FLUID DYNAMICS

Coupling the large and the small

Solid objects generally produce a splash upon entering water. Surprisingly, a small change in the surface chemistry of an object can turn a big splash into an inconspicuous disappearance and vice versa.

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Throw an object into a pond: the expected result is a big splash — a loud noise accompanied by a substantial column of water rising from the impact site. But then sometimes, a similar toss only produces a harmless 'plop', after which the projectile sinks almost without a trace (Fig 1). What distinguishes these two cases? In this issue, Cyril Duez and co-workers[1] show that the impact velocity has to be above a critical value to produce a splash, which is perhaps not surprising. The truly remarkable part of their discovery is that the value of the critical velocity is controlled by the molecular interactions between the fluid and the solid. Thus by changing physical properties on a nanometre scale, it is possible to profoundly change the behaviour on a much larger scale, up to centimetres and beyond.

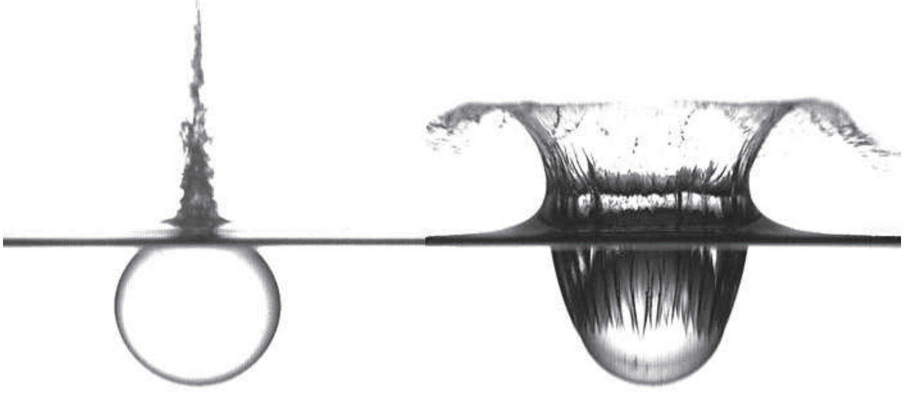


Figure 1 From no splash to a large splash. As the two spheres strike the water at the same velocity, the only difference between the two cases is the surface treatment. (Image courtesy of the authors of ref. 1.)

The problem addressed by Duez *et al.* has a considerable pedigree — A. M. Worthington[2] made it one of the centrepieces of his observations in his ground-breaking book, *A Study of Splashes*. Worthington began as a science master at Clifton College, Bristol[3], and ended his career, fittingly given his interest in watery impacts, at the Royal Naval College, Greenwich. He discovered that a carefully polished sphere can sink without a trace, whereas the same sphere under the same conditions, but roughened by the application of dust, produces a splash. This is beautifully illustrated by Fig. 2, taken from the final chapter of his book, which shows a sphere whose surface has been roughened on one side only. As a result, a symmetric impact results in a completely asymmetric splashing event.

These qualitative observations, obtained in the best British naturalist's tradition without the use of any mathematics or detailed physical models, have now been beautifully explained by Duez and co-workers within a simple and fully quantitative model[1]. They find that the flow, as viewed macroscopically, consists of a film of liquid racing along the upper surface of the sphere to close the void, and thus to produce a 'plop'. Indeed, significant sound production depends on air bubbles being trapped inside the fluid: the compressibility of air allows bubbles to 'breath' (something an incompressible fluid can't do), and thus to generate sound.

The crucial observation is that the fluid flow near the tip of this film, the so-called contact line where the solid, liquid and surrounding gas meet, must be compatible with such a rapidly advancing motion. Namely, if contact lines are forced to move too fast, they become unstable so that the film detaches from the sphere, leaving a void. Using this simple idea the authors determine the critical splash velocity, which for a given liquid is determined by wetting — the tendency of the liquid to spread on a material. Specifically, the critical speed is determined by the angle at which an isolated drop of liquid contacts the solid. In the extreme case of a contact angle close to 180 degrees, which is produced by a roughening of the solid[4], there is always a splash, regardless of how small the velocity of entry.

The coupling of vastly disparate length scales, which here proves crucial to splashing, might be thought a rare exception. But in fact more and more examples are being found, illustrating the important role of such coupling in our understanding of the natural world and presenting us with huge challenges for mathematical analysis and computer modelling. One of the oldest and most important of such problems is turbulence, which can extend across global scales. Yet no description of turbulence is complete without a mechanism for the ultimate demise of the energy contained in the turbulent motion, which occurs on length scales below a millimetre in the atmosphere[5]. Thus the smallest and the largest scales are coupled in turbulence, making it so hard a problem that most of its critical features remain unexplained.

Another more tractable example is the 'egg-beater' problem: the entrainment of air bubbles into a fluid by vigorous stirring. If the liquid surface were to remain smooth, no air could ever enter it. Instead, the interface locally deforms into sharp folds, whose size decreases rapidly with the strength of the stirring[6]. These singularities of the surface eventually become unstable, providing the channels through which air can enter[7]. Unless all scales down to that of a micrometre are properly modelled, there is no hope of describing air entrainment even qualitatively.

Two ideas have proved their worth in the quest for a deeper mathematical understanding of these multiscaled phenomena. First, the observation that there is a rather generic tendency of hydrodynamics to generate small-scale motion naturally couples small and large scales. Mathematically, this tendency is best captured in the limit that the formation of small scales continues with no end, so the underlying equations form a singularity[8,9]. The description of singularities is aided by the fact that they are self-similar, meaning that their structure remains the same under a change of length scale.

Second, it still remains to be established how the largest and the smallest features (the impacting sphere and the molecular interactions in the splashing problem) are to be joined together. Problems of this nature are captured by the mathematical technique of 'matched asymptotic expansions'[10,11], which describes the rules by which two functions characterizing two different length scales can be made compatible. In general this will be impossible, unless parameters of both parts of the solution are 'just right'. In other words, though separated by many orders of magnitude in scale, the microscopic and the macroscopic part of the solution are intimately related. The reality of more and more of today's problems requires us to combine the very large and the very small. Fortunately, some powerful mathematical tools open avenues for significant progress.

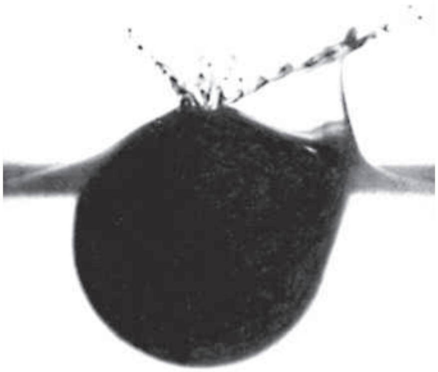


Figure 2 Wonky splash. A sphere of polished serpentine (2.57 cm in diameter), dusted on one side, falling into water from a height of 14 cm. Despite the perfect symmetry of the impact, the splash is plainly asymmetric. Image reproduced from ref. 2.

References

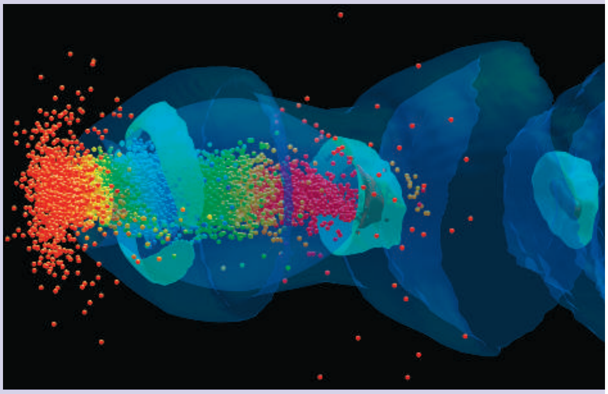
1. Duez, C. *et al. Nature Phys.* **3,** 180–183 (2007).
2. Worthington, A. M. *A Study of Splashes* (Longmans, London, 1908).
3. Pippard, B. *Notes Rec. R. Soc. London* **56,** 63–81 (2002).
4. Quéré, D. *Rep. Prog. Phys.* **68,** 2495–2532 (2005).
5. Davidson, P. *Turbulence* (Oxford Univ. Press, Oxford, 2004).
6. Jeong, J.-T. & Moffatt, H. K. *J. Fluid Mech.* **241,** 1–22 (1992).
7. Lorenceau, È., Quéré, D. & Eggers, J. *Phys. Rev. Lett.* **93,** 254501 (2004).
8. Kadanoff, L. P. *Phys. Today* **50,** 11–13 (1997).
9. Berry, M. V. *Phys. Today* **55,** 10–11 (2002).
10. Berry, M. V. *Philos. Sci.* **9,** 597–607 (1994).
11. Bender, C. & Orszag, S. *Advanced Mathematical Methods for Scientists and Engineers* (McGraw–Hill, New York, 1978).

WAKEFIELD ACCELERATORS

Hybrid particle drive

The electric field within any given section of a conventional particle accelerator is limited by the breakdown field of the materials from which it is made. Consequently, getting to the relativistic energies where interesting particle physics takes place requires multiple accelerator stages spanning many kilometres. The extreme fields generated when a high-power laser is focused into a plasma have been suggested as a way of accelerating particles in a much smaller space, and potentially with less expense. On their own, accelerators that operate on this principle — known as plasma wakefield accelerators — can generate electron beams with energies exceeding 1 GeV, in a plasma just a few centimetres long. This is, of course, far below the energy demanded by particle physicists. But by using wakefield accelerator techniques in tandem with the conventional linear accelerator at the Stanford Linear Accelerator Centre, Ian Blumenfield and colleagues show they can use them to double the energy of electrons from a 42 GeV electron beam over a distance of less than a metre (*Nature* **445,** 741–744; 2007).

Plasma wakefield accelerators typically operate by focusing a high-power laser pulse on a tight spot in some medium, which can be a gas, liquid or solid. This creates a fully ionized plasma and drives freed electrons through the plasma at close to the speed of light. It is the wake left behind by this burst of ultra-relativistic electrons that generates the large fields used for particle acceleration. But a high-power laser is not the only means to drive electrons through a plasma to produce such a wake, a fact that Blumenfield *et al.* clearly demonstrate by focusing their conventionally accelerated electron beam into an 85-cm-long column of lithium vapour. When the beam interacts with the vapour it transfers most of its energy to the resulting plasma, and the wake it produces accelerates a proportion of electrons in the tail of the beam to twice their original energy (see 'particle in cell' simulation pictured). Simulations of this interaction confirm an implied accelerating field of 53 GV m^{-1} — a thousand times greater than can be achieved in a typical linear accelerator stage.



Particle-in-cell simulation of the wake generated in a lithium plasma by a 42 GeV electron beam.

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