Special relativity problem sheet 1

In the following questions, S and S' will denote inertial frames in standard configuration.

1. The following relativistic equations are written in units defined so that the speed of light $c = 1$. Insert the correct factors of c. t is time, x is distance, v is velocity, E is energy, p is momentum, and m is mass.
(a)$t' = (t - vx)/\sqrt{1 - v^2}$
(b)$E = \sqrt{p^2 + m^2}$
(c)$v = p/E$

2. In S, events occur at the origin and at $(X, 0, 0)$ simultaneously at time $t = 0$. The time interval between the events in S' is T. Show that the spatial distance between the events in S' is $\sqrt{X^2 + c^2T^2}$ and determine the relative velocity of the two frames in terms of X and T.

3. If two events occur at the same point in some inertial frame S, prove that their temporal order is the same in all inertial frames and that the least time separation is assigned to them in S.

4. Prove that at any instant there is just one plane in S on which the clocks of S agree with the clocks of S', and that this plane moves with velocity $(1 - \gamma^{-1})\frac{c^2}{v}$.

5. Show that the circle $(x')^2 + (y')^2 = a^2$ in S' is measured to be an ellipse in S.