desired transmission and an outage occurs. The probability of outage, $\mathbb{P}\left(\text{SINR} \leq q\right)$, given the location of the desired transmitter at x_t is

$$\begin{aligned}\mathbb{P}(\mathcal{E}) &= \mathbb{P}\left(\frac{P_t g(x_t-x_p)h_{tx}}{P_N+\mathcal{I}(x_p)} \leq q\right)\\ &= \mathbb{P}\left(h_{tx} \leq \frac{q\left(P_N+\mathcal{I}(x_p)\right)}{P_t g(x_t-x_p)}\right).\end{aligned}$$

Taking the average with respect to all possible spatial, fading and activity realizations of the interferers, and keeping in mind that the fading in the access link is Rayleigh we get

$$\begin{aligned}\mathbb{P}(\mathcal{E}) &= \mathbb{E}_{\mathcal{I}(x_p)}\left\{\mathbb{P}\left(h_{tx} \leq \frac{q(P_N+\mathcal{I}(x_p))}{P_t g(x_t-x_p)}\middle|\mathcal{I}(x_p)\right)\right\}\\ &= 1-e^{-\mathcal{P}_N}\mathbb{E}_{\mathcal{I}(x_p)}\left\{e^{-s\mathcal{I}(x_p)}\right\},\end{aligned} \tag{23}$$

where $s=\frac{q}{P_t g(x_t-x_p)}$ and $\mathcal{P}_N=\frac{qP_N}{P_t g(x_t-x_p)}$.

For simplicity in the notation, let define the discrete-valued function $G(x_k)=\frac{1}{1+sP_t g(x_k-x_p)}$. The Laplace Transform of the interference, $\mathcal{L}_\mathcal{I}=\mathbb{E}_{\mathcal{I}(x_p)}\left\{e^{-s\mathcal{I}(x_p)}\right\}$ may take the following form

$$\begin{aligned}\mathcal{L}_\mathcal{I} &= \mathbb{E}_{x_k,\xi_k,h_k}\left\{e^{-sP_t\sum_k \xi_k h_k g(x_k-x_p)}\right\}\\ &\overset{(a)}{=} \mathbb{E}_{x_k,\xi_k}\left\{\prod_k \frac{1}{1+sP_t\xi_k g(x_k-x_p)}\right\}\\ &\overset{(b)}{=} \mathbb{E}_{x_k}\left\{\prod_k 1-\xi+\xi G(x_k)\right\}\\ &\overset{(c)}{=} \left(\mathbb{E}_{x_k}\{1-\xi+\xi G(x_k)\}\right)^K\\ &= \left(1-\xi+\xi\sum_{n=1}^{N}G(n)f(n)\right)^K.\end{aligned} \tag{24}$$

In the above equations, (a) follows from the i.i.d. Rayleigh fading in the interfering links which is independent of the locations and activities of the users, (b) follows from averaging over ALOHA, and (c) from the i.i.d. locations of interferers. After substituting equation (24) into (23), the unconditional outage probability becomes

$$\mathbb{P}(\mathcal{E})=1-e^{-\mathcal{P}_N}\left(1-\xi+\xi\sum_{n=1}^{N}G(n)f(n)\right)^K. \tag{25}$$

6.2 Conditional outage probability

The conditional probability of outage can be written in terms of the joint probability of outage at $t=0$ and $t=\tau$, $\mathbb{P}\left(\mathcal{E}_\tau|\mathcal{E}_0\right)=\frac{\mathbb{P}(\mathcal{E}_\tau,\mathcal{E}_0)}{\mathbb{P}(\mathcal{E})}$. The joint probability of outage is

$$\begin{aligned}\mathbb{P}\left(\mathcal{E}_\tau,\mathcal{E}_0\right) &= \mathbb{P}\left(h_{tx}\leq H_{tx}, h_{tx}(\tau)\leq H_{tx}(\tau)\right)\\ &= \mathbb{E}_{\mathcal{I}(x_p),\mathcal{I}(x_p,\tau)}\Big\{\Big(1-e^{-\mathcal{P}_N}e^{-s\mathcal{I}(x_p)}-\\ &\qquad e^{-\mathcal{P}_N}e^{-s\mathcal{I}(x_p,\tau)}+e^{-2\mathcal{P}_N}e^{-s\mathcal{I}(x_p)}e^{-s\mathcal{I}(x_p,\tau)}\Big)\Big\}\\ &= 1-2(1-\mathbb{P}\left(\mathcal{E}\right))+e^{-2\mathcal{P}_N}\mathcal{L}_\mathcal{I}(\tau),\end{aligned} \tag{26}$$

where $H_{tx}=\frac{q(P_N+\mathcal{I}(x_p))}{P_t g(x_t-x_p)}$, $H_{tx}(\tau)=\frac{q(P_N+\mathcal{I}(x_p,\tau))}{P_t g(x_t-x_p)}$, and the joint Laplace functional of the interference at time slots $t=0$ and $t=\tau$, $\mathcal{L}_\mathcal{I}(\tau)=\mathbb{E}_{\mathcal{I}(x_p),\mathcal{I}(x_p,\tau)}\left\{e^{-s\mathcal{I}(x_p)}e^{-s\mathcal{I}(x_p,\tau)}\right\}$ may take the following form

$$\begin{aligned}\mathcal{L}_\mathcal{I}(\tau) &= \mathbb{E}_{\mathrm{x}_k,\xi_k,\mathrm{h}_k}\Big\{\exp\Big(-sP_t\sum_k\xi_k h_k g(x_k-x_p)-\\ &\qquad sP_t\sum_k\xi_k(\tau)h_k(\tau)g(x_k(\tau)-x_p)\Big)\Big\}\\ &\overset{(a)}{=} \mathbb{E}_{\mathrm{x}_k,\xi_k}\Big\{\prod_k\frac{1}{1+sP_t\xi_k\, g(x_k-x_p)}\times\\ &\qquad \prod_k\frac{1}{1+s\,P_t\,\xi_k(\tau)g(x_k(\tau)-x_p)}\Big\}\\ &\overset{(b)}{=} \mathbb{E}_{\mathrm{x}_k}\Big\{\prod_k 1-\xi+\xi G(x_k)\prod_k 1-\xi+\xi G(x_k(\tau))\Big\}\\ &\overset{(c)}{=} \Big(\mathbb{E}_{\mathrm{x}_k}\{(1-\xi+\xi G(x_k))\,(1-\xi+\xi G(x_k(\tau)))\}\Big)^K\\ &= \Big((1-\xi)^2+2\xi(1-\xi)\sum_{n=1}^{N}G(n)f(n)+\xi^2\sigma_G(\tau)\Big)^K\end{aligned} \tag{27}$$

where (a) follows from i.i.d. Rayleigh fading over users and time slots, (b) from i.i.d. activities over users and time slots, (c) from i.i.d. locations of different users, and the cross-correlation of the function $G(x_k)$ at time-lag τ, $\sigma_G(\tau)\triangleq\mathbb{E}_{\mathrm{x}_k}\{G(x_k)G(x_k(\tau))\}$, can be computed after averaging over all possible user locations and user displacements

$$\sigma_G(\tau)=\sum_{n=1}^{N}G(n)f(n)\sum_{k=-\tau}^{\tau}G(n+k)\mathbb{P}(n+k,\tau). \tag{28}$$

Equations (26)−(28) are also main results of the paper showing how to incorporate the displacement law of a

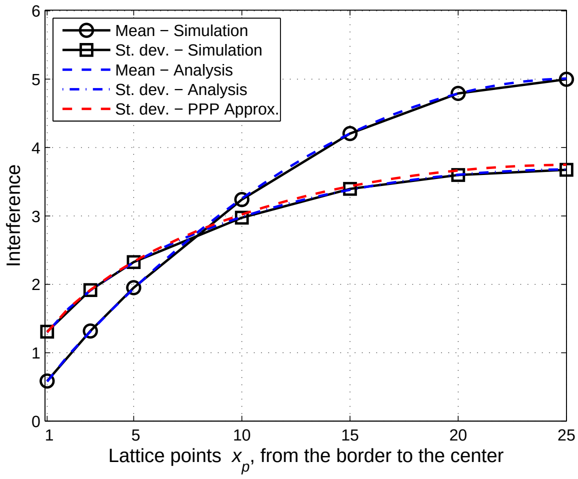

Fig. 1. Mean and standard deviation of the interference at different points of the lattice. Pathloss exponent $a=4$ and maximum think time $M=5$ time slots. The rest of the parameter settings can be found in the beggining of Section 7.

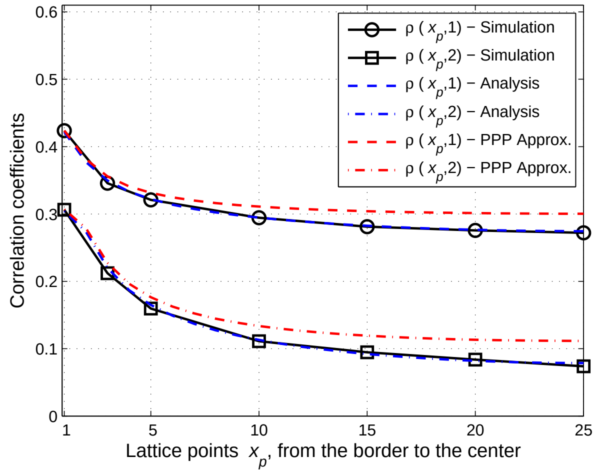

Fig. 2. Correlation coefficients for $\tau=1$ and $\tau=2$ at different points of the lattice. Same parameter settings used to generate Fig. 1.

mobility model over one-dimensional lattice into the joint probability of outage for two time slots at time-lag τ. For instance, for the mobility model considered in the paper and $\tau=1$, the joint probability of outage becomes

$$\mathbb{P}\left(\mathcal{E}_1,\mathcal{E}\right) = 2\mathbb{P}\left(\mathcal{E}\right)-1+e^{-2\mathcal{P}_N}\Bigg((1-\xi)^2 + 2\xi(1-\xi)\sum_{n=1}^{N}G(n)f(n)+\xi^2\sum_{n=1}^{N}G(n)f(n)\Big(G(n)\mathbb{P}(n,1)+G(n+1)\mathbb{P}(n+1,1)+ G(n-1)\mathbb{P}(n-1,1)\Big)\Bigg)^K, \tag{29}$$

and the conditional probability of outage $\mathbb{P}(\mathcal{E}_1|\mathcal{E})$ is computed after dividing equation (29) by equation (25).

7 Numerical examples

We consider $K = 50$ users moving over a lattice of size $N = 50$ according to the RWPM model described in Section 2. Initially, we let the network run for $10\,000$ time slots to converge to its stationary state. Alternative methods to obtain the stationary user distribution are discussed in [17]. The transmission probability in slotted ALOHA is taken equal to one, $\xi=1$. The fast fading channel is Rayleigh with mean unity, $\mathbb{E}\left\{h\right\}=1$, and the distance-based propagation pathloss model is parameterized with $\epsilon=0.5$. The validation of the computation of the mean and standard deviation of interference, see equations (18) and (19), is illustrated in Fig. 1. As expected, receivers close to the center of the lattice are more exposed to interference. Also, we see that approximating the BPP by the equi-dense PPP slightly overestimates the standard deviation. Since the number of users is high, neglecting the last term in equation (19) results in a small approximation error.

The computation of the correlation coefficient for $\tau=1$ and $\tau=2$ is validated in Fig. 2. The correlation is in general higher at the border than at the center of the lattice. Also, even within a single time slot, the interference correlation reduces rapidly, particularly close to the center. In order to explain this behaviour, we recall that close to the boundaries a higher fraction of users think, see equation (6), making the interference pattern more correlated over there. Also, users may approach or leave the center from either direction while, close to the boundaries, one of the probabilities $\mathbb{P}(n+1,1), \mathbb{P}(n-1,1)$ will dominate the other. This means that the user distribution changes more rapidly at the center, making the interference pattern uncorrelated within few time slots.

In Fig. 2, we also see that the PPP approximation results in non-negligible approximation errors for the correlation coefficients. The PPP overestimates both the variance and the first-order cross-moment, see equations (19) and (20). The absolute error is the same, $\frac{1}{K}\mathbb{E}\left\{I(x_p)\right\}^2$, but it affects more the cross-moment, resulting in an overestimation of the correlation coefficients. Even though the number of users is already high, the PPP approximation might not be reliable for computing the cross-moments of the interference generated by a BPP.

Next, we study the impact of propagation pathloss exponent and maximum think time on the correlation coefficient of interference for $\tau=1$. A lower pathloss exponent reduces the impact of dominant interferers on the interference pattern. Therefore the local interference feature, i.e., mobility of dominant interferers, does not affect the interference profile as much as in the case of a high pathloss exponent. Because of that, lower pathloss exponents are associated with higher interference correlation, see Fig. 3. In addition, a higher think time makes the network more static, thus increasing the interference correlation too.

In Fig. 4, we examine how long it takes for the interference to become uncorrelated at different locations assuming fast mobility, i.e., zero think time, see Section 4.3. Due to the fact that the user distribution changes rapidly at the center, it takes only four time slots to see uncorrelated interference over there, while at the boundary, 10 time slots are needed.

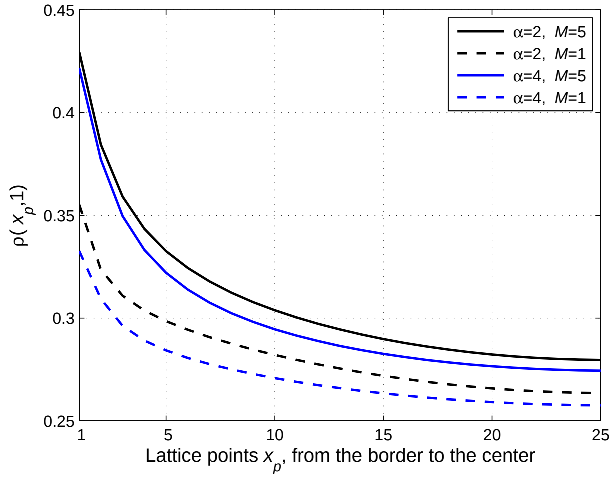

Fig. 3. Correlation coefficient $\rho(x_p, 1)$ for different think time M and propagation pathloss exponent a. The rest of the parameter settings are the same used to generate Fig. 1.

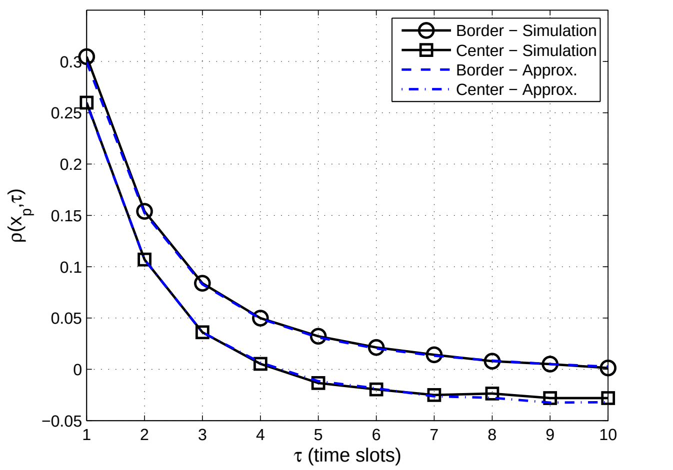

Fig. 4. Correlation coefficient $\rho(x_p, \tau)$ with respect to the time-lag τ at the border of the lattice, $x_p = 1$, and at the center, $x_p = N/2$, for zero think time, $M = 0$, and propagation pathloss exponent $a = 2$. The rest of the parameter settings are the same used to generate Fig. 1.

When the pathloss exponent is higher, $a = 4$, the correlation becomes zero after three time slots at the center and four time slots at the boundary. For a larger lattice, $N = 100$, the related values are six time slots at the center and 16 time slots at the boundary. We see that even though the user mobility is correlated, the interference becomes uncorrelated in few time slots, the users have to travel only within a part of the lattice before the correlation drops to zero.

In Fig. 4 we also see that small negative correlation coefficients may arise. For instance, the correlation coefficient at the center will oscillate in the interval $[-0.03, 0.02]$ and progressively converge to zero for $\tau > 100$. This sort of behaviour cannot be captured by the model proposed in Section 4.3. Nevertheless, the model is useful because it gives an accurate estimate about the amount of time needed and subsequently about the distance covered before the interference correlation becomes negligible. From Fig. 4, we also deduce that the approximations for the probabilities of the user displacement, see equations (14)−(17), introduce only small errors in the calculation of interference correlation. Actually, the method presented in Section 4.3 slightly underestimates the probabilities for the user displacement because it does not account for all the paths reaching at the point $(n+k)$ after τ time slots. As a result, the cross-correlation in equation (20), and subsequently the correlation coefficient are slightly underestimated too.

In Fig. 5, we study interference correlation under two extreme scenarios, RWPM with highly moving users, $M = 0$, and static users with a uniform density, $M \to \infty$. For the latter, the correlation coefficient is calculated in equation (22), and it is only slightly higher at the boundaries as compared to the center, see Fig. 5. On the other hand, with mobile users, the correlation is clearly location-dependent, because of the different mobility levels at different locations. In addition, the propagation pathloss exponent does not seem to affect much the correlation in either scenario. When there is no correlation in the user traffic and the fading channel, mobility is the key factor reducing the interference correlation.

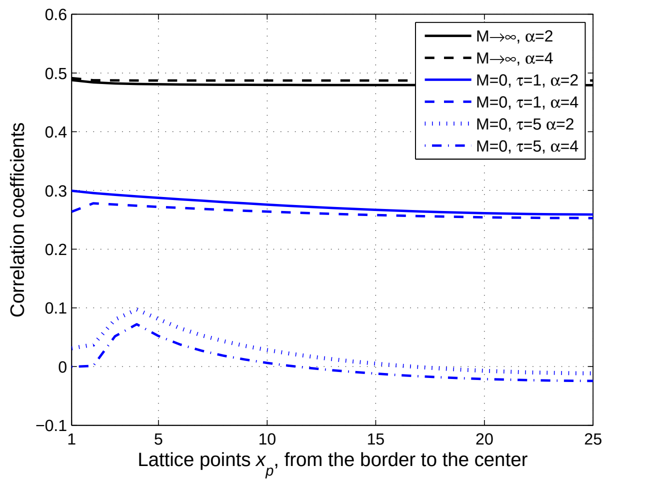

Fig. 5. Correlation coefficient in static networks with uniform density of users, $M \to \infty$, and in highly moving networks, $M = 0$. The rest of the parameter settings are the same used to generate Fig. 1.

We have so far seen that in mobile networks the interference profile and interference correlation are in general location-dependent. Under the RWPM model, receivers close to the boundaries experience less interference, but the interference correlation is also higher over there. Because of that, when the location of the desired transmitter is fixed and known, the outage probability becomes lower close to the boundary but at the same time, the conditional outage probability is clearly higher than the unconditional, see Fig. 6. On the other hand, close to the center, the outage events in subsequent time slots are essentially independent.

Finally, in Fig. 7, we assess the outage probability under the two extreme cases. In the static case with uniform distribution of users, $M \to \infty$, the interference and subsequently the outage are still lower close to the border, however, the variations with respect to the location are smoother. The interference correlation is reduced only due

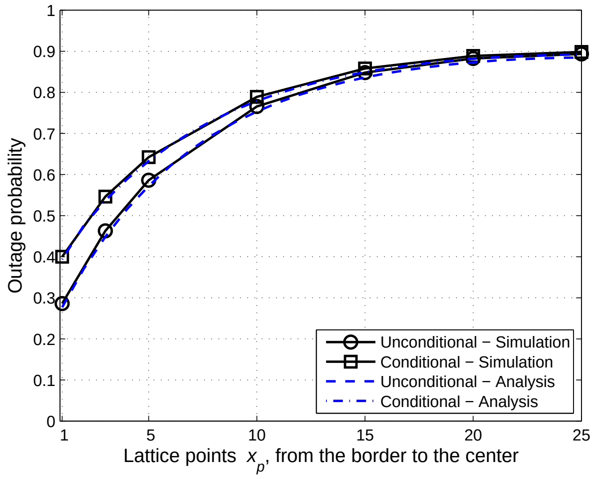

Fig. 6. Conditional, $\mathbb{P}(\mathcal{E})$, and unconditional outage probability, $\mathbb{P}(\mathcal{E}_1|\mathcal{E})$, assuming the desired transmitter is located at $x_t=x_p$. SINR target $q=1$, noise power level $P_N=10^{-3}$, maximum think time $M=5$ time slots, and propagation pathloss exponent $a=2$. The rest of the parameter settings are the same used to generate Fig. 1.

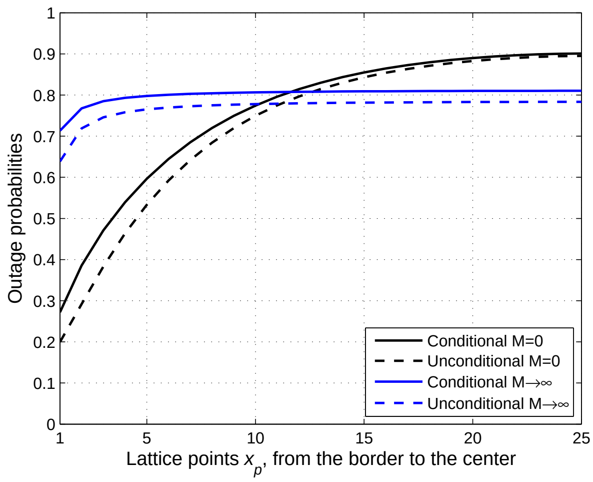

Fig. 7. Conditional, $\mathbb{P}(\mathcal{E})$, and unconditional outage probability, $\mathbb{P}(\mathcal{E}_1|\mathcal{E})$, in highly moving, $M=0$, and static networks, $M\to\infty$. The rest of the parameter settings are the same used to generate Fig. 6.

to the randomness in the channel because the users transmit continuously, $\xi=1$, and they are also static. Therefore, the conditional outage probability becomes clearly higher than the unconditional at all locations. In the Appendix, we show that for $M\to\infty$, the conditional outage probability is higher than the unconditional $\mathbb{P}(\mathcal{E}_\tau|\mathcal{E})\geq\mathbb{P}(\mathcal{E})$ at all locations and time-lags. On the other hand, for $M=0$, the distribution of users is highly non-uniform, making the interference and outage location-dependent, while the mobility helps reduce the interference correlation particularly close to the center.

8 Conclusion

In practice, wireless networks have finite boundaries, and the user mobility is not a random walk. Nevertheless, existing interference models assume infinite deployment areas and/or i.i.d. user mobility. In order to capture the impact of mobility correlation on interference and outage correlation, we showed that one has to identify the user distribution, along with the user displacement law. As an example case, we used the RWPM model, and we identified its displacement law in the discrete one-dimensional space. We illustrated that the temporal correlation of interference is location-dependent, being higher close to the boundaries, where the level of mobility is lower than near the center. Because of that, the interference needs more time to become uncorrelated at the boundaries than at the center. Our findings reveal that neglecting the particular features of deployment and user mobility may lead to erroneous calculation for the interference. In the future, it is important to study exactly how the correlation properties of interference and outage may impact network performance metrics, e.g., local delay, packet travel times over multiple hops, etc. at different locations.

Appendix

For a uniform user distribution, $f(n)=\frac{1}{N}\,\forall n$, the outage probability in equation (25) can be written as

$$\mathbb{P}(\mathcal{E})=1-e^{-\mathcal{P}_N}\left(1-\xi+\frac{\xi}{N}\sum_{n=1}^{N}G(n)\right)^K. \tag{30}$$

Also, for a static network, $\mathbb{P}(n,1)=1\,\forall n$, the cross-correlation in equation (28) is simplified to

$$\mathbb{E}_{\mathrm{x}_k}\{G(x_k)G(x_k(\tau))\}=\frac{1}{N}\sum_{n=1}^{N}G^2(n). \tag{31}$$

After substituting equation (31) into (27) we get

$$\begin{aligned}\mathcal{L}_{\mathcal{I}}(\tau) &= \left((1-\xi)^2+2\frac{\xi(1-\xi)}{N}\sum_{n=1}^{N}G(n)+\frac{\xi^2}{N}\sum_{n=1}^{N}G^2(n)\right)^K\\ &\overset{(a)}{\geq} \Bigg((1-\xi)^2+2\frac{\xi(1-\xi)}{N}\sum_{n=1}^{N}G(n)+\\ &\qquad \frac{\xi^2}{N^2}\left(\sum_{n=1}^{N}G(n)\right)^2\Bigg)^K\\ &= \left(1-\xi+\frac{\xi}{N}\sum_{n=1}^{N}G(n)\right)^{2K}.\end{aligned} \tag{32}$$

Here, (a) follows from the Cauchy-Schwarz inequality. Using (32), the joint outage probability in equation (26) can be upper-bounded as

$$\begin{aligned}\mathbb{P}(\mathcal{E}_\tau,\mathcal{E}) &\geq 2\mathbb{P}(\mathcal{E})-1+\left(e^{-\mathcal{P}_N}\left(1-\xi+\frac{\xi}{N}\sum_{n=1}^{N}G(n)\right)^K\right)^2\\ &\overset{(a)}{=} 2\mathbb{P}(\mathcal{E})-1+(1-\mathbb{P}(\mathcal{E}))^2=\mathbb{P}(\mathcal{E})^2,\end{aligned} \tag{33}$$

where (a) follows from (30). As a result, the conditional outage probability is always higher than the unconditional $\mathbb{P}(\mathcal{E}_\tau|\mathcal{E})=\frac{\mathbb{P}(\mathcal{E}_\tau,\mathcal{E})}{\mathbb{P}(\mathcal{E})}\geq\mathbb{P}(\mathcal{E})$.

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