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Motion of a drop driven by substrate vibrations

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Abstract. We report an experimental study of liquid drops moving against gravity, when placed on a vertically vibrating inclined plate, which is partially wet by the drop. Frequency of vibrations ranges from 30 to 200 Hz, and above a threshold in vibration acceleration, drops experience an upward motion. We attribute this surprising motion to the deformations of the drop, as a consequence of an up/down symmetry-breaking induced by the presence of the substrate. We relate the direction of motion to contact angle measurements.

1 Introduction

A drop of liquid on a substrate experiences retention forces due to the hysteresis in the values of the static contact angle. For instance, when the substrate is inclined, the drop deforms but does not yield until the contact angles at its front and at its back overcome respectively the advancing θ_a and the receding contact angle θ_r. The origin of hysteresis are imperfections in the surface which make the motion of the contact-line energetically costly [1]. When the hysteresis is small enough, the drop slides for moderate inclination angle and can exhibit transition to highly asymmetric shapes like corners and cusps, and even pearling at the trail of the drop [2,3]. When the hysteresis is large, the drop remains pinned on the substrate even for a vertical inclination. This pinning is problematic for microfluidics batches and one seeks for various strategies to overcome the effect of hysteresis: for instance a wettability gradient [4], an interplay between thermal effects and ratcheting [5] or asymmetric vibrations [6]. In the same fashion as a solid held by friction can be dislodged with substrate vibrations [7], one might expect that the sessile drop will slide downwards if the incline is shaken up and down. Here we report for the first time that a drop on a vertically-vibrated incline can in fact climb up the surface, providing that the acceleration field is large enough.

2 Setup and phenomenological description

The substrate sits on a shaker that prescribe the sinusoidal vibrations. The natural control parameter is the maximal value of the acceleration $a = (2\pi f)^2 A$, A and f being the amplitude and frequency. As a result, a sessile drop in partial wetting conditions, of typical size of the capillary length, exhibits a rocking motion along its equilibrium position. The frequency ranges around the frequencies of the first modes of resonance: from 30 to 200 Hz, for drops of volume $2\,\mu$l to $10\,\mu$l used here. α denotes the angle formed by the axis of vibration and the incline.

In order to achieve partial wetting conditions, the liquid/substrates combinations are: mixtures of water and glycerol on plexiglass and silicon oil on fluorad-coated glass (see Table 1).

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Table 1. Properties of the liquids.

Liquid	ν (cSt)	ρ (g/cm^3)	γ (mN/m)	θ_a (°)	θ_r (°)
Water + Glycerin	35	1.19	66	77 ± 2	44 ± 3
Water + Glycerin	55	1.21	66	77 ± 2	44 ± 3
Silicon Oil	56	0.95	20.5	57 ± 1	46 ± 1

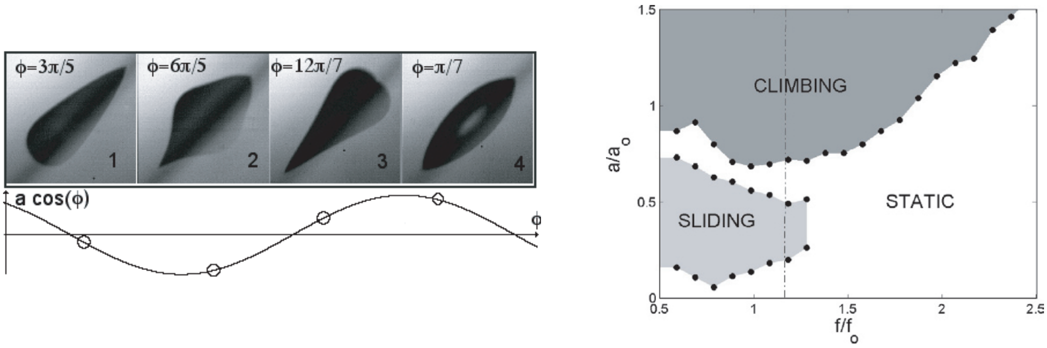

Fig. 1. Left: four snapshots of side views of a climbing drop (and its reflection) on a vibrating plate inclined at $\alpha = 45°$. Parameters are $V = 5\,\mu$l, $f = 60$ Hz, $\nu = 31\,\text{mm}^2/\text{s}$. The lower plot shows the phase of the acceleration. The phase origin ($\phi = 0$) is taken when the acceleration is maximal in the upward direction. Right: phase diagram of the drop's behavior for $V = 5\,\mu$l, $\alpha = 45°$, and $\nu = 31\,\text{mm}^2/\text{s}$.

The kinematic viscosity ν of the various liquids ranges between 31 and 55 mm^2/s. For lower viscosities the drop can break up before the onset of climbing; for higher viscosities, drops move slower and thus their dynamics is more tedious to access. Table 1 gives the properties of liquids as well as the advancing θ_a and receding θ_r contact angles. These angles are measured by inflating or deflating a drop with a syringe and by observing the yielding point of the contact line.

We extracted side-view pictures in the climbing situation, see Fig. 1-*Left*. The snapshots show qualitatively how the broken symmetries of the problem lead to peculiar deformations of the drop during one cycle. During the downward phase of the plate motion (panel 2 and 3 in Fig. 1-*Left*), the drop is more compliant to lateral forcing and hence the maximum contact angle achieved is greater than during the upward phase of the plate motion (panel 1 and 4 in Fig. 1-*Left*). The asymmetry of this rocking motion just makes this motion prevalent in one direction, i.e. the upward/right one at large a. Thus, we attribute the climbing motion to the presence of the plate that breaks the up/down and left/right symmetries around the drop. This interpretation is consistent with complementary observations of similar lateral motions of drops on a horizontal substrate that is diagonally shaken [8].

The situation investigated the most quantitatively is the large hysteresis case, i.e. water + glycerin drops on plexiglass. We obtained 'phase diagrams' that picture out the domains in frequency and acceleration for the different modes: *Static*, *Sliding* and *Climbing* (Fig. 1-*Right*). We used different viscosities, drop volume and angle of inclination. Except for very small drops (smaller than $5\,\mu$l), where the sliding zone does not exist, the diagrams remain qualitatively similar. The dimensionless frequency and acceleration in the axis of these diagrams are obtained by dividing f and a by f_0 and a_0, with f_0 is the resonance frequency of the rocking mode as determined in [9,10] and a_0 is the corresponding characteristic acceleration $a_0 = (2\pi f_0)^2 V^{1/3}$. For glycerol/water drops of $5\,\mu$l, these are equal to: $f_0 = 50.77$ Hz and $a_0 = 174\,\text{m/s}^2$. With these parameters, we obtained Fig. 2. Although the center of mass of the drop moves back and forth during a period (and subsequently lead to variations in the contact angles), the drop's contour does not vary much over one period. However the contour shape does evolve with the drop's global velocity, and becomes more and more asymmetric as the speed increases. Actually, the contours are comparable to those found by Podgorski et al. [2] for sliding drops, i.e. the rear of the drop can turn from round to cornered, and from cornered to pearling (see the two right pictures of Fig. 2).

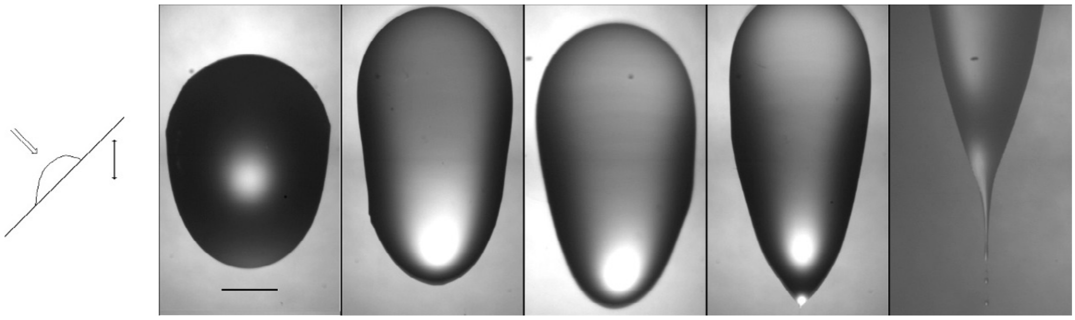

Fig. 2. Snapshots of a moving drop from above. The drop at the left slides, and the four other on the right climb. Velocity increases from left to right. From left to right: $a = 80.1\,\mathrm{m}^2/\mathrm{s}$, $a = 142.1\,\mathrm{m}^2/\mathrm{s}$, $a = 167.2\,\mathrm{m}^2/\mathrm{s}$, $a = 196.2\,\mathrm{m}^2/\mathrm{s}$, $a = 242.2\,\mathrm{m}^2/\mathrm{s}$. The horizontal bar on the left picture scales 1 mm.

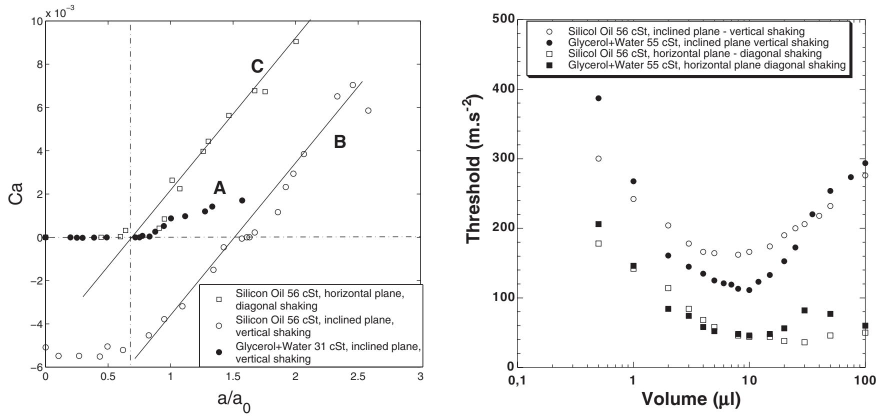

Fig. 3. Left: dimensionless drop velocity versus dimensionless acceleration. Right: threshold for climbing versus drop volume.

A study of local mechanisms for the drop's displacements is reported elsewhere [8]. Briefly, we found that the motion of contact lines compares well with the evolution of contact angles over one period [8]. Nonetheless, we find no local and instantaneous law relating the contact angle measurements to the contact line speed. Most notable, the two are not related by the Cox-Voinov law [11] found for steady motion [3]. This was indicated by the absence of pinning when the contact angle lies between the advancing and receding contact angle. The absence of pinning is consistent with the observations of [12] that contact angle hysteresis is obliterated by vibrations comparable to ours. The resulting unbalanced Young force, built on the average over one period of the contact angles at the upper (θ_u) and the lower (θ_l) ends of the drop, equals [8]: $F = \frac{1}{T}\int_0^T (\cos\theta_l - \cos\theta_u)dt$. The up/down and left/right asymmetries observed in the drop shapes (Fig. 1-*Left*) produce a net force F oriented upwards over one period.

3 Results

Velocity measurements are carried out with drops of volume $5\,\mu$l, and we used a frequency of $f = 60\,$Hz, a value around the resonance frequency of the rocking mode ($f_0 = 50.77\,$Hz for glycerin mixtures and $f_0 = 37.54\,$Hz for silicon oil). To investigate the influence of the contact angle hysteresis $\Delta\theta = \theta_a - \theta_r$ (see table 1), we conduct measurements for both glycerin mixtures and silicon oil. As the retention force due to capillary effects increases with $\Delta\theta$, a $5\,\mu$l drop of silicon oil slides down the fluorad-coated substrate in the absence of shaking, whereas a $5\,\mu$l drop of glycerin mixture remains pinned. The hysteresis also influences the width of the *Static* stripe between the *Sliding* and *Climbing* regions (see Fig. 1-*Right*): the displayed phase diagram corresponds to a situation of a large hysteresis, whereas this stripe barely exists for the low-hysteresis case. Experiments are conducted for both a vertically-shaken inclined substrate, and a diagonally shaken horizontal substrate. The later configuration allows to suppress the offset due to gravity, and therefore no sliding motion is observed.

Our first measurements consisted in measuring the threshold for climbing with liquids of different viscosities and similar surface tension (obtained by mixing glycerol and water at various shares): it turns out that the threshold does not depend on the liquid viscosity. However once moving, a drop of small viscosity moves faster than a more viscous one. Then, the capillary number $Ca = \rho\nu U/\sigma$ appears to be the relevant parameter for building a dimensionless velocity. The number Ca is plotted on Figure 3-*Left* versus dimensionless acceleration for different liquids and substrates. (A): Glycerol + water $\nu = 31$ cSt on an inclined plexiglass substrate. (B): Silicon oil $\nu = 56$ cSt on an inclined flurad-coated substrate. (C): Silicon oil $\nu = 56$ cSt on a diagonally shaken horizontal substrate. Particularly instructive is the comparison between situations (B) and (C): the two series of points have a roughly constant offset between them. Also, the velocity turns from constant to increasing with a/a_0 beyond the same value of a/a_0, around 0.65 (dash-dotted line). This suggests that gravity introduces an offset which is constant with acceleration, and that the capillary forces due to contact-angle differences are not influenced by the tilting of the plate. Also, it is noticeable that the transition to pearling for cases (B) and (C) occurs at about $Ca = 8 \times 10^{-3}$, then at around the same value found for sliding drops [3] for similar liquids and substrate. The situation (A) is more peculiar: there exists a sliding motion with very slow speed ($Ca \simeq 10^{-5}$) and the value of Ca for climbing motions remains smaller than 2×10^{-3} (above which the pearling occurs). This suggests a subtle influence of θ_r and θ_a in the capillary forces that provoke the displacement, which remains to be understood.

Figure 3-*Right* shows the evolution of the climbing threshold versus the drop's volume, for diagonal and vertical shaking and with two different liquids/substrates. As no obvious dimensionless number appeared relevant for the collapse of data, we kept them dimensionalized. Small drops are difficult to move and there exists an optimum for climbing between 6 and 10 μl. For larger drops, the threshold slightly increases if the substrate is horizontal, whereas this increase can become very significant for inclined substrates, when the drop has to overcome gravity.

4 Discussion, conclusion

Our experiments have shown that the motion of a drop on a substrate can be triggered by symmetrical vibrations which are prescribed with a certain angle from the substrate. This motion is due to the averaged Young capillary forces induced by the peculiar motion of the center of mass of the drop during its rocking motion. Particularly when the substrate is inclined, the vertical vibrations lead to a spectacular climbing motion. Although the shape transitions look similar to those observed in gravity-driven sliding drops [3], the local dynamics are drastically different: the substrate vibrations induce the ceaseless depinning of the contact-line [8] even when contact angles lie between θ_r and θ_a. This depinning occurs everywhere around the drop during sliding and climbing motions, but some regions remain stuck for static drops.

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