PUB QUIZ REVIEW

1. **DEFINTIONS ROUND (10 points, 1 point for each correct answer)**
   (i) Besides Eisenstein's Criterion, Gauss' Lemma, field extension, field homomorphism/isomorphism/automorphism, list 5 words or phrases from Algebra 2 that we use in Galois Theory.
   (ii). Besides Galois group, Galois extension, K-homomorphism, multiple/repeated root, list 5 new words or phrases we use in Galois Theory.

2. **TRUE/FALSE ROUND (15 points, 1 point for each correct answer)**:
   (i) There are field homomorpisms that are not injective.
   (ii) If $L : K$ is a field extension then an algebraic closure $\overline{L}$ of L is also an algebraic closure of K.
   (iii) Suppose that $\gamma \in \mathbb{C}$ is transcendental over $\mathbb{Q}$. Then there exist infinitely many fields M with $\mathbb{Q} \subseteq M \subseteq \mathbb{Q}(\gamma)$.
   (iv) There is a normal extension $L : K$ with some $\alpha \in L$ so that $m_\alpha(K)$ does not split over L.
   (v) Suppose that $L : K$ is a field extension with $K \subseteq L$, $\alpha \in L$, and $E_\alpha : K[t] \to L$ the evaluation map. Then α is transcendental over K if and only if E_α is injective.
   (vi) Suppose that K, L_1, L_2 are fields with $K \subseteq L_1$, $K \subseteq L_2$, and $[L_1 : K] = [L_2 : K] < \infty$. Then $L_1 \simeq L_2$.
   (vii) With $\mathbb{F}_3$ a field with 3 elements, $\mathbb{F}_3[t]/(t^2 + t + 1)$ is a field with 9 elements.
   (viii) The polynomial $t^5 + 3$ is separable over $\mathbb{F}_{25}$.
   (ix) $\mathbb{C}$ is an algebraic closure of $\mathbb{Q}$.
   (x) Suppose that $L : K$ is a splitting field extension for $f \in K[t] \setminus K$. If $\alpha \in L$ then $m_\alpha(K)$ splits in $L[t]$.
   (xi) With K a field and $f \in K[t] \setminus K$, if f has one multiple root in $\overline{K}$ then all the roots of f are multiple roots.
   (xii) Suppose $L : M : K$ is a tower of fields with $K \subseteq M \subseteq L$. We have that $L : K$ is an algebraic extension if and only if $L : M$ and $M : K$ are algebraic extensions.
   (xiii) Suppose $L : M : K$ is a tower of fields with $K \subseteq M \subseteq L$. We have that $L : K$ is a normal extension if and only if $L : M$ and $M : K$ are normal extensions.
   (xiv) Suppose $L : M : K$ is a tower of fields with $K \subseteq M \subseteq L$. We have that $L : K$ is a Galois extension if and only if $L : M$ and $M : K$ are Galois extensions.
   (xv) Suppose $L : K$ is an algebraic extension, $\alpha \in L$, and $\tau : L \to L$ is a homomorphism. Then $\tau(\alpha)$ is a root of $m_\alpha(K)$.

3. **SHORT ANSWER ROUND (5 points, 1 point for each correct answer)**. Throughout, K is a field with $K \subseteq \overline{K}$.
   (i) Suppose that $\alpha \in \overline{K}$ with α algebraic over K. How does $[K(\alpha) : K]$ relate to data attached to α?
   (ii) What is the difference between a splitting field extension and a normal extension?
   (iii) Suppose that $f \in K[t]$ is irreducible over K and $\deg f = 4$. Let $L : K$ be a splitting field extension for f. Can $Gal(L : K)$ be cyclic of order 8?
   (iv) Suppose that $L : K$ is a field extension. What condition(s) ensures that $L : K$ is a simple extension?
   (v) Suppose $\sigma : K \to \overline{K}$ is a field homomorphism, and $L : K$ is a field extension. What condition(s) ensures that we can extend σ to a homomorphism $\tau : L \to \overline{K}$? You may assume that $K \subseteq L$.
4. **SHORTISH ANSWER ROUND (10 points, 2 points for each correct answer)**. Throughout, $L : K$ is a finite field extension with $K \subseteq L$, and let $G = Gal(L : K)$.
   (i) How does $|G|$ always compare to $[L : K]$?
   (ii) Set $K_0 = Fix_L(G)$. How does K compare to K_0?
   (iii) Without using the terms 'normal', 'splitting field', or 'separable', give **two** conditions that are equivalent to $L : K$ being a Galois extension.
   (iv) Suppose that $L : K$ is a Galois extension and H is a subgroup of G. Define $Fix_L(H)$.
   (v) Again suppose that $L : K$ is a Galois extension, H is a subgroup of G, and $M = Fix_L(H)$. How does $[M : K]$ compare to H?