# Comment on "Force Balance at the Transition from Selective Withdrawal to Viscous Entrainment"

In a recent Letter [1], Blanchette and Zhang (BZ) proposed a theory for the critical flow rate at the entrainment transition in the selective withdrawal experiment. Their theory, which uses a hydrodynamic description, is based on the assumption of failure of the interface on a large scale, presumed insensitive to the nature of the entrained phase (such as its viscosity $\mu_0$). We show that BZ's theory is untenable for two reasons. First, it disagrees with earlier experiments done for a fluid-air system with very small viscosity ratio $\mu_0/\mu = 3 \times 10^{-7}$ [2]. No failure was observed (as confirmed recently [3]), and entrainment of air occurs only when the highly deformed tip of the interface enters the orifice; see Fig. 1 (left). Thus, there is no transition if the entrained phase is air, contradicting one of BZ's key assertions.

Second, BZ's theory also fails for $\mu_0/\mu$ being closer to unity, as is the case for typical two-fluid experiments. This we demonstrate by repeating BZ's numerical simulations using our own method, described in detail in Ref. [4]. We used an infinite domain, as appropriate for the very large tank used in the two-fluid experiment [5]. As the flow rate $Q$ is increased, we observe the interface undergoing a saddle-node bifurcation, in agreement with BZ. At the bifurcation, a single eigenvalue crosses the real axis, leading to a linear instability. In Fig. 1 (right), we show the spatial shape $\delta z(r)$ of the eigenmode that first turns linearly unstable at the transition. Even for $\mu_0/\mu = 1$, the unstable mode is localized relative to the capillary length $\ell_\sigma$, while for $\mu_0/\mu = 0.1$, the width of the peak becomes even narrower. This proves that the interface fails only in the highly localized region inside the peak of $\delta z$, while the interface does not move outside of it; no net force is acting on fluid elements outside of the central peak. Thus the failure mechanism is local, not global, and in addition depends strongly on $\mu_0/\mu$. This invalidates the basis of BZ's theoretical arguments.

Finally, we also believe the agreement between simulation and experiment reported by BZ to be an artifact of the unphysical boundary conditions imposed at the edge of a very small domain, chosen arbitrarily to be of size $\ell_\sigma$. This

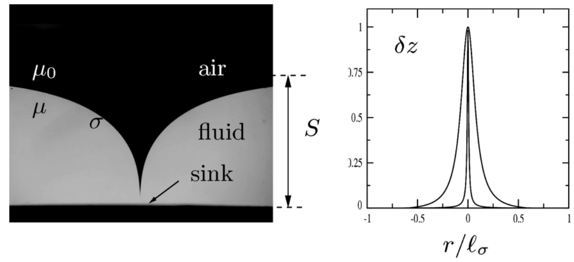


FIG. 1. Left: Image of fluid-air interface as the tip enters the tube [2], $\mu_0/\mu = 3 \times 10^{-7}$. Right: Failure mode $\delta z$ for $S/\ell_\sigma = 1.3$ and $\mu_0/\mu = 1$ as well as $\mu_0/\mu = 0.1$ (narrow peak).

amounts to an adjustable parameter and, thus, an arbitrary shift in the $x$ direction. BZ also introduce a second adjustable parameter by allowing for a shift (assumed to be the tube diameter) between the experimental tube position and the point sink of their numerical model. No argument is advanced for this particular choice. In conclusion, neither BZ's theoretical arguments nor their numerical simulations explain the experimental data.

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[1] F. Blanchette and W. W. Zhang, Phys. Rev. Lett. **102**, 144501 (2009).
[2] S. Courrech du Pont and J. Eggers, Phys. Rev. Lett. **96**, 034501 (2006).
[3] D. Zhou and J. J. Feng, J. Non-Newtonian Fluid Mech. **165**, 829 (2010).
[4] J. Eggers and S. Courrech du Pont, Phys. Rev. E **79**, 066311 (2009).
[5] I. Cohen, Phys. Rev. E **70**, 026302 (2004).

 