Corrections on
"Multiple change-point detection for high-dimensional time series via Sparsified Binary Segmentation" [1]

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We correct Lemma 2 in Appendix of [1] as below.

Lemma 2' Suppose (15) holds and let $\eta \equiv \eta_{q_1+q} \in [s, e]$ for some q, denote a true change-point. Then there exists $c_0 \in (0, \infty)$ such that for b satisfying $|\eta - b| \geq c_0\epsilon_T$ and $\mathbb{S}_{s,b,e} < \mathbb{S}_{s,\eta,e}$, we have $\mathbb{S}_{s,\eta,e} \geq \mathbb{S}_{s,b,e} + C'\epsilon_T\delta_T T^{-2}\mathbb{S}_{s,\eta,e}$.

Proof. The proof follows directly from the proof of Lemma 2.6 in [2]. We only consider Case 2 of Lemma 2.6, since adapting the proof of Case 1 (when there is a single change-point within $[s, e]$) to that of the current lemma takes analogous arguments.
Using the notations therein, it is shown that the term E_{1l} is dominant over E_{2l} and E_{3l} in $\mathbb{S}_{s,\eta,e} - \mathbb{S}_{s,b,e}$, where $l = c_0\epsilon_T$. Noting further that $i = \eta - s + 1$, $h = \delta_T$, $j = e - \eta - h$ and $a = \sum_{t=s}^{\eta} \sigma(t/T) - (e - s + 1)^{-1} \sum_{t=s}^{e} \sigma(t/T)$, and that $h \geq 2l$,

$$\begin{aligned} E_{1l} &= \frac{la\sqrt{i+j+h}}{\sqrt{i}\sqrt{j+h}} \cdot \frac{h-l}{\sqrt{i+l}\sqrt{j+h-l}\{\sqrt{(i+l)(j+h-l)} + \sqrt{i(j+l)}\}} \\ &\geq \mathbb{S}_{s,\eta,e} \cdot C\epsilon_T\delta_T T^{-2}. \end{aligned}$$

□

Applying the above Lemma 2' to Lemma 4 of [1], the upper bound on $(|\langle f, \widetilde{\psi}\rangle| - |\langle f, \psi^0\rangle|)|\langle f, \psi^0\rangle|$

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is corrected to $-C\epsilon_T\delta_T^3 T^{-3}$. Also, we can refine the upper bound imposed on III and IV, since

$$\begin{aligned}
& |(\widehat{\eta}-s+1)^{-1}\sum_{t=s}^{\widehat{\eta}} f_t - (\eta-s+1)^{-1}\sum_{t=s}^{\eta} f_t| \\
= & \left|\frac{1}{\widehat{\eta}-s+1}\sqrt{\frac{(\widehat{\eta}-s+1)(e-\widehat{\eta})}{e-s+1}}\mathbb{S}_{s,\widehat{\eta},e} - \frac{1}{\eta-s+1}\sqrt{\frac{(\eta-s+1)(e-\eta)}{e-s+1}}\mathbb{S}_{s,\eta,e}\right| \\
= & \frac{1}{\sqrt{e-s+1}}\left|\sqrt{\frac{e-\widehat{\eta}}{\widehat{\eta}-s+1}}(\mathbb{S}_{s,\widehat{\eta},e}-\mathbb{S}_{s,\eta,e}) - \left(\sqrt{\frac{e-\eta}{\eta-s+1}} - \sqrt{\frac{e-\widehat{\eta}}{\widehat{\eta}-s+1}}\right)\mathbb{S}_{s,\eta,e}\right| \\
\leq & \sqrt{\frac{e-\eta}{(e-s+1)(\eta-s+1)}}\left\{\left|1-\frac{\sqrt{1-\frac{\widehat{\eta}-\eta}{e-\eta}}}{\sqrt{1+\frac{\widehat{\eta}-\eta}{\eta-s+1}}}\right|\mathbb{S}_{s,\eta,e} + \left|\mathbb{S}_{s,\widehat{\eta},e}-\mathbb{S}_{s,\eta,e}\right|\right\} \\
\leq & \sqrt{\frac{e-\eta}{(e-s+1)(\eta-s+1)}}\left\{C\epsilon_T\delta_T^{-1}\mathbb{S}_{s,\eta,e} + \left|\mathbb{S}_{s,\widehat{\eta},e}-\mathbb{S}_{s,\eta,e}\right|\right\} \\
\leq & 2\sqrt{\frac{e-\eta}{(e-s+1)(\eta-s+1)}}\cdot C\epsilon_T\delta_T^{-1}\mathbb{S}_{s,\eta,e},
\end{aligned}$$

and

$$\begin{aligned}
& |(\widehat{\eta}-s+1)^{-1}\sum_{t=s}^{\widehat{\eta}} f_t - (e-\eta)^{-1}\sum_{t=\eta+1}^{e} f_t| \\
= & \left|\frac{1}{\widehat{\eta}-s+1}\sqrt{\frac{(\widehat{\eta}-s+1)(e-\widehat{\eta})}{e-s+1}}\mathbb{S}_{s,\widehat{\eta},e} + \frac{1}{e-\eta}\sqrt{\frac{(\eta-s+1)(e-\eta)}{e-s+1}}\mathbb{S}_{s,\eta,e}\right| \\
= & \frac{1}{\sqrt{e-s+1}}\left|\left(\sqrt{\frac{e-\widehat{\eta}}{\widehat{\eta}-s+1}} + \sqrt{\frac{\eta-s+1}{e-\eta}}\right)\mathbb{S}_{s,\eta,e} + \sqrt{\frac{e-\widehat{\eta}}{\widehat{\eta}-s+1}}(\mathbb{S}_{s,\widehat{\eta},e}-\mathbb{S}_{s,\eta,e})\right| \\
\leq & 2\sqrt{\frac{e-s+1}{(\eta-s+1)(e-\eta)}}\mathbb{S}_{s,\eta,e},
\end{aligned}$$

and thus $|III| \leq C\epsilon_T\delta_T^{-1}\log T\cdot\mathbb{S}_{s,\eta,e}$ and $|IV| \leq C'\epsilon_T^{1/2}\delta_T^{-1/2}\log T\cdot\mathbb{S}_{s,\eta,e}$.
In summary, the inequality in (24) is modified to

$$\frac{\epsilon_T\delta_T^3}{T^3} > (\epsilon_T T^{-1/2}\log T)\vee(\epsilon_T^{1/2}\delta_T^{1/2}T^{-1/2}\log T)\vee(\log^2 T). \tag{24'}$$

Together with the fact that Lemmas 5–6 require that $\delta_T^{-5/2}T^{5/2}\log T < \epsilon_T^{1/2} \ll \pi_T \ll \delta_T/\sqrt{T}$, we derive Theorems 1'–2' below with assumptions (A1') and (B2') which modify (A1) and (B2) in [1], respectively.

(A1') The distance between any two adjacent change-points is bounded from below by $\delta_T \asymp T^{\Theta}$ for $\Theta \in (6/7, 1]$.

(B2') $\nu_r \in \mathbb{B},\ r = 1, \ldots, N$ satisfy (A1') in place of $\eta_q,\ q = 1, \ldots, N$.

Theorem 1' Let $\Delta_T \asymp \epsilon_T$ in the SBS algorithm. Under (A1') and (A2)–(A4), there exists $C_1 > 0$ such that $\widehat{\eta}_q,\ q = 1, \ldots, \widehat{N}$ satisfy

$$\mathbb{P}\left\{\widehat{N} = N;\ |\widehat{\eta}_q - \eta_q| < C_1\epsilon_T \text{ for } q = 1, \ldots, N\right\} \to 1$$

as $T \to \infty$, where

- if $\delta_T \asymp T$, there exists some positive constant κ such that we have $\epsilon_T = \log^{2+\vartheta} T$ with $\pi_T = \kappa \log^{1+\omega} T$ for any positive constants ϑ and $\omega > \vartheta/2$.
- if $\delta_T \asymp T^{\Theta}$ for $\Theta \in (6/7, 1)$, we have $\epsilon_T = T^{\theta} \log^2 T$ for $\theta = 5 - 5\Theta$ with $\pi_T = \kappa T^{\gamma}$ for some $\kappa > 0$ and any $\gamma \in (5(1-\Theta)/2, \Theta - 1/2)$.

Theorem 2' Let $\Delta_T \asymp \epsilon_T$ in the SBS algorithm and $\Lambda_T \asymp \epsilon_T$ in the across-scales post-processing. Under (B1), (B2') and (B3)–(B5), there exists $C_2 > 0$ such that $\widehat{\nu}_r,\ r = 1, \ldots, \widehat{N}$ estimated with $I_T^* = -\lfloor \alpha \log\log T \rfloor$ for $\alpha \in (0, 2+\vartheta]$, satisfy

$$\mathbb{P}\left\{\widehat{N} = N;\ |\widehat{\nu}_r - \nu_r| < C_2\epsilon_T \text{ for } r = 1, \ldots, N\right\} \to 1$$

as $T \to \infty$, where

- if $\delta_T \asymp T$, there exists some positive constant κ such that we have $\epsilon_T = \log^{2+\vartheta} T$ with $\pi_T = \kappa \log^{1+\omega} T$ for any positive constants ϑ and $\omega > \vartheta/2$.
- if $\delta_T \asymp T^{\Theta}$ for $\Theta \in (6/7, 1)$, we have $\epsilon_T = T^{\theta} \log^2 T$ for $\theta = 5 - 5\Theta$ with $\pi_T = \kappa T^{\gamma}$ for some $\kappa > 0$ and any $\gamma \in (5(1-\Theta)/2, \Theta - 1/2)$.

References

[1] Cho, H. and Fryzlewicz, P. (2015), "Multiple change-point detection for high-dimensional time series via Sparsified Binary Segmentation," *Journal of the Royal Statistical Society, Series B (to appear).*

[2] Venkatraman, E. S. (1992), "Consistency results in multiple change-point problems," *Technical Report No. 24, Department of Statistics, Stanford University.*