ANALYSIS 1A: FIRST ASSESSED HOMEWORK

Deadline: To be handed in by 2 pm on Friday October 27th

Hand in: You should hand in your work along with a completed cover sheet in to the marked cabinet on the ground floor of the School of mathematics. Your work should be stapled together with the cover page at the front. Cover sheets are available from the ground floor of the mathematics building, the unit website and blackboard. Solutions will be released online by the 10th of November.

Assessment: This homework will count for 5% of your total mark for Analysis 1A.

Collaboration: The work you hand in should be your own work. You are welcome to discuss the problems other students but the solutions you hand in should be written solely by you.

Questions

1. **(3+3+3 marks)**
   (a) Find all $x \in \mathbb{R}$ so that $|x - 7| \leq 2$.
   (b) Find all $x \in \mathbb{R}$ so that $|x - 3| \geq 5$.
   (c) Find all $x \in \mathbb{R}$ with $x \neq -2$ so that $\left|\frac{x-1}{x+2}\right| < 1$.

2. **(4+4 marks)**
   (a) Let
   $$A = \left\{ \frac{n}{n+2} \ : \ n \in \mathbb{N} \right\}.$$
   Find $\inf A$, and justify your answer.
   (b) Let
   $$B = \left\{ \frac{n^2}{n+1} \ : \ n \in \mathbb{Z}, \ n \neq -1 \right\}.$$
   Show that $\sup B = \infty$.

3. **(2+2+4 marks)**
   (Here you complete part of the proof of Proposition 2.8. You are only to use properties (A1)-(A11), (O1)-(O4), the Completeness Axiom, and the Archimedean Principle. **In particular, given $x \in \mathbb{R}$ with $x > 0$, you are not to assume that $\sqrt{x}$ exists,** as this is what we are trying to prove in Proposition 2.8.)

   Suppose that $x, y \in \mathbb{R}$ with $x, y > 0$ and $y^2 > x$.

(a) Suppose that $\varepsilon \in \mathbb{R}$ with $0 < \varepsilon < y$ and $x + 2y\varepsilon < y^2$. Use this inequality to find $\alpha \in \mathbb{R}$ so that $\varepsilon < \alpha$.

(b) With α as in (a), find $\varepsilon \in \mathbb{R}$ with $0 < \varepsilon < y$ and $\varepsilon < \alpha$. Clearly explain your reasoning, using complete sentences.

(c) With ε as in (b), clearly explain why $x + 2y\varepsilon < y^2$, and from this deduce that $x < (y - \varepsilon)^2$.