GALOIS THEORY 2019: HW 1 Feedback
For assessment: Problems 1, 2, 3
Due by noon Tuesday, week 3 of the term

Here your are to remember that with $K \subseteq L$ fields and $\alpha \in L$, α is algebraic over K if and only if $[K(\alpha) : K] < \infty$. Please see the posted solutions to the problem set. (These solutions include solutions to the non-assessed problems, and they identify which problems were previous exam problems.)

The symbol $\implies$ means "implies". So $A \implies B$ means that A implies B, or equivalently, that **if** A is true **then** B is true, but it does **not** mean that B is true. The word "implies" is not interchangeable with "so" or "thus" or "hence".

1. (a) Suppose you are wanting to use induction to prove a statement $P(n)$ holds for all $n \in \mathbb{Z}_+$. *Should you write: "Suppose $P(n)$ is true for $n = k \in \mathbb{Z}_+$"?* **No!** This statement only requires that n is in $\mathbb{Z}_+$, so this statement is **supposing** that $P(n)$ (or $P(k)$) holds for any $n \in \mathbb{Z}_+$, which is what you want to **prove**. Your induction hypothesis needs to **quantify** $n \in \mathbb{Z}_+$ (or $k \in \mathbb{Z}_+$). After proving the base case, being that $P(1)$ is true, then you begin the induction step by supposing that $P(n)$ (or $P(k)$) holds for **some** [fixed] $n \in \mathbb{Z}_+$ (or for some $k \in \mathbb{Z}_+$). This assumption is logically sound as you have already shown that $P(n)$ holds for $n = 1$ (or that $P(k)$ holds for $k = 1$).
   (b) In (b)(ii), you need to recognise that $f(\alpha^p) = (f(\alpha))^p$ so that you can then use the assumption $f(\alpha) = 0$ to get $f(\alpha^p) = 0$.

2. You should explain why $[K(\alpha) : K] < \infty$ implies $[K(\alpha, \beta) : K(\beta)] < \infty$.

3. Quite a few students did not explain why γ being a root of g implies that $[K(\gamma) : K(h(\gamma)] \leq 3$.